66

$$\vec{a} = \begin{bmatrix} a_r \\ a_\theta \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$a = \sqrt{36+4} = \sqrt{40} = 6{,}32 \text{ m/s}^2$$

$$\boxed{a = 6{,}32 \text{ m/s}^2}$$

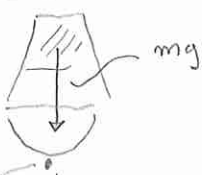
$$\vec{v} = \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} = \begin{bmatrix} 0 \\ r\omega \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{24} \end{bmatrix}$$

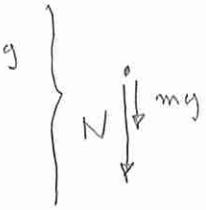
ON SAIT QUE $\boxed{a_r = r\omega^2 \quad v_\theta = r\omega}$ $\Rightarrow$ $\boxed{r\,a_r = (r^2\omega^2) \quad \sqrt{r\,a_r} = v}$

4 6

$$\boxed{v = 4{,}9 \text{ m/s}}$$

67





$$\boxed{\sum \vec{F} = m\vec{a}}$$

VITESSE MINIMALE $N = 0$

$$mg = m\,r\,\omega^2 \qquad \omega^2 = \frac{v^2}{r}$$

$$\boxed{v = \sqrt{gr} = 2{,}8 \text{ m/s}}$$

9,81 0,8

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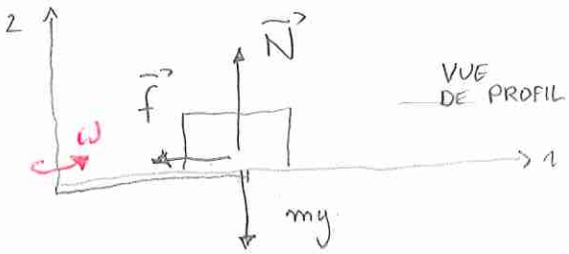
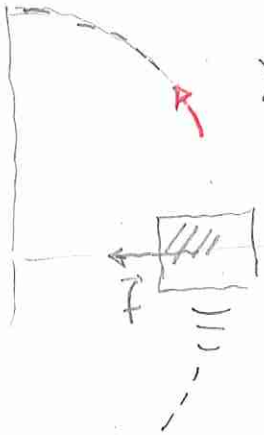


$$\boxed{N = mg \qquad f = \mu_S\, mg}$$



$$\boxed{\sum \vec{F} = m\vec{a}}$$

$\mu_S\, mg = m\,r\,\omega^2$

FORCE DE FROTTEMENT — ACCELERATION CENTRIPETE

0,15

$$\boxed{\mu_S = \frac{r\omega^2}{g} - 4{,}7^2}$$

9,81

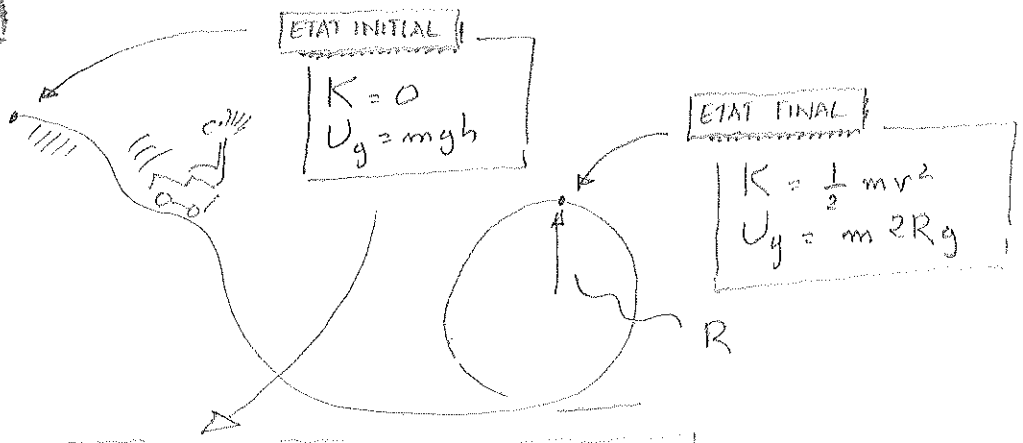
$$\omega = 45\,\frac{2\pi}{60} = 4{,}7 \left[\frac{\text{rad}}{\text{s}}\right]$$

$$\left[\frac{\text{tour}}{\text{minute}}\right]\left[\frac{\text{rad}}{\text{tour}}\right]\left[\frac{\text{minute}}{\text{sec}}\right]$$

VALEUR NUMERIQUE

$\mu_S = 0{,}34$

69



ETAPE 1
CONSERVATION
ENERGIE
MECANIQUE

$$\frac{1}{2} m v^2 + mg\,2R = mgh$$

$$v^2 = 2g(h-2R)$$

ETAPE 2

$$mg = mR\omega^2$$

GRAVITE — ACCELERATION CENTRIPETE

$\sum \vec{F} = m\vec{a}$:-)

CAS LIMITE
$N = 0$ CAR ON EST À LA LIMITE DE QUITTER LE CERCLE

$$g = \frac{2g(h-2R)}{R}$$

$R\omega^2 = \frac{v^2}{R}$

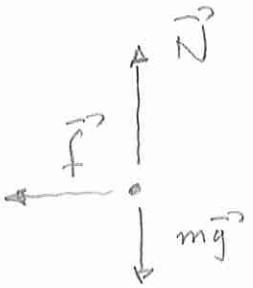
$$R = 2h - 4R$$

$$5R = 2h$$

IL FAUT QUE $h > 5R/2$

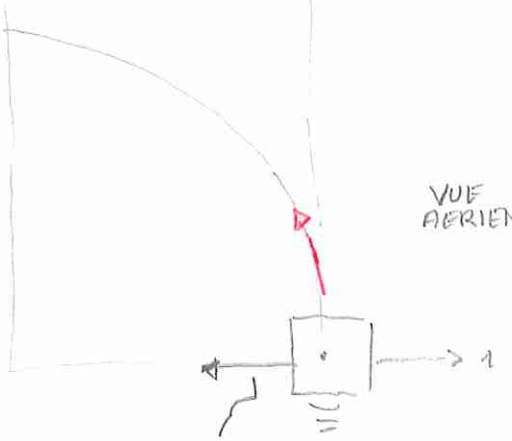
70

VUE DE PROFIL



$N = mg$

$f = \mu_S mg$



VUE AERIENNE

$\vec{a}$ = ACCELERATION CENTRIPETE

$$\frac{mv^2}{R} = \mu_S mg$$

$\frac{mv^2}{R}$: ACCELERATION CENTRIPETE

$\mu_S mg$: FORCE DE FROTTEMENT

$$\mu_S = \frac{v^2}{Rg}$$

v^2 : $16{,}7^2$ [m²/s²]

R : 60 [m]

g : 9,81 [m/s²]

60 Km/h = $\frac{60.000}{3600}$ m/s

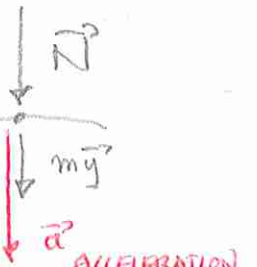
$\frac{60.000}{3600}$ = 16,7

71

VALEUR NUMERIQUE

$\mu_S > 0{,}47$

C'EST UNE VALEUR REALISTE :-)



$$(N + mg) = m\frac{v^2}{R}$$

$(N + mg)$: FORCES

$m\frac{v^2}{R}$: ACCELERATION CENTRIPETE

VITESSE MINIMALE

$N = 0$

$$mg = m\frac{v^2}{R}$$

$$v = \sqrt{gR}$$

g : 9,81

R : 6,5

$\simeq$ 8 m/s

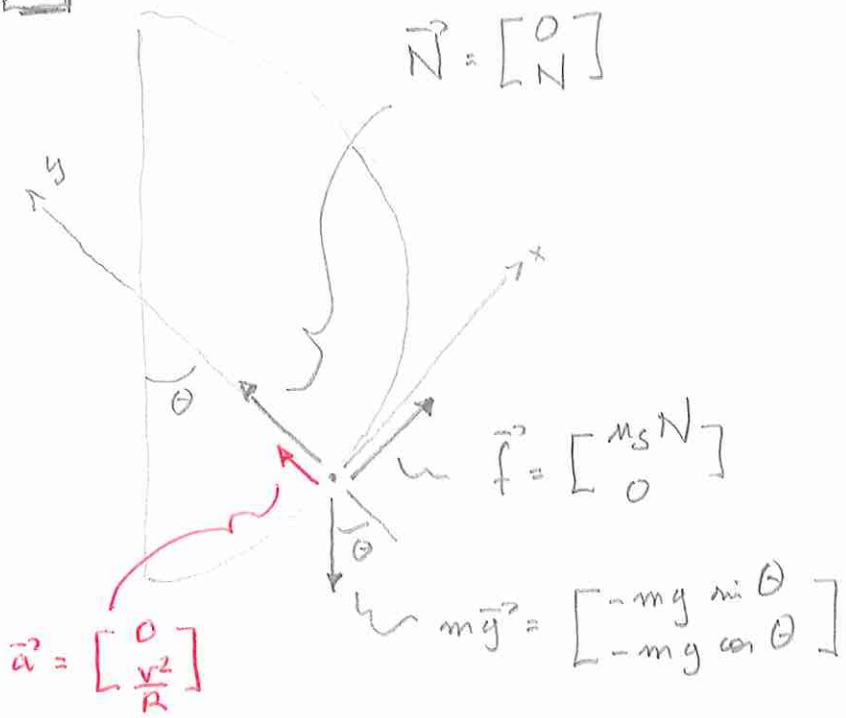
VITESSE MINIMALE REQUISE :-)

POIDS APPARENT

$$= m\left(\frac{v^2}{R} - g\right)$$

LE POIDS APPARENT ↗ SI LA VITESSE ↗ :-)

72



$$\sum \vec{F} = m\vec{a}$$

$$\begin{bmatrix} \mu_s N - mg\sin\theta \\ N - mg\cos\theta \end{bmatrix} = m \begin{bmatrix} 0 \\ \frac{v^2}{R} \end{bmatrix}$$

$$\frac{mv^2}{R} = \frac{mg\sin\theta}{\mu_s} - mg\cos\theta$$

$$g\sin\theta = \mu_s \left(g\cos\theta + \frac{v^2}{R} \right)$$

COMMENT OBTENIR θ ?

$$\sin^2\theta = \mu_s^2 \left(\cos\theta + \frac{v^2}{Rg} \right)^2$$

$$1 - \cos^2\theta \qquad \mu_s^2 \cos^2\theta + \mu_s^2 \frac{v^4}{R^2 g^2} + 2\mu_s^2 \cos\theta \frac{v^2}{Rg}$$

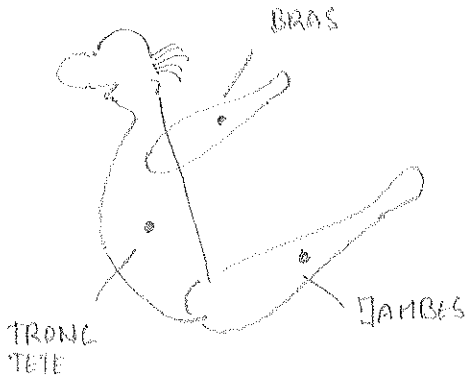
EQUATION DU SECOND DEGRE EN $\cos(\theta)$

VALEUR NUMERIQUE ACCEPTABLE

$\cos\theta = 0{,}631$

$\theta = 51°$

73



60 kg

2 JAMBES 32 %
2 BRAS 10 %
LE RESTE 100 % − 42 % = 58 %

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0,3 \\ 0,32 \end{bmatrix} 0,32 + \begin{bmatrix} 0,21 \\ 0,53 \end{bmatrix} 0,1 + \underbrace{\begin{bmatrix} 0,08 \\ 0,27 \end{bmatrix}}_{\vec{x}_3} \underbrace{0,58}_{\frac{m_3}{m}}$$

$$\begin{bmatrix} 0,16 \\ 0,31 \end{bmatrix}$$

$$\vec{x} = \sum \frac{m_i}{m} \vec{x}_i$$

LE CENTRE DE MASSE NE DEPEND PAS DE m !
UNIQUEMENT DES RAPPORTS ENTRE LES MASSES DU CORPS

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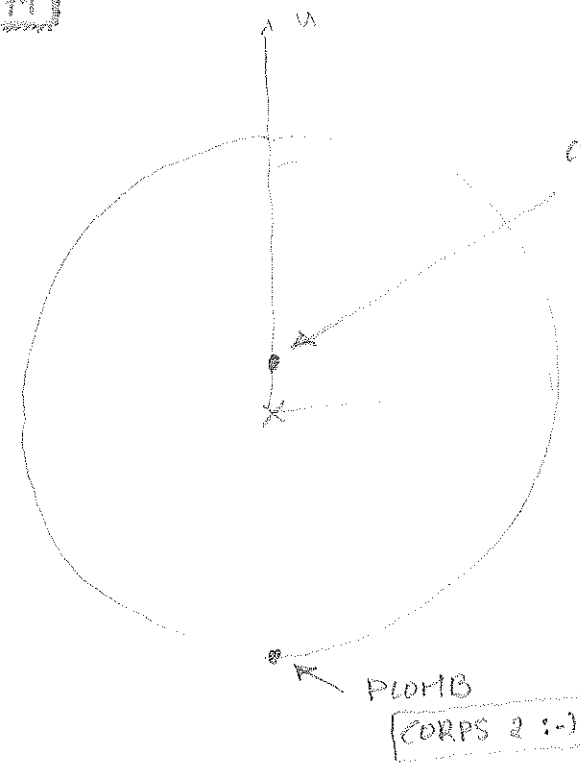
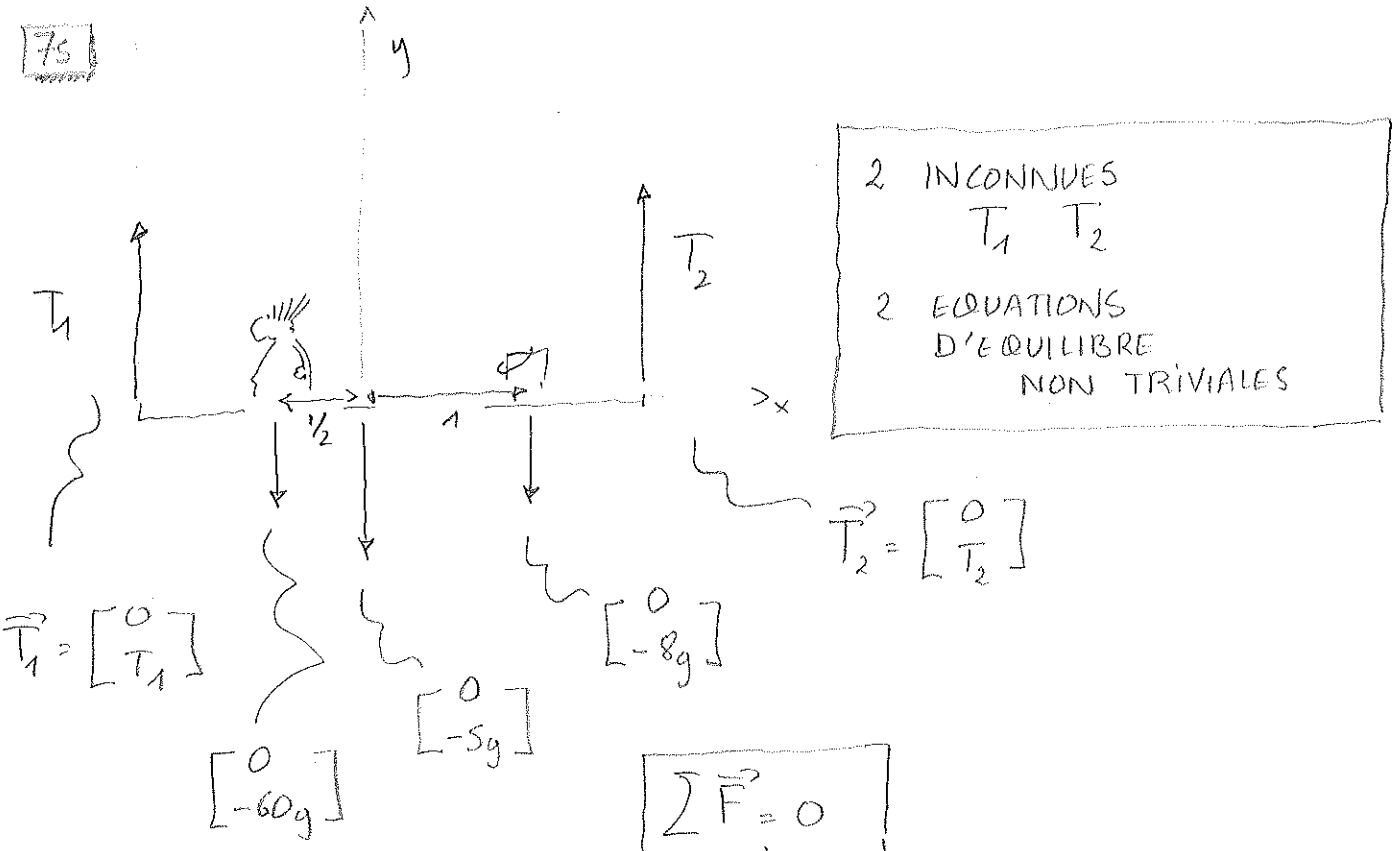


0,033 kg = 33 g

$$m_2 = 20 \frac{0,0003}{0,18}$$

$$\underbrace{m \begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\text{ON VEUT QUE LE CENTRE DE MASSE DE LA ROUE EQUILIBREE SE TROUVE AU CENTRE GEOMETRIQUE}} = 20 \underbrace{\begin{bmatrix} 0 \\ 0,0003 \end{bmatrix}}_{\vec{x}_1} + m_2 \underbrace{\begin{bmatrix} 0 \\ -0,18 \end{bmatrix}}_{\vec{x}_2}$$

75



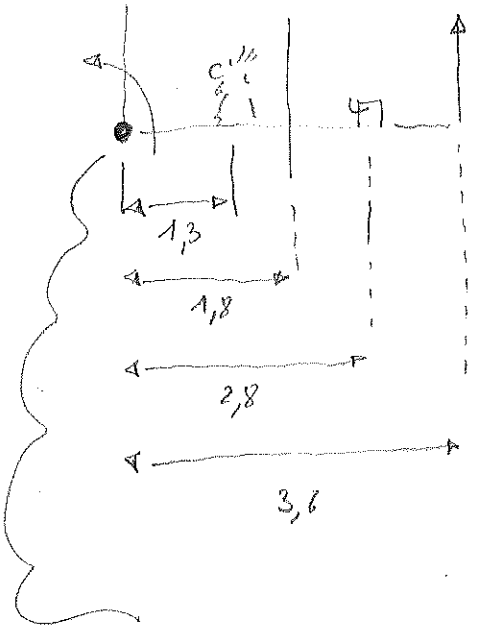
2 INCONNUES
T_1 T_2

2 EQUATIONS D'EQUILIBRE NON TRIVIALES

$\sum \vec{F} = 0$

$T_1 + T_2 = 73\,g$

$\sum \vec{M} = 0$



EQUILIBRE DE ROTATION PAR RAPPORT A CE POINT !

$T_2\ 3{,}6 = -60g \times 1{,}3 - 5g \times 1{,}8 - 8g \times 2{,}8$

$T_2 = 298\ N$

$T_1 = 73 \times 9{,}81 - 298$

$418\ N$

C'EST UNE BONNE IDEE DE CHOISIR LE POINT D'APPLICATION D'UNE FORCE INCONNUE !

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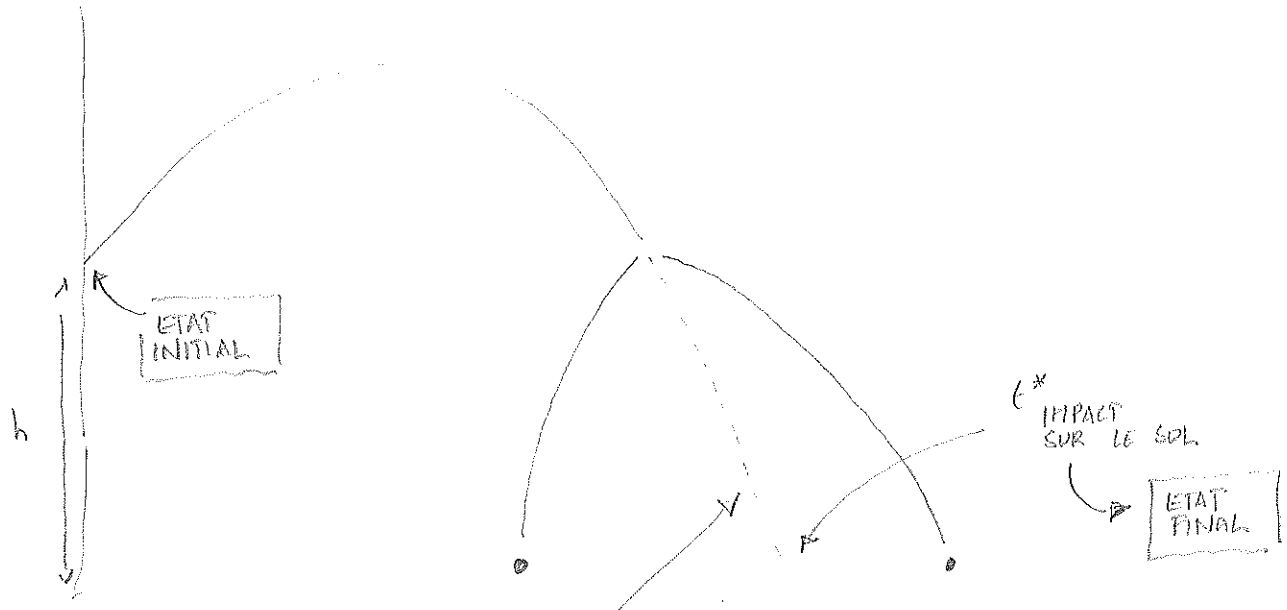

TRAJECTOIRE CENTRE DE MASSE = MRUA

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 0 + v_0 \cos\theta\, t \\ h + v_0 \sin\theta\, t - g t^2/2 \end{bmatrix}$$

ETAPE 1
CALCOL DE t^*

$$2h = g t_*^2 - 2 v_0 \sin\theta\, t_*$$

$$t^* = \frac{2 v_0 \sin\theta \pm \sqrt{4 v_0^2 \sin^2\theta + 8gh}}{2g}$$

$$= \begin{cases} 10{,}15 \text{ sec} \\ -2{,}91 \text{ sec} \end{cases}$$

ETAPE 2

$$x(t^*) = \underbrace{v_0 \cos\theta\, t^*}_{305{,}4 \text{ m}}$$

$$m x = m_1 x_1 + m_2 x_2$$

m: 6 — x: 305 — m_1: 4 — x_1: 200 — m_2: 2

$$x_2 = \frac{6 \times 305 - 4 \times 200}{2}$$

$$x_2 = 515 \text{ m}$$

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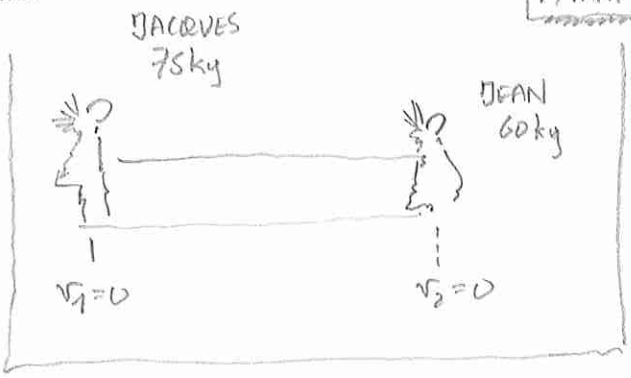
AVANT

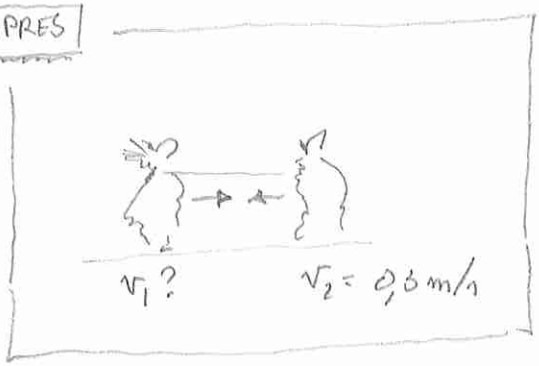


PAS DE FORCES EXTERNES AU SYSTEME JEAN + JACQUES !

$$\Delta [m \vec{v}] = 0$$

$$m_1 \vec{v_1} + m_2 \vec{v_2}$$

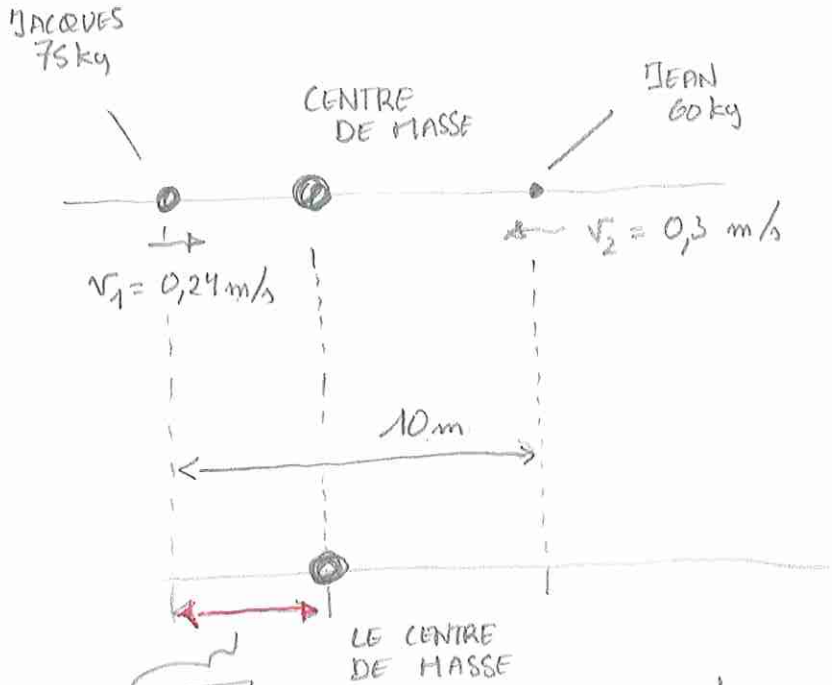
APRES



CONSERVATION QUANTITE DE MVT

$$0 = m_1 v_1 - m_2 0,3$$

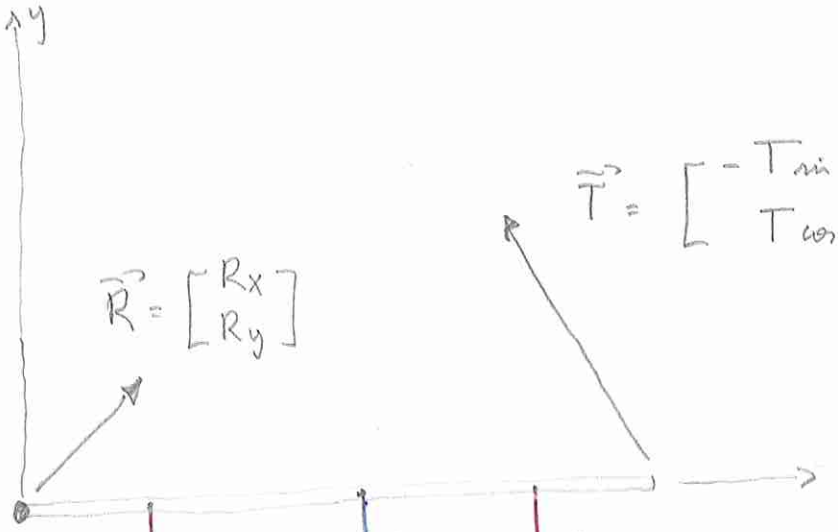
$$v_1 = \frac{60}{75} 0,3 = 0,24 \text{ m/s}$$



LE CENTRE DE MASSE NE BOUGE PAS !

$$\frac{60}{135} 10 = 4,4 \text{ m}$$

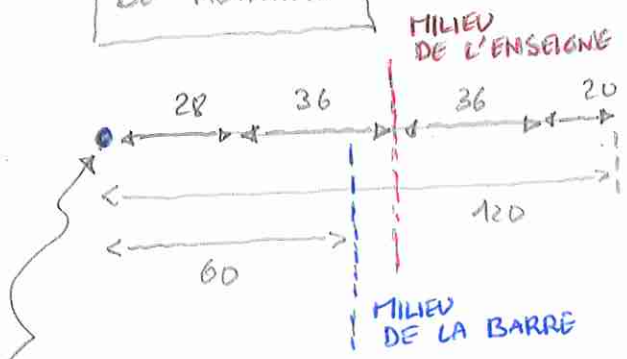
78



3 INCONNUES
R_x, R_y, T

3 EQUATIONS D'EQUILIBRE

ECQUILIBRE DE ROTATION



$$-0{,}6 * 2g - 0{,}28 * 1{,}5g - 1{,}00 * 1{,}5g + 1{,}2\, T\cos\theta = 0$$

$(-1{,}2 - 0{,}42 - 1{,}5) = -3{,}12$

EQUILIBRE DE ROTATION PAR RAPPORT A CE POINT

$$T = \frac{3{,}12\,g}{1{,}2\cos\theta} \quad (\approx 51\ N)$$

$$R_x = T\sin\theta = 44{,}2\ N$$
$$R_y = 5g - T\cos\theta = 23{,}5\ N$$

EQUILIBRE DE TRANSLATION

$$R = 50{,}6\ N$$