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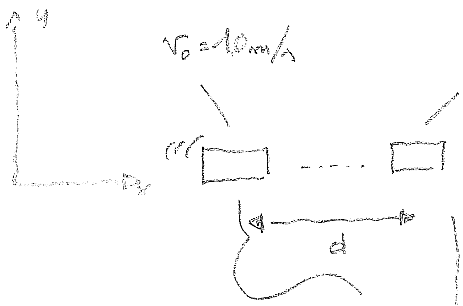
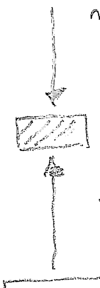


ETAPE 1



$$m\vec{g} = \begin{bmatrix} 0 \\ -mg \end{bmatrix}$$

$$\vec{f} = \begin{bmatrix} -\mu m g \\ 0 \end{bmatrix}$$

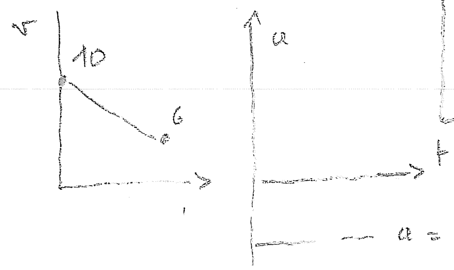
$$\vec{N} = \begin{bmatrix} 0 \\ mg \end{bmatrix}$$

DYNAMIQUE

$$\boxed{\sum \vec{F} = m\vec{a}}$$

$$-\mu g = a$$

$$\vec{a} = \begin{bmatrix} -\mu g \\ 0 \end{bmatrix} = \begin{bmatrix} -a \\ 0 \end{bmatrix}$$



ON DECCELERE ! L'ACCELERATION EST NEGATIVE

ETAPE 2

CINEMATIQUE

$$x(0) = 0$$
$$v(0) = v_0$$

$$x(t) = d$$
$$v(t) = v_f$$

$$x(t) = v_0 t - a t^2/2 \quad (1)$$
$$v(t) = v_0 - a t \quad (2)$$

$$\boxed{t = \frac{v_0 - v_f}{a}}$$

EN INJECTANT DANS (1)

$$d = v_0 \left(\frac{v_0 - v_f}{a}\right) - \frac{a}{2}\left(\frac{v_0 - v_f}{a}\right)^2$$

$$= \frac{1}{2a}\left[2 v_0^2 - 2 \cancel{v_0 v_f} - v_0^2 + v_f^2 + 2 \cancel{v_0 v_f}\right]$$

$$\boxed{a = \frac{v_0^2 - v_f^2}{2d}}$$

VALEURS NUMERIQUES

$$a = \frac{100 - 64}{2 \times 12} = \frac{36}{24} = 1{,}5 \text{ m/s}^2$$

$$f = 0{,}09 \times 1{,}5 = 0{,}135 \text{ N}$$

$$\mu = \frac{0{,}135}{0{,}09} = 0{,}15$$ ASSEZ COHERENT

$$\vec{a} = \begin{bmatrix} -1{,}5 \\ 0 \end{bmatrix}$$

DECCELERATION

CELA NE SERT A RIEN D'ETUDIER CETTE RELATION !
IL VAUT MIEUX S'ENTRAINER
A LA DEDUIRE CHAQUE FOIS (si, si !)

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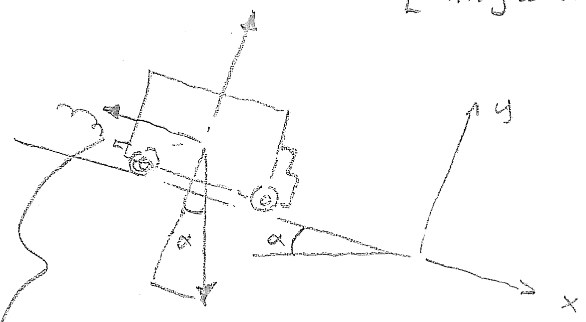
$$\vec{N} = \begin{bmatrix} 0 \\ mg\cos\alpha \end{bmatrix}$$



$$m\vec{g} = \begin{bmatrix} mg\sin\alpha \\ -mg\cos\alpha \end{bmatrix}$$

$$\vec{f} = \begin{bmatrix} -f \\ 0 \end{bmatrix}$$

EN DESCENTE

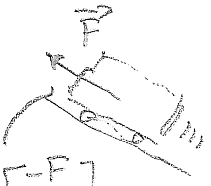
$$\underbrace{ma}_{=0} = -f + mg\sin\alpha$$

$$\boxed{f = mg\sin\alpha}$$

EN MONTEE

$$\underbrace{ma}_{=0} = \underbrace{-F}_{} + \underbrace{f}_{= mg\sin\alpha\ :-)} + mg\sin\alpha$$

LA FORCE EST NEGATIVE POUR QUE LE CAMION REMONTE

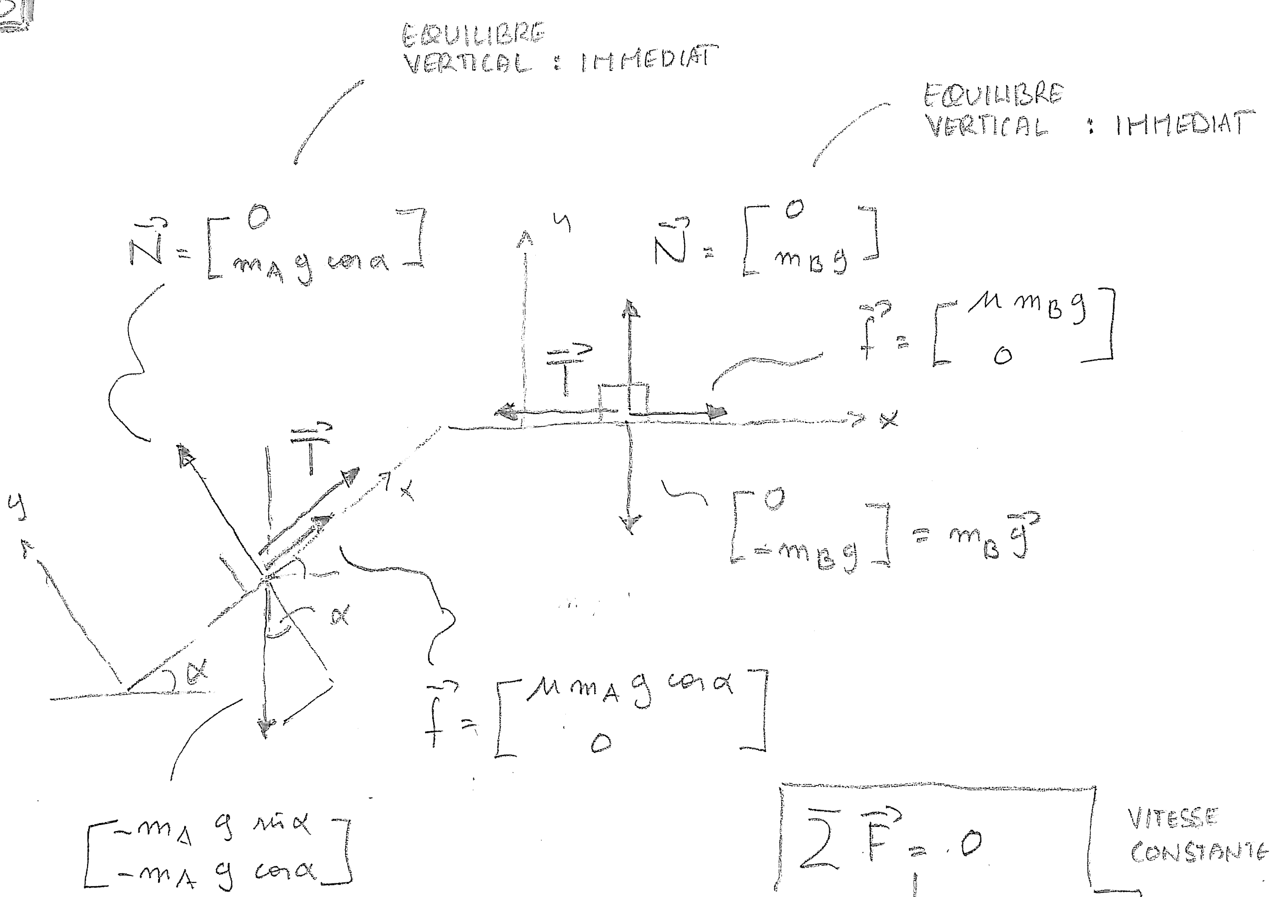


$$\vec{F} = \begin{bmatrix} -F \\ 0 \end{bmatrix} \ :-)$$

$$\boxed{F = 2mg\sin\alpha}$$

VALEUR NUMERIQUE

$$F = \underbrace{2 * 3000 * 9{,}81 * \sin(5^\circ)}_{5130\ [N]}$$



$$\boxed{\sum \vec{F} = 0}$$ VITESSE CONSTANTE

CORPS B

$$\boxed{-T + \mu m_B g = 0}$$

$$\boxed{\sum \vec{F} = 0}$$ VITESSE CONSTANTE

CORPS A

$$\boxed{T - m_A g \sin\alpha + \mu m_A g \cos\alpha = 0}$$

$$\mu \cancel{g} (m_B + m_A \cos\alpha) = m_A \cancel{g} \sin\alpha$$

$$\boxed{\mu = \frac{m_A \sin\alpha}{(m_B + m_A \cos\alpha)}}$$

$$\boxed{T = \mu m_B g}$$

VALEURS NUMERIQUES

$\sin(30°) = 1/2$
$\cos(30°) = \sqrt{3}/2$ } A CONNAITRE

$$\mu = \frac{1/2}{(2/5 + \sqrt{3}/2)} = \frac{1}{(4/5 + \sqrt{3})} = 0{,}395$$

$$T = 0{,}395 * 9{,}81 * 2 = 7{,}75 \ [N]$$

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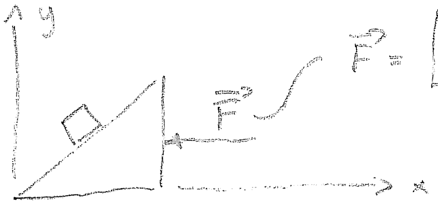
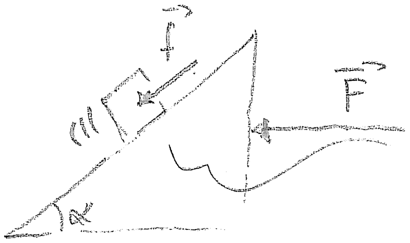
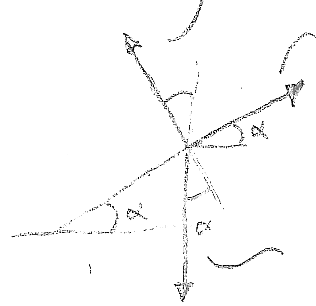


$$\vec{F} = \begin{bmatrix} -F \\ 0 \end{bmatrix}$$

$$\boxed{a = \frac{F}{(M+m)}} \qquad \vec{a} = \begin{bmatrix} -a \\ 0 \end{bmatrix}$$

$\vec{F}$ EST MAXIMAL
LE BLOC DU DESSUS VOUDRAIT MONTER !



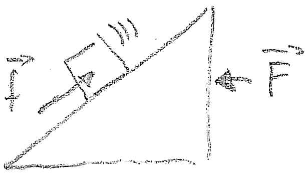


$$\vec{N} = \begin{bmatrix} -N \sin\alpha \\ N \cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} \pm \mu_S N \cos\alpha \\ \pm \mu_S N \sin\alpha \end{bmatrix} = \vec{f}$$

INCONNU!

$$\begin{bmatrix} 0 \\ -mg \end{bmatrix}$$



$\vec{F}$ MINIMAL
LE BLOC VOUDRAIT DESCENDRE

$$\boxed{\sum \vec{F} = m\vec{a}}$$

BLOC DU DESSUS !

$$\begin{cases} -m a = \mu_S N \cos\alpha - N \sin\alpha \\ 0 = -mg + \mu_S N \sin\alpha + N \cos\alpha \end{cases}$$

$$\boxed{N = \frac{mg}{(\cos\alpha + \mu_S \sin\alpha)}}$$

EN INJECTANT

$$\boxed{a = \frac{F}{(M+m)}}$$

ET

DANS L'EQUATION HORIZONTALE ...

$$-\cancel{m} \frac{F}{(M+m)} = \cancel{m} g \frac{(\mu_S \cos\alpha - \sin\alpha)}{(\cos\alpha + \mu_S \sin\alpha)}$$

$$F = (M+m) \frac{(\sin\alpha - \mu_S \cos\alpha)}{(\cos\alpha + \mu_S \sin\alpha)} g = 3{,}9 \text{ [N]}$$

$$F = (M+m) \frac{(\sin\alpha + \mu_S \cos\alpha)}{(\cos\alpha - \mu_S \sin\alpha)} g = 71{,}0 \text{ [N]}$$

CONCLUSION

$$3{,}9 \leq F \leq 71{,}0 \text{ [N]}$$

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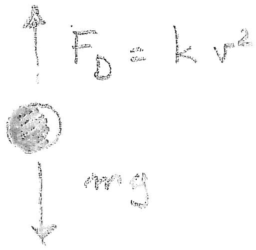

$$\underbrace{m a}_{=0} = mg - k v_L^2 \quad\Rightarrow\quad v_L = \sqrt{\frac{mg}{k}}$$

LORSQUE $v = v_L/2$

$$F = k\left(\frac{v_L}{2}\right)^2 = k\,\frac{mg}{4k} = \frac{\overset{20}{m}\,\overset{9,8}{g}}{4} = 49\ [N]$$

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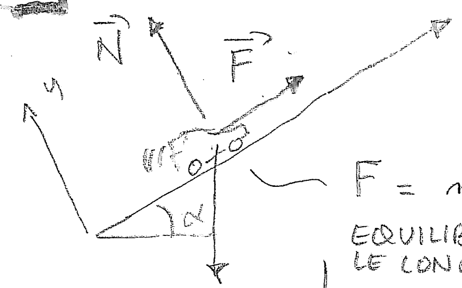
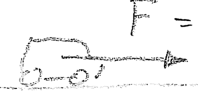


$F = mg \sin\alpha$

EQUILIBRE LE LONG DE X

OBSERVONS QU'ON PEUT OUBLIER LE FROTTEMENT CAR IL EST IDENTIQUE DANS LES DEUX CAS (*) SI ! SI !



$F = mg \sin\alpha$

ACCELERATION SUR ROUTE HORIZONTALE

$$a = \underset{9{,}81}{g} \sin \underset{\alpha = 10° \text{ PAR EXEMPLE}}{\alpha} = 1{,}7\ [m/s^2]$$

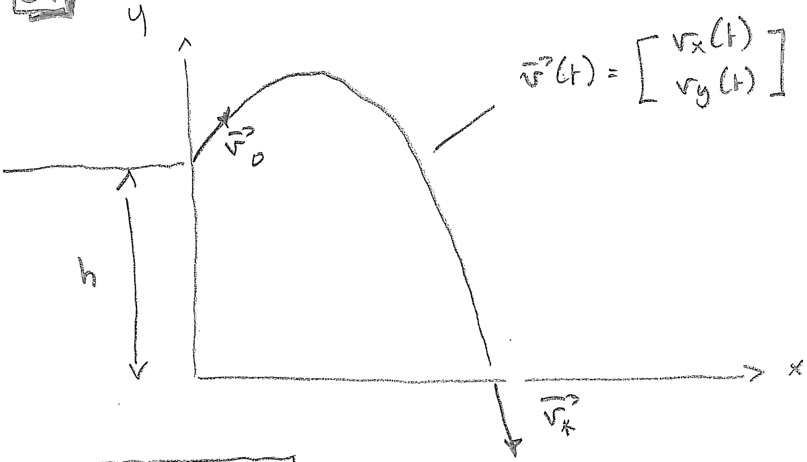
(*) DANS LA MONTEE

$$\underbrace{F}_{\text{SUR ROUTE HORIZONTALE}} = mg \sin\alpha + f$$

$$ma = F - f = mg \sin\alpha + \cancel{f} - \cancel{f}$$

$$a = g \sin\alpha$$

AVEC LE FROTTEMENT !



QUESTION

A QUELLE VITESSE v_* LA BALLE TOUCHE LE SOL ?

CINEMATIQUE

MRUA :-)

$$\frac{d}{dt}(m\vec{v}) = \sum \vec{F}$$

$$\frac{d}{dt}\begin{bmatrix} \cancel{m}\, v_x(t) \\ \cancel{m}\, v_y(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -\cancel{m}g \end{bmatrix}$$

VITESSE INITIALE $\vec{v}(0) = \begin{bmatrix} v_{x0} \\ v_{y0} \end{bmatrix}$

$$\begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix} = \begin{bmatrix} v_{x0} \\ v_{y0} - gt \end{bmatrix}$$

POSITION INITIALE $\vec{x}(0) = \begin{bmatrix} 0 \\ h \end{bmatrix}$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} v_{x0}t \\ h + v_{y0}t - gt^2/2 \end{bmatrix}$$

QUAND EST-CE QUE LA BALLE TOUCHE LE SOL ?

$$\underbrace{y(t^*)}_{=0} = h + v_{y0}t^* - g\frac{t^{*2}}{2}$$

ET

$$\underbrace{v_y(t^*)}_{v_{y*}} = v_{y0} - gt_*$$

$$t_* = \frac{v_{y0} - v_{y*}}{g}$$

$$0 = h + v_{y0}\underbrace{\left(\frac{v_{y0} - v_{y*}}{g}\right)}_{t^*} - \frac{g}{2}\underbrace{\left(\frac{v_{y0} - v_{y*}}{g}\right)^2}_{(t^*)^2}$$

$$0 = h + \frac{1}{g} v_{y0}(v_{y0} - v_{y*}) - \frac{g}{2}\frac{1}{g^2}(v_{y0} - v_{y*})^2$$

EN MULTIPLIANT TOUT PAR $2g$!

$$0 = 2hg + 2v_{y0}^2 - \cancel{2v_{y0}v_{y*}} - v_{y0}^2 - v_{y*}^2 + \cancel{2v_{y0}v_{yx}}$$

$$v_{y*}^2 = v_{y0}^2 + 2hg$$

ON CONCLUT FINALEMENT

$$\boxed{v_* = \sqrt{\underbrace{v_{x*}^2}_{\substack{v_{x0}^2 \\ \text{MRU :-)}}} + \underbrace{v_{y*}^2}_{v_{y0}^2 + 2hg}} \qquad v_* = \sqrt{v_0^2 + 2gh}}$$

BILAN ENERGIE

CONFIGURATION INITIALE

EN $t = 0$

ENERGIE CINETIQUE $= \frac{1}{2} m v_0^2$

CONFIGURATION FINALE

EN $t = t^*$

ENERGIE CINETIQUE $= \frac{1}{2} m v_*^2$

IL Y A UNE UNIQUE FORCE CONSTANTE $\vec{F} = m\vec{g}$!

$$\boxed{\Delta\left[\frac{1}{2} m v^2\right] = \vec{F} \cdot \Delta\vec{x}}$$

$$\begin{bmatrix} 0 \\ -mg \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -h \end{bmatrix}$$

L'ALGEBRE EST BEAUCOUP PLUS SIMPLE !

$$\frac{1}{2}\cancel{m} v_*^2 - \frac{1}{2}\cancel{m} v_0^2 = \cancel{m} g h$$

$$\boxed{v_*^2 = v_0^2 + 2gh} \quad \text{:-)}$$

BILAN D'ENERGIE

CONFIGURATION INITIALE

EN $t=0$

ENERGIE CINETIQUE $= \frac{1}{2} m v^2$

CONFIGURATION FINALE

ENERGIE CINETIQUE $= 0$

$\Delta \vec{x} = \begin{bmatrix} 0 \\ d \end{bmatrix}$

3 FORCES CONSTANTES

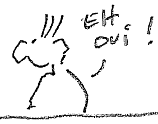
$m\vec{g}$ $\vec{f}$ $\vec{N}$

IL N'Y A DU TRAVAIL QUE POUR LE FROTTEMENT !

$W = -d \mu m g$

$\underbrace{\begin{bmatrix} -\mu m g \\ 0 \end{bmatrix}}_{\text{FORCE}} \cdot \underbrace{\begin{bmatrix} d \\ 0 \end{bmatrix}}_{\text{DEPLACEMENT}}$

LE TRAVAIL A UN SIGNE !



LE TRAVAIL DE LA FORCE DE GRAVITE EST NUL !

$\underbrace{\begin{bmatrix} 0 \\ -mg \end{bmatrix} \cdot \begin{bmatrix} d \\ 0 \end{bmatrix}}_{=0\,!}$

IDEM POUR LA REACTION !

$$\boxed{\Delta\left(\frac{1}{2} m v^2\right) = \sum \vec{F} \cdot \Delta \vec{x}}$$

$\Delta\left(\frac{1}{2} m v^2\right)$: $0 - \frac{1}{2} m v^2$

$\sum \vec{F} \cdot \Delta \vec{x}$: $-d \mu m g$

$$\frac{1}{2} \cancel{m} v^2 = d \mu \cancel{m} g$$

$$\boxed{d = \frac{v^2}{2 \mu g}}$$

$[m^2/s^2]$

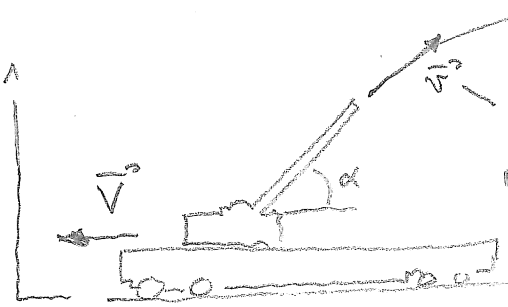
$[m/s^2]$

CHECK DIMENSIONS !

VALEURS NUMERIQUES

$d = \frac{4}{\underbrace{2 \times 0{,}6 \times 9{,}81}}$

1,36 m



VITESSE RELATIVE DE L'OBUS PAR RAPPORT AU CANON

LA VITESSE ABSOLUE RÉELLE EST DONC $\vec{v} + \vec{V}$!! POUR UN REPÈRE INERTIEL

CONFIGURATION AVANT

$$\boxed{\text{QUANTITÉ DE MOUVEMENT HORIZONTALE} = 0}$$

CONFIGURATION APRÈS LE TIR

$$\boxed{\text{QUANTITÉ DE MOUVEMENT HORIZONTALE} = m(v\cos\alpha - V) - MV}$$

PAS DE FORCES HORIZONTALES

ON SUPPOSE QUE LE WAGON ROULE SANS AUCUN FROTTEMENT SUR LES RAILS

$$\boxed{\Delta(m v_x) = 0 \quad !}$$

$$m(v\cos\alpha - V) - MV = 0$$

$$\boxed{V = \frac{m v \cos\alpha}{(M+m)}}$$

$$\boxed{\mathrm{tg}\,\Theta = \frac{v_y}{v_x} = \frac{v\sin\alpha}{v\cos\alpha - V}}$$

$$= \frac{v\sin\alpha}{v\cos\alpha\left(1 - \frac{m}{(M+m)}\right)}$$

$$\boxed{\mathrm{tg}\,\Theta = \frac{(M+m)}{M}\,\mathrm{tg}\,\alpha}$$

ON OBSERVE BIEN QUE $\Theta \neq \alpha$!

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JUSTE APRES LE CHOC

ENERGIE CINETIQUE

$\frac{1}{2}(m_1 + m_2)\, v_c^2$

À L'ARRET

ENERGIE CINETIQUE

0 !

TRAVAIL DU FROTTEMENT

$W = -\mu_c (m_1 + m_2)\, g\, d$

BILAN D'ENERGIE

$\frac{1}{2} v_c^2 = \mu_c g d$

$$\boxed{v_c = \sqrt{2 \underset{0{,}6}{\mu_c} \underset{9{,}81}{g} \underset{4}{d}}}$$

$= 6{,}86 \text{ m/s}$

DIMENSION CHECK $\sqrt{[\frac{m}{s^2}][m]}$ OK :-)

BILAN DE QUANTITE DE MOUVEMENT HORIZONTALE

AVANT LE CHOC

QUANTITE DE MVT $m_1 v_1$

APRES LE CHOC

QUANTITE DE MVT $(m_1 + m_2)\, v_c$

$$\boxed{v_1 = \frac{(m_1 + m_2)\, v_c}{m_1}}$$

VALEURS NUMERIQUE

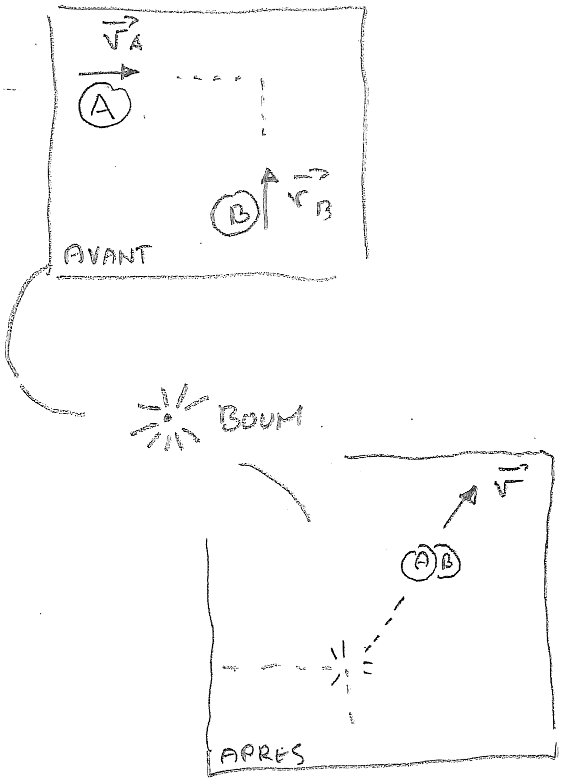
$v_c = \underbrace{\frac{2400}{1400}\ 6{,}86}_{11{,}76 \text{ m/s}}$

AVANT LE CHOC

APRES LE CHOC

QUANTITE DE MVT

$$m_A \begin{bmatrix} v_A \\ 0 \end{bmatrix} + m_B \begin{bmatrix} 0 \\ v_B \end{bmatrix} \longrightarrow m_A + m_B \underbrace{\begin{bmatrix} v_x \\ v_y \end{bmatrix}}_{\vec{v}}$$



$$\begin{cases} m_A v_A = (m_A + m_B)\, v_x \\ m_B v_B = (m_A + m_B)\, v_y \end{cases}$$

$$\vec{v} = \begin{bmatrix} \frac{90}{200} \; 8 \\ \\ \frac{110}{200} \; 7{,}5 \end{bmatrix} = \begin{bmatrix} 3{,}6 \\ \\ 4{,}13 \end{bmatrix}$$

PERTE ENERGIE CINETIQUE

$$= \underbrace{\underbrace{\left[\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2\right]}_{5974 \text{ J}} - \underbrace{\left[\frac{1}{2}(m_A + m_B)(v_x^2 + v_y^2)\right]}_{3003 \text{ J}}}_{2971 \text{ J}}$$

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220 [Km/h]

$$\boxed{220 \; \frac{1000}{3600} \; [m/s] = 61,1 \; [m/s]}$$

A SAVOIR FAIRE !

AVANT

$$\boxed{\text{QUANTITE DE MOUVEMENT} = 0}$$

APRES

$$\boxed{\text{QUANTITE DE MOUVEMENT} = m\,v}$$

m : 0,046 ; v : 61,1

$$\boxed{\Delta[m\,v] = \underbrace{F\,\Delta t}_{\text{IMPULSION}}}$$

$$F = \frac{m\,v}{\Delta t}$$

m : $46 \; 10^{-3}$; v : 61,1 ; Δt : $5 \; 10^{-4}$

$$F = 5620 \; N$$

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AVANT

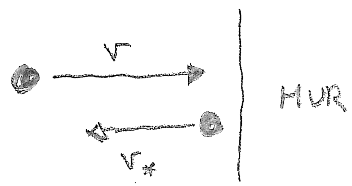
$$\boxed{\text{ENERGIE CINETIQUE} = \frac{1}{2} m\,v^2}$$

APRES

$$\boxed{\underbrace{0,81 \; \frac{1}{2} m\,v^2}_{\frac{1}{2} m\,v_*^2} = \text{ENERGIE}}$$

$$v_*^2 = 0,81\,v^2$$

$$v_* = 0,9\,v$$



$$\boxed{\Delta[m\,v] = \underbrace{F\,\Delta T}_{\text{IMPULSION}}}$$

$$\text{IMPULSION} = 1,9 \; m\,v$$

m : $60 \; 10^{-3}$; v : 30

$$= 3,42 \; [kg\,m/s]$$

ATTENTION LA VITESSE CHANGE D'ORIENTATION

$$\underbrace{m\begin{bmatrix} -0,9\,v \\ 0 \end{bmatrix}}_{\text{APRES !}} - \underbrace{m\begin{bmatrix} v \\ 0 \end{bmatrix}}_{\text{AVANT !}}$$