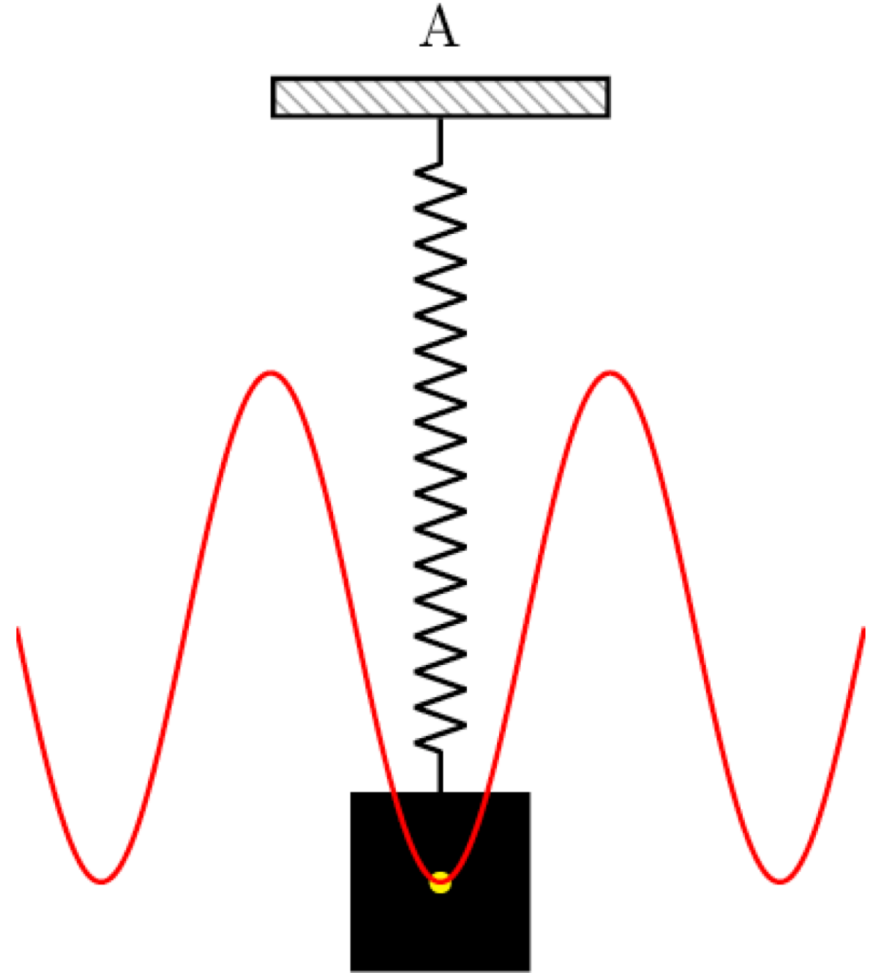
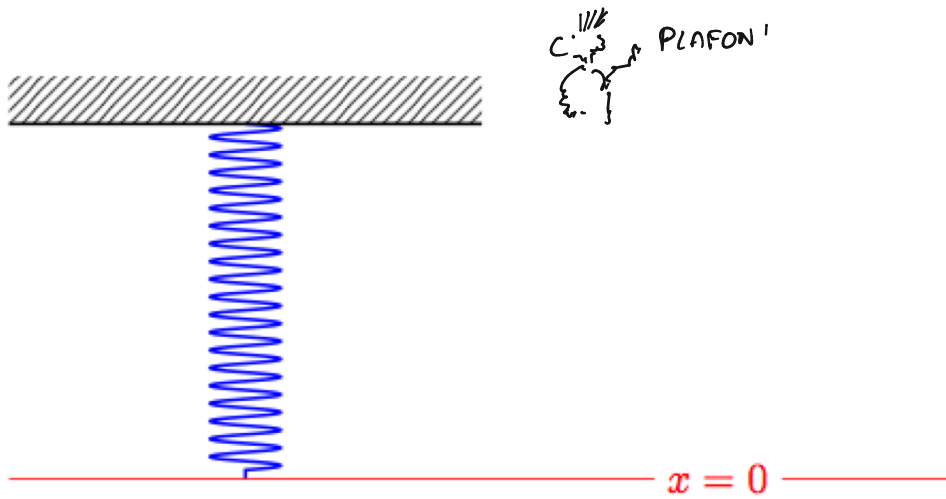


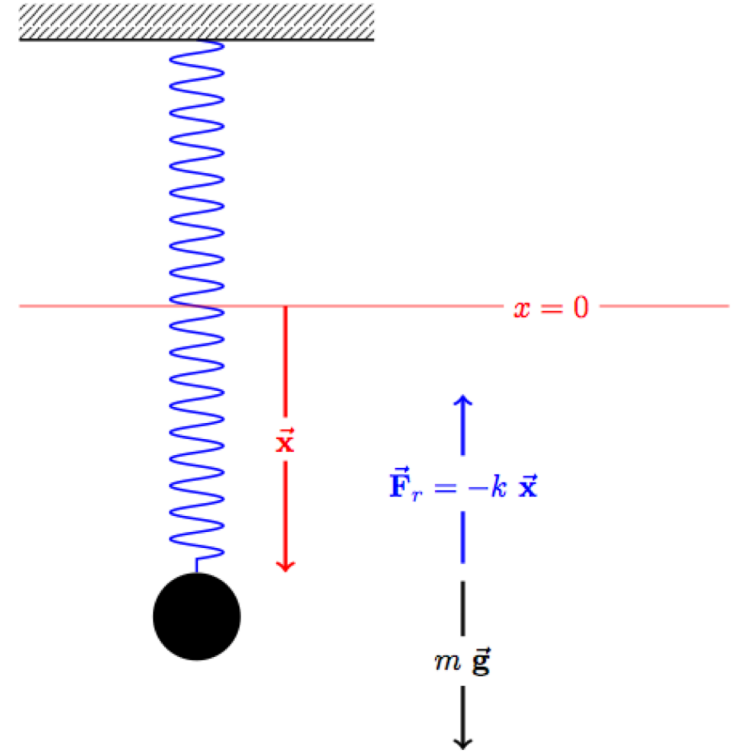
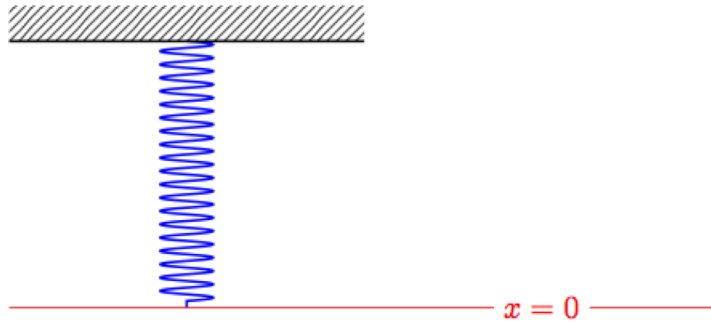
La force
de rappel
d'un ressort !



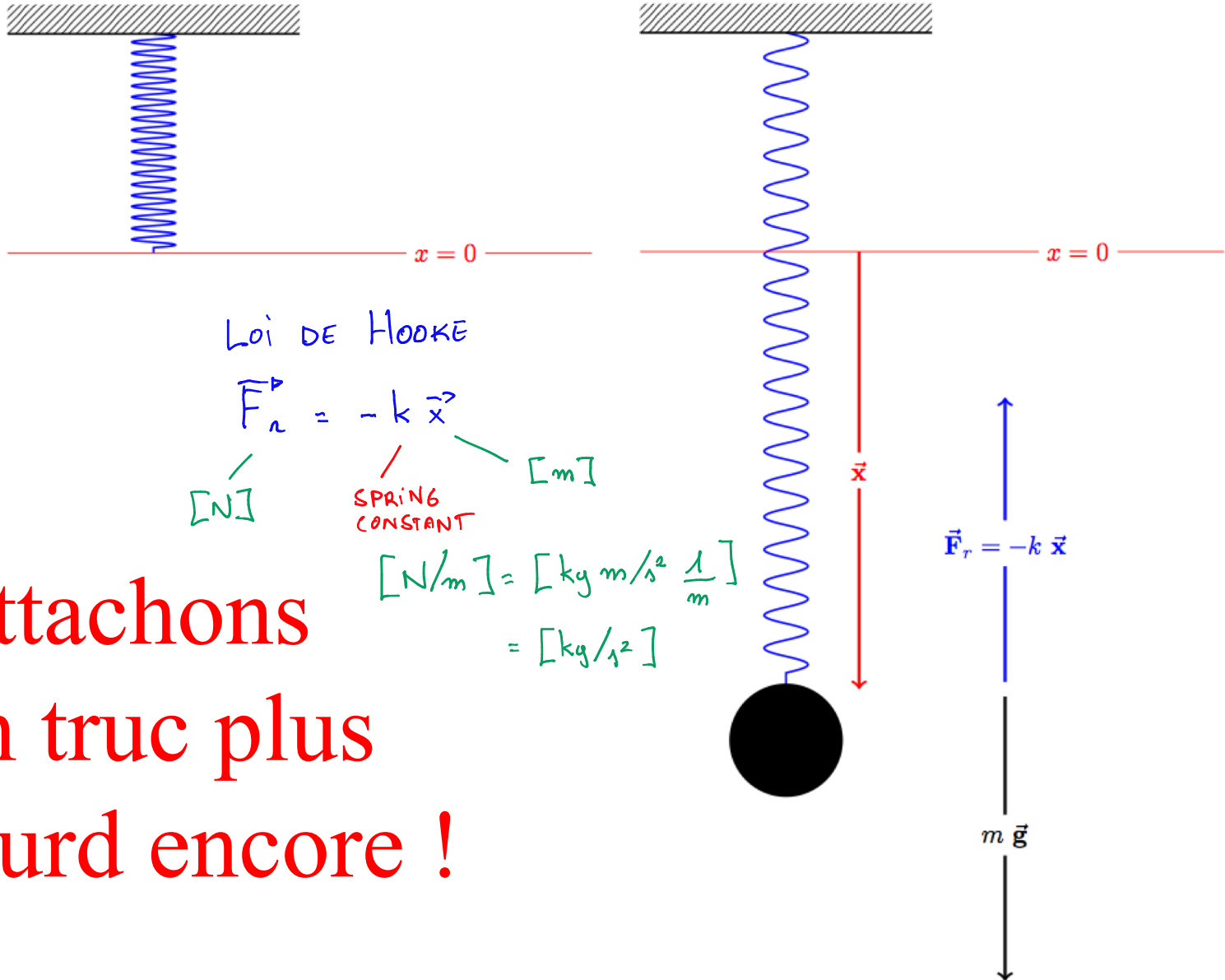


Considérons
un ressort !





Attachons
un poids !



Loi DE Hooke

$$\vec{F}_r = -k \vec{x}$$

[N]

SPRING
CONSTANT

[m]

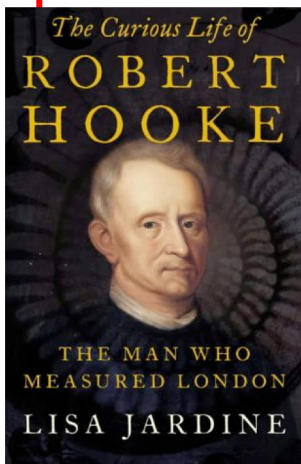
$$[N/m] = [kg \cdot m/s^2 \cdot \frac{1}{m}]$$

$$= [kg/s^2]$$

Attachons
un truc plus
lourd encore !

Loi de Hooke

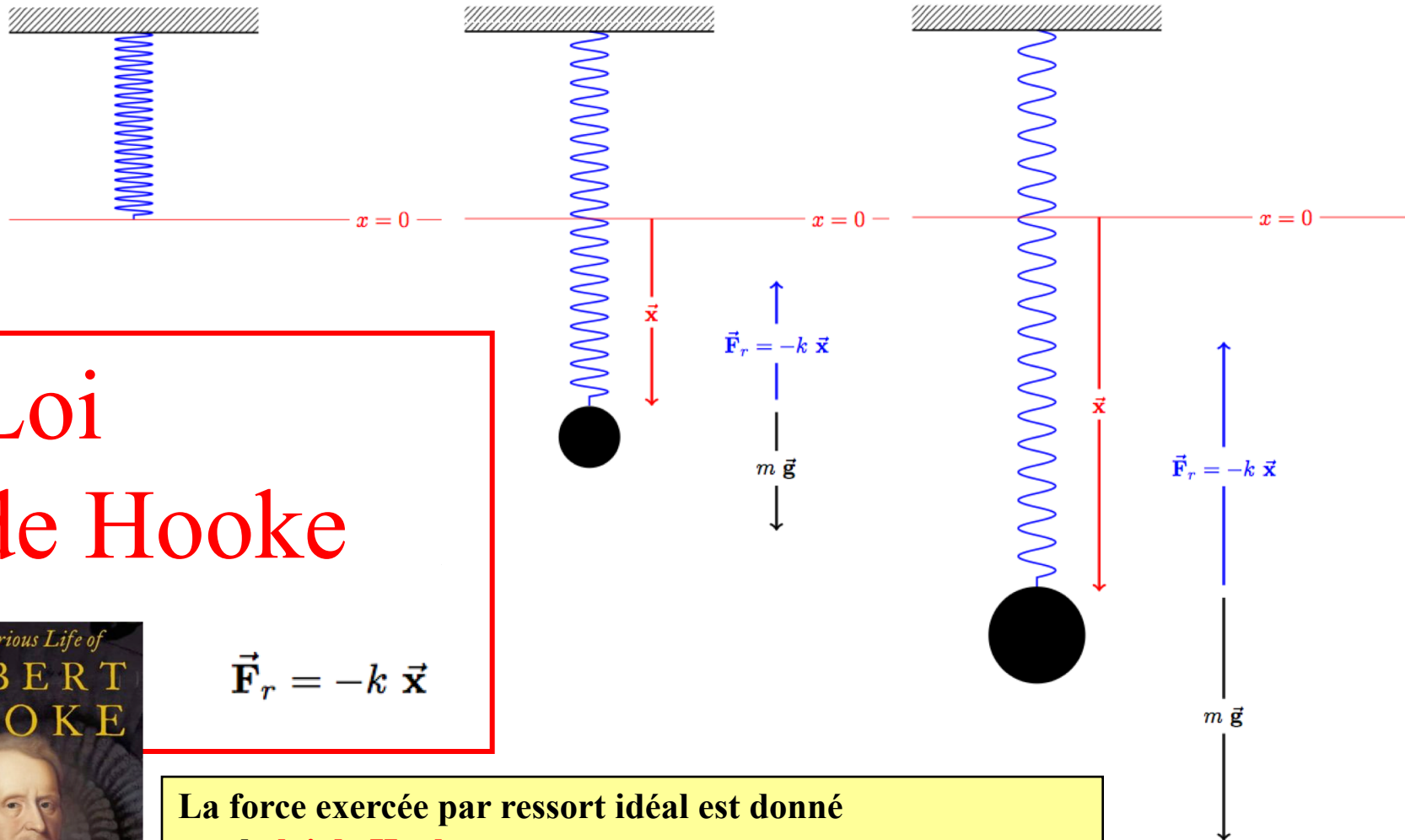
$$\vec{F}_r = -k \vec{x}$$



La force exercée par ressort idéal est donné
par la **loi de Hooke**.

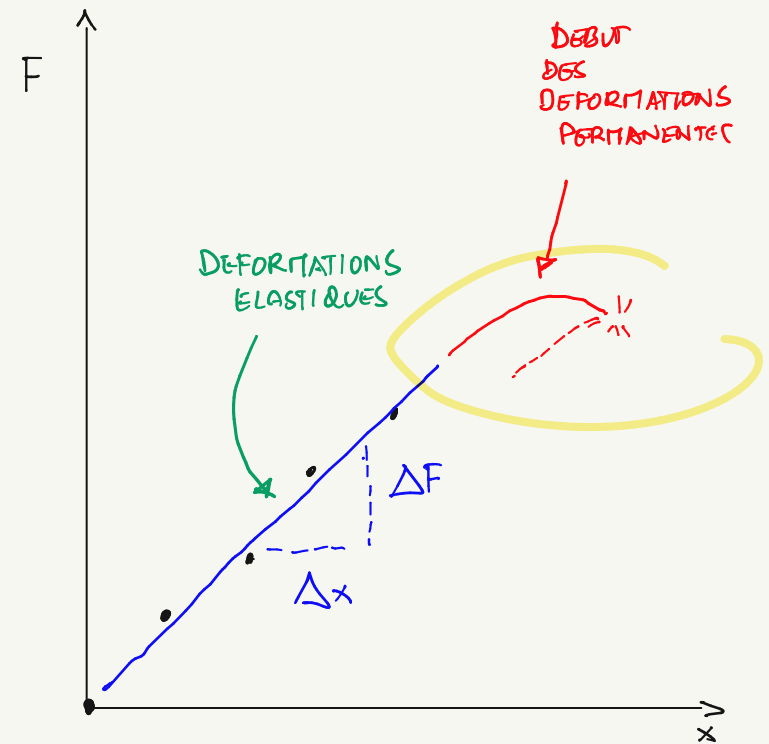
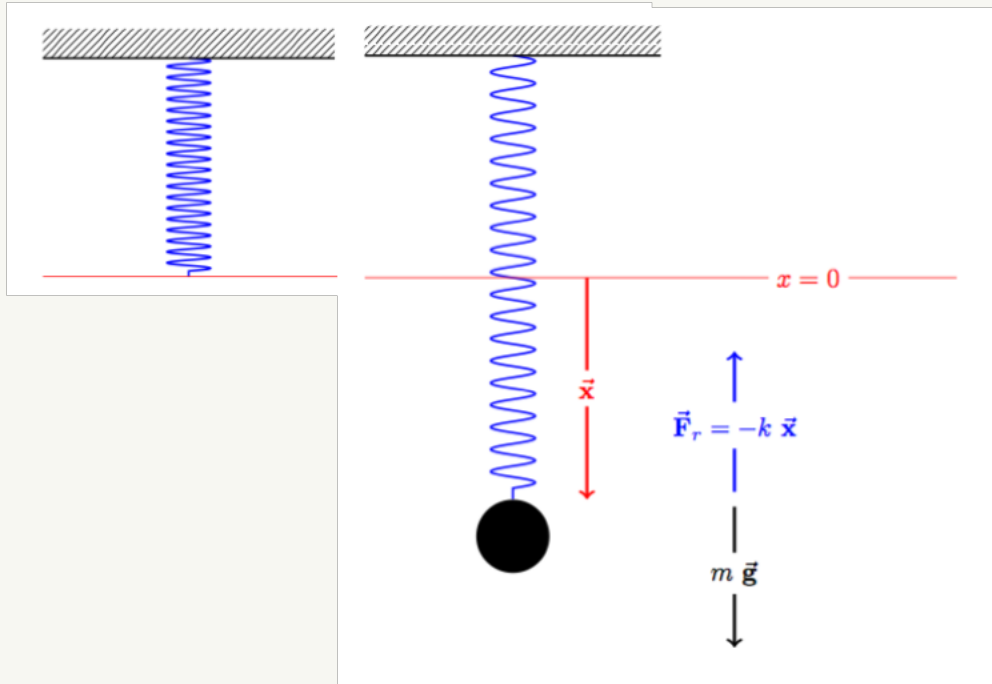
La force s'oppose toujours à l'allongement du ressort et
est proportionnelle à cet allongement.

Le facteur k est appelé **constante de raideur du ressort**.



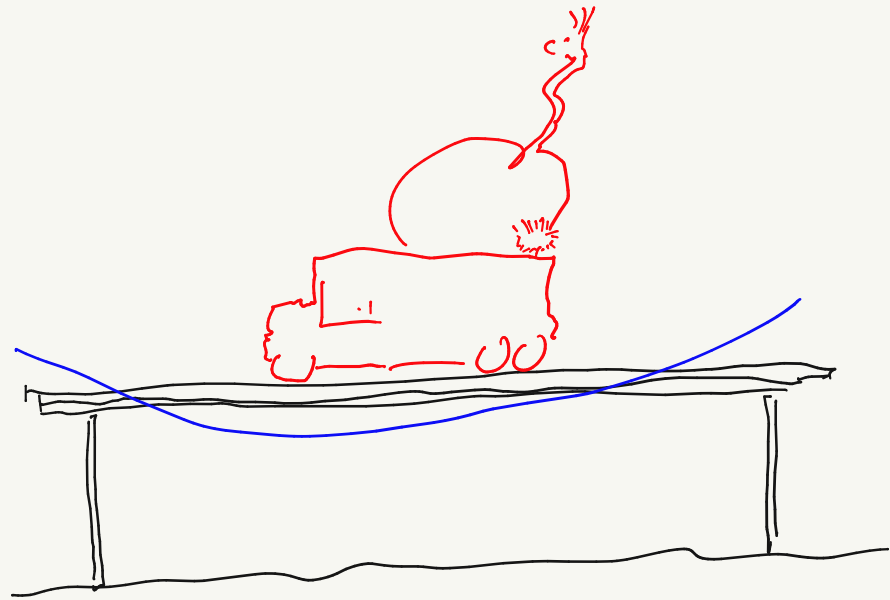


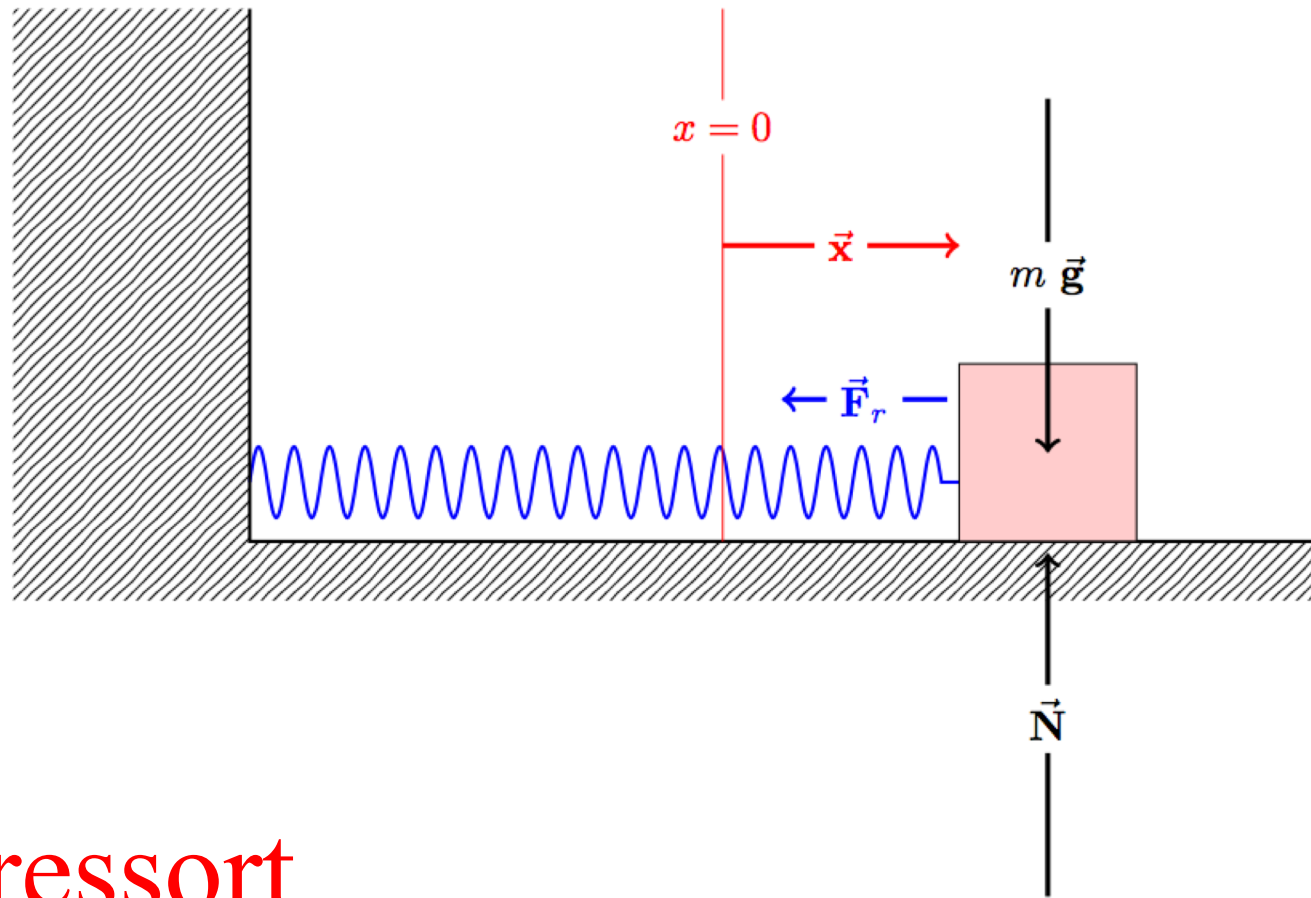
COMMENT
OBTENIR k ?



$$k = \frac{\Delta F}{\Delta x}$$

$$\vec{F}_r = -k \vec{x}$$

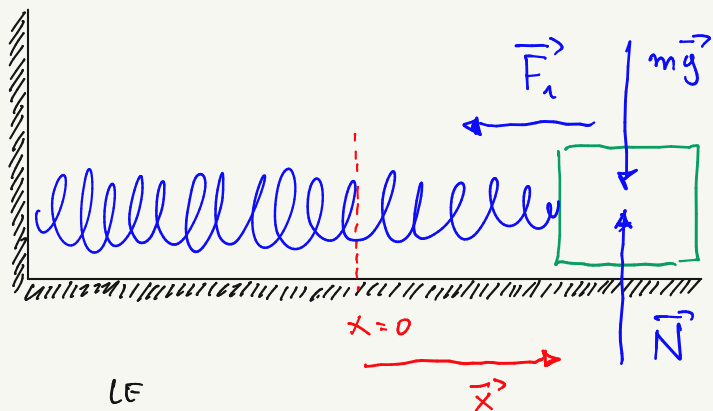




Un ressort
cela oscille !

2

UN RESSORT
CELA OSCILLE !



LE
BLOC
GLISSE
PARFAITEMENT
SUR LE SOL

$$m a(t) = -k x(t)$$

$$m x''(t) = -k x(t)$$

$$x''(t) = -\frac{k}{m} x(t)$$

EQUATION
DIFFERENTIELLE
ORDINAIRE

$\sin(\dots)$
 $\cos(\dots)$

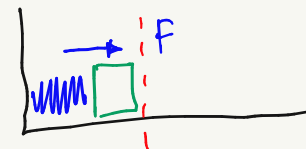
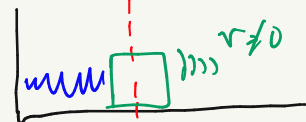
AMPLITUDE

ANGLE
DE PHASE

$$x(t) = A \cos(\omega t + \varphi)$$

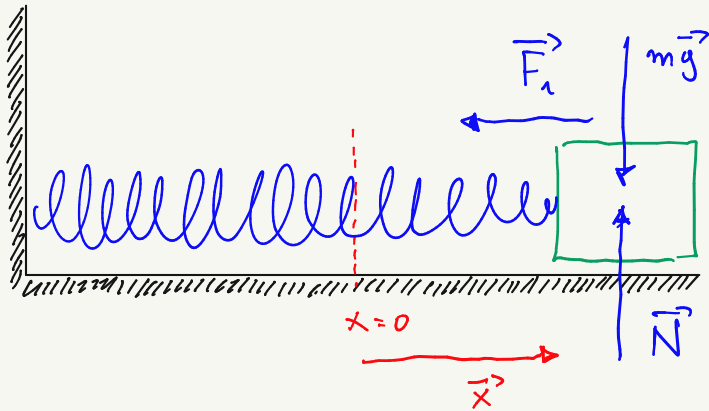
$$x'(t) = -A \omega \sin(\omega t + \varphi)$$

$$x''(t) = -A \omega^2 \cos(\omega t + \varphi) = -\omega^2 x(t)$$



$$\omega = \sqrt{\frac{k}{m}}$$

FREQUENCE
ANGULAIRE



$$x''(t) = -\frac{k}{m} x(t)$$

EQUATION
DIFFERENTIELLE
ORDINAIRE

FREQUENCE
OU VITESSE
ANGULAIRE

$$\omega = \sqrt{\frac{k}{m}} \quad \left[\frac{\text{kg}}{\text{s}^2} \right] \quad \underbrace{\quad}_{\left[\text{kg} \right]} \quad \underbrace{\quad}_{\left[\text{s}^2 \right]}$$

$$x(t) = A \cos(\omega t + \varphi)$$

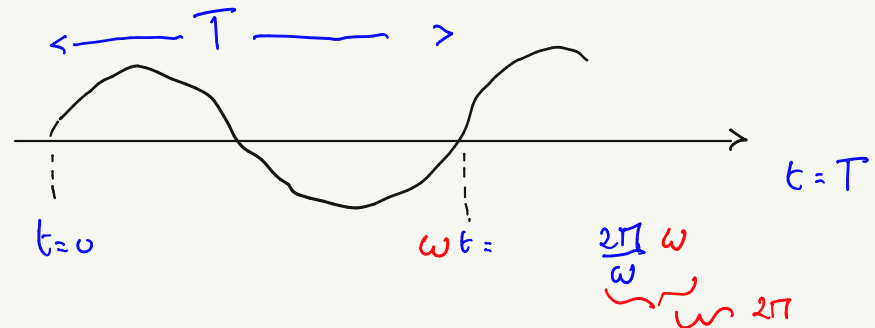
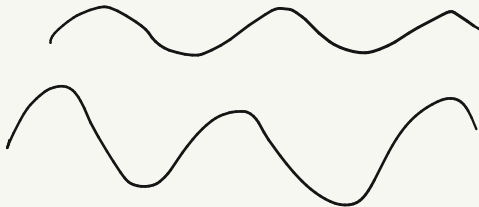
AMPLITUDE
[m]

VITESSE
ANGULAIRE
[rad/s]

ANGLE
DE PHASE
[rad]

PERIODE

$$T = 2\pi \sqrt{\frac{m}{k}} \\ = 2\pi / \omega$$



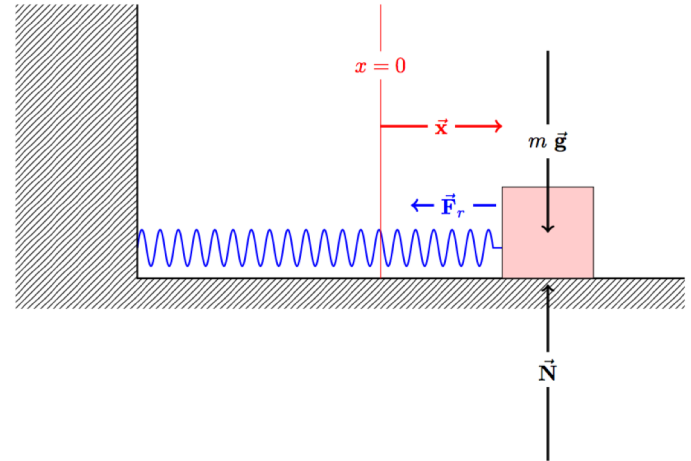
Le mouvement harmonique !

$$m a(t) = -k x(t)$$

$$m \frac{d^2 x}{dt^2}(t) = -k x(t)$$



$$\frac{d^2 x}{dt^2}(t) = -\frac{k}{m} x(t)$$



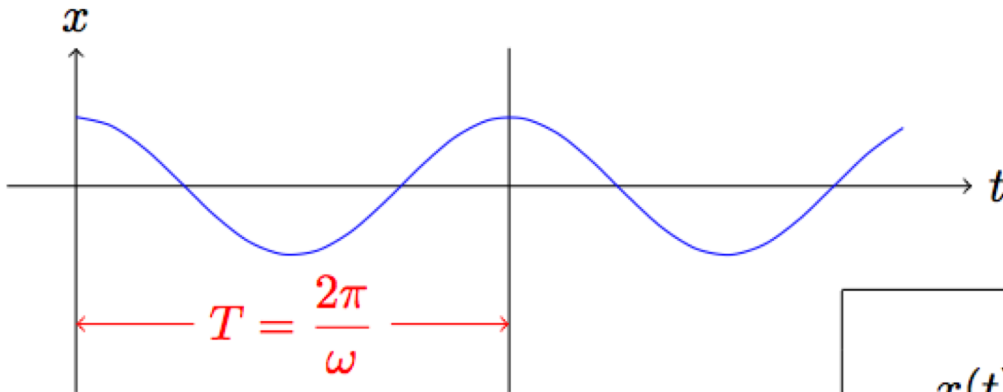
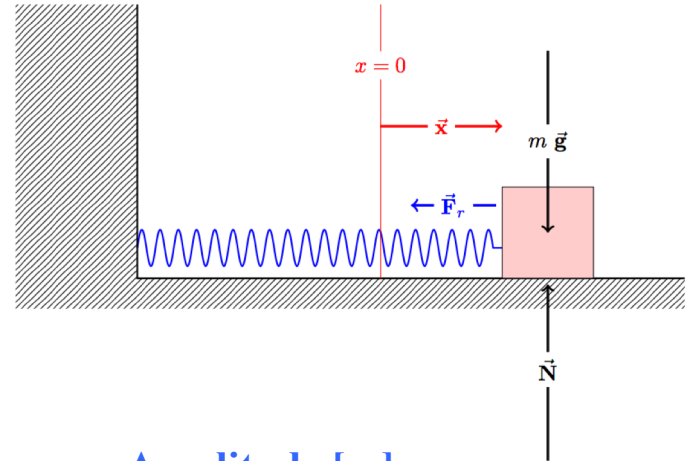
$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A \cos(\omega t + \phi)$$

$$x'(t) = -\omega A \sin(\omega t + \phi)$$

$$x''(t) = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x(t)$$

Le mouvement harmonique !



$$\omega = \sqrt{\frac{k}{m}}$$

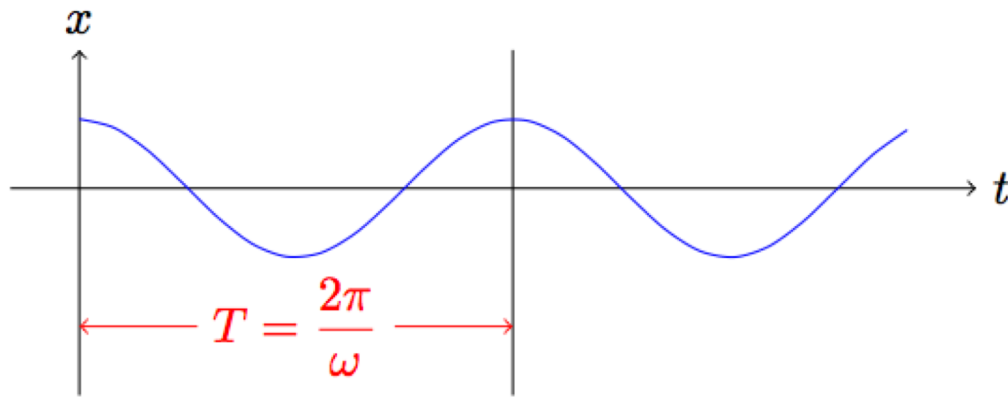
Amplitude [m]

$$x(t) = A \cos(\omega t + \phi)$$

Fréquence angulaire
[radian / sec]

Angle de phase
[radian]

*L'amplitude et l'angle de phase sont déterminés
par les conditions initiales de position et de vitesse !*



$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Période : T

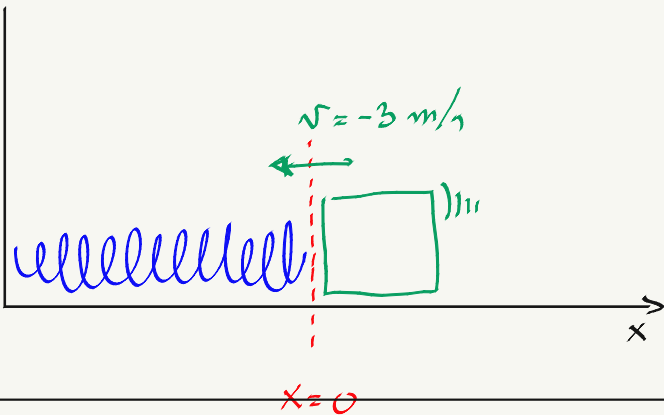
Fréquence : f

Fréquence angulaire : ω

3

UN PETIT
EXEMPLE :-)

$$\begin{aligned} t &= 0 & k &= 10 \text{ N/m} \\ x &= 0 & m &= 0.1 \text{ kg} \\ v &= -3 \text{ m/s} \end{aligned}$$



3

$$x'(0) = -3$$

$$\begin{aligned} -A \omega \sin(\omega t + \varphi) &= -3 \\ \begin{matrix} \swarrow & \searrow \\ = 10 & 0 \end{matrix} & \quad \begin{matrix} \swarrow & \searrow \\ & \pi/2 \end{matrix} \\ \underbrace{\hspace{1cm}} &= 1 \end{aligned} \quad \left. \vphantom{\begin{matrix} -A \omega \sin(\omega t + \varphi) \\ = 10 \\ 0 \end{matrix}} \right\} \begin{aligned} -10A &= -3 \\ A &= 0.3 \end{aligned}$$

$$x(t) = A \cos(\omega t + \varphi)$$

QUESTION

QUE VALENT A, ω, φ ?

1

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} = 10 \text{ [rad/s]} \\ &\quad \left\{ \begin{matrix} \swarrow \\ 0.1 \end{matrix} \right. \sqrt{\frac{10 \cdot 10}{1}} = \sqrt{100} = 10 \text{ :-)} \end{aligned}$$

2

$$x(0) = 0$$

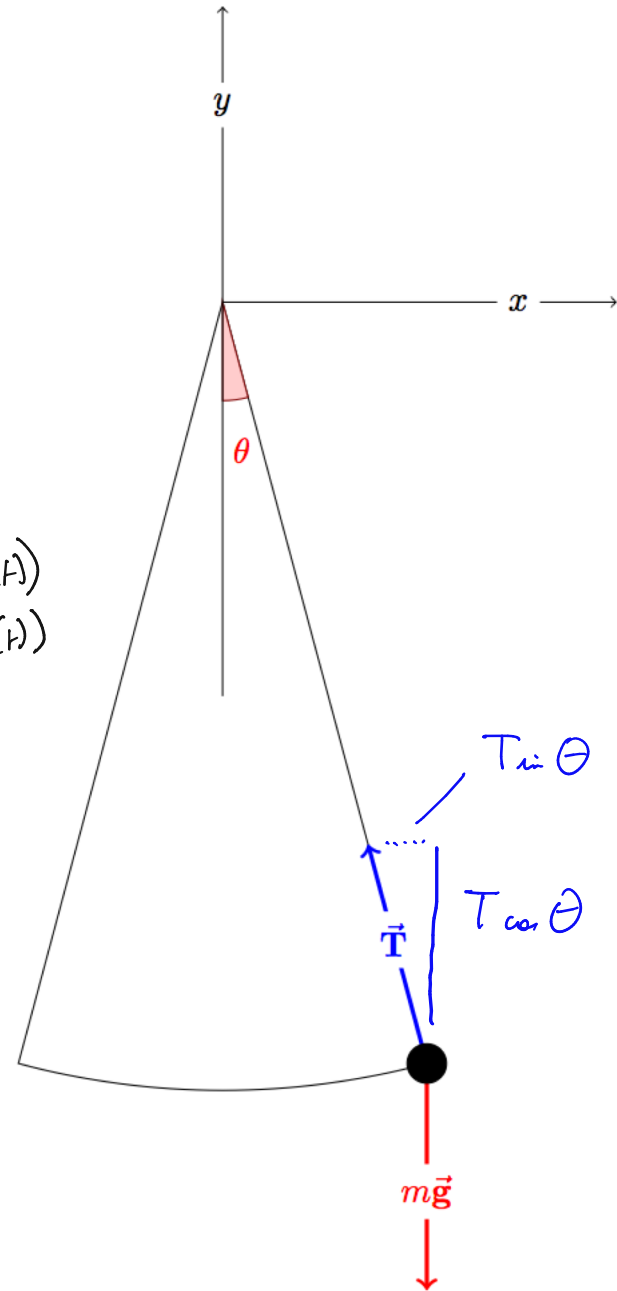
$$\begin{aligned} \omega t + \varphi &= \pi/2 \\ \underbrace{\omega t}_{=0} & \quad \searrow \quad \varphi = \pi/2 \end{aligned}$$

$$x(t) = 0.3 \cos(10t + \pi/2)$$



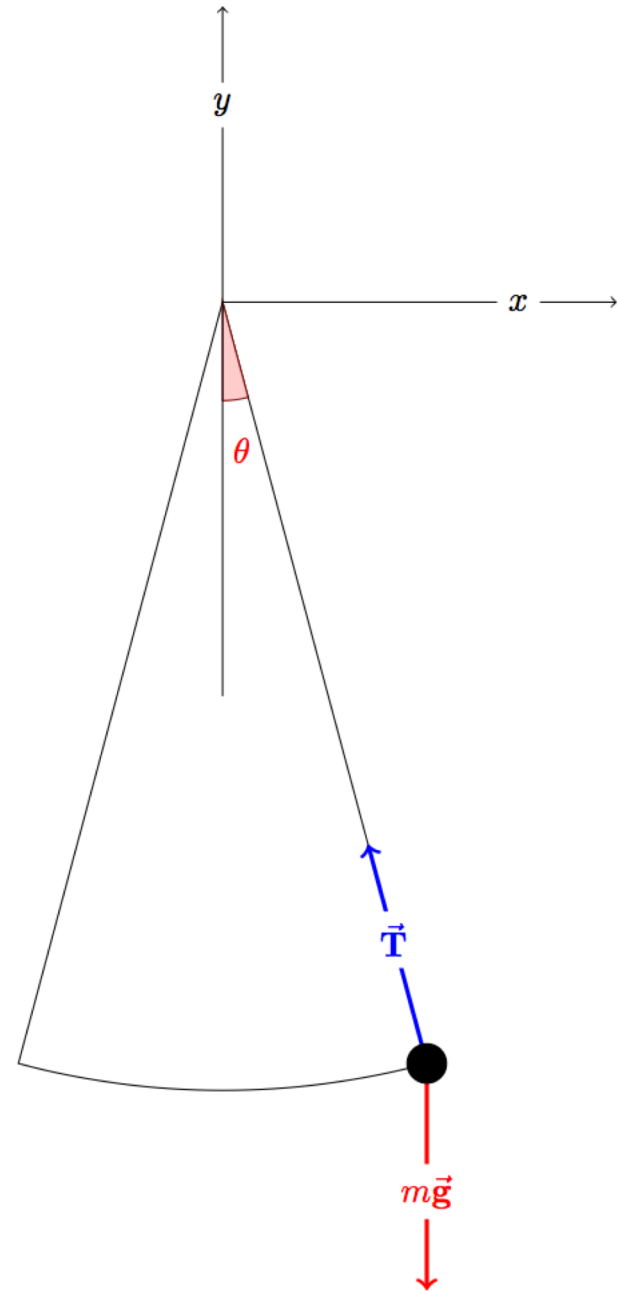
$$\begin{aligned} m x''(t) &= -T \sin(\theta(t)) \\ m y''(t) &= T \cos(\theta(t)) - mg \end{aligned}$$

Un pendule
cela oscille !



Dynamique du pendule !

$$\begin{cases} m \frac{d^2 x}{dt^2}(t) &= -T \sin(\theta(t)) \\ m \frac{d^2 y}{dt^2}(t) &= T \cos(\theta(t)) - mg \end{cases}$$



4

SMALL
ANGLES
APPROXIMATION

$$\theta \ll 1 !$$

$$\theta \ll 1$$

1 $\cos(\theta) \approx 1$

5° $\cos = 0,996$

10° $\cos = 0,985$

$$m x''(t) = -T \sin(\theta(t))$$

$$m y''(t) = T \cos(\theta(t)) - mg$$

$\underbrace{m y''(t)}_{=0}$ $\underbrace{\cos(\theta(t))}_{\approx 1}$

$$mg = T$$

$$m x''(t) = -\frac{mg}{L} x(t)$$

3

$$y \approx L$$

$$y'' = 0$$

2 $\sin(\theta) \approx \frac{x}{L}$

5° $\frac{1}{180} = 0,0873$ $0,0872$

10° $= 0,175$ $0,172$

$$x''(t) = -\frac{g}{L} x(t)$$

$$\omega = \sqrt{\frac{g}{L}}$$

$[m/s^2]$
 $[m]$
 $[1/s]$

$$x(t) = A \cos(\omega t + \varphi)$$

Pour des petits angles
- pas si petits d'ailleurs -
on peut simplifier !

$$\left\{ \begin{array}{l} m \frac{d^2 x}{dt^2}(t) = -T \overbrace{\sin(\theta(t))}^{=\frac{x(t)}{L}} \\ m \underbrace{\frac{d^2 y}{dt^2}(t)}_{\approx 0} = T \underbrace{\cos(\theta(t))}_{\approx 1} - mg \end{array} \right.$$

$$\frac{d^2 x}{dt^2}(t) = -\frac{g}{L} x(t)$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$



NOT INTUITIVE !



$$\frac{d^2x}{dt^2}(t) = -\frac{g}{L} x(t)$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Le pendule

$$T = 2\pi \sqrt{\frac{L}{g}}$$

PENDULE

RESSORT

$$T = 2\pi \sqrt{\frac{m}{k}}$$



Le ressort

$$\frac{d^2x}{dt^2}(t) = -\frac{k}{m} x(t)$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

5

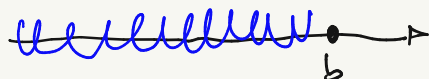
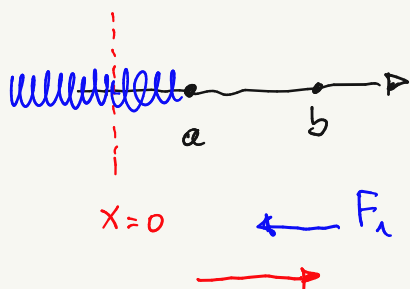
LA FORCE
DU RESSORT
EST CONSERVATIVE

.... ENERGIE
POTENTIELLE
DU RESSORT

ENERGIE
POTENTIELLE
DU
RESSORT EN x_b

....
EN x_a

$$W = \int_a^b -kx \, dx = \left[-\frac{kx^2}{2} \right]_a^b = - \left(\left[\frac{kx_b^2}{2} \right] - \left[\frac{kx_a^2}{2} \right] \right) = -\Delta U_r$$



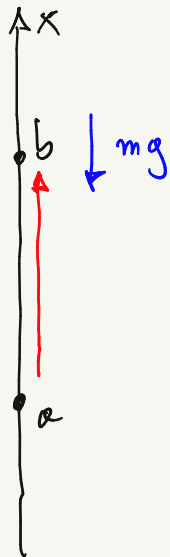
$$U_r = \frac{kx^2}{2}$$

ENERGIE
POTENTIELLE

6

LA GRAVITE
EST CONSERVATIVE

.... ENERGIE
POTENTIELLE
DE GRAVITE !



$$W_g = \int_a^b -mg dx = [-mgx]_a^b = - \left([mgx_b] - [mgx_a] \right)$$

ENERGIE
POTENTIELLE
DE GRAVITE

$$U_g = mgx$$

ENERGIE
POTENTIELLE
EN x_b

... EN x_a

$$-\Delta U_g$$

$$\frac{mv^2}{2}$$

ENERGIE
CINETIQUE



ENERGIE
POTENTIELLE

$$U_g + U_r$$

La force de rappel
du ressort est
conservative !

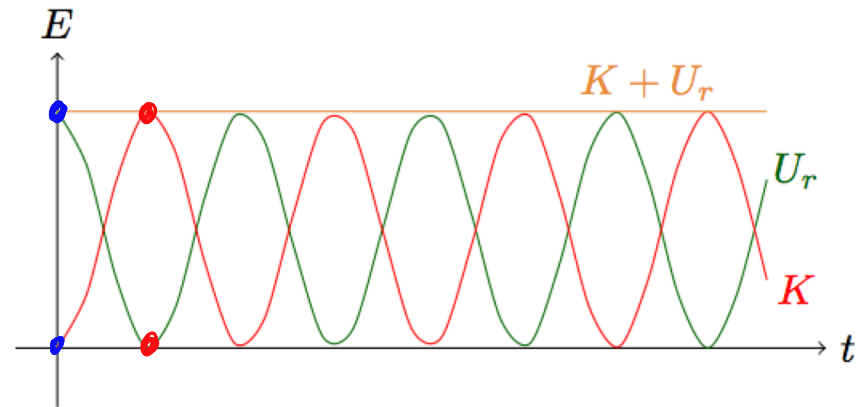
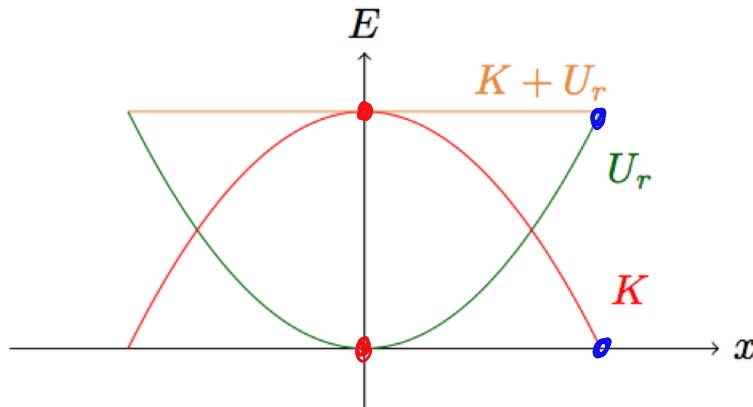
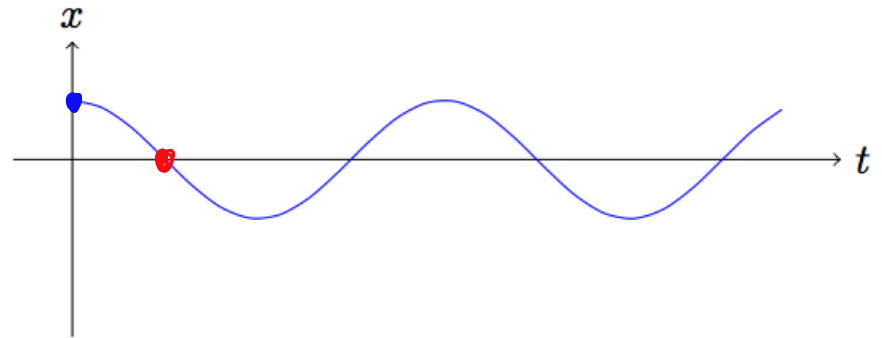
$$\underbrace{\Delta K}_{\text{ENERGIE}} + \underbrace{\Delta U_r}_{\text{MECANIQUE}} = 0$$

$$\underbrace{\left[\frac{1}{2} m v^2 \right]_a^b}_{\Delta K} = \underbrace{\int_a^b -kx \, dx}_{W_r} = - \underbrace{\left[\frac{1}{2} k x^2 \right]_a^b}_{\Delta U_r}$$

↑
**Energie
cinétique**

↑
**Energie
potentielle**

On conserve l'énergie mécanique



$$\underbrace{\left[\frac{1}{2} m v^2 \right]_a^b}_{\Delta K} = \underbrace{\int_a^b -kx \, dx}_{W_r} = - \underbrace{\left[\frac{1}{2} k x^2 \right]_a^b}_{\Delta U_r}$$

**Energie
cinétique**

**Energie
potentielle**

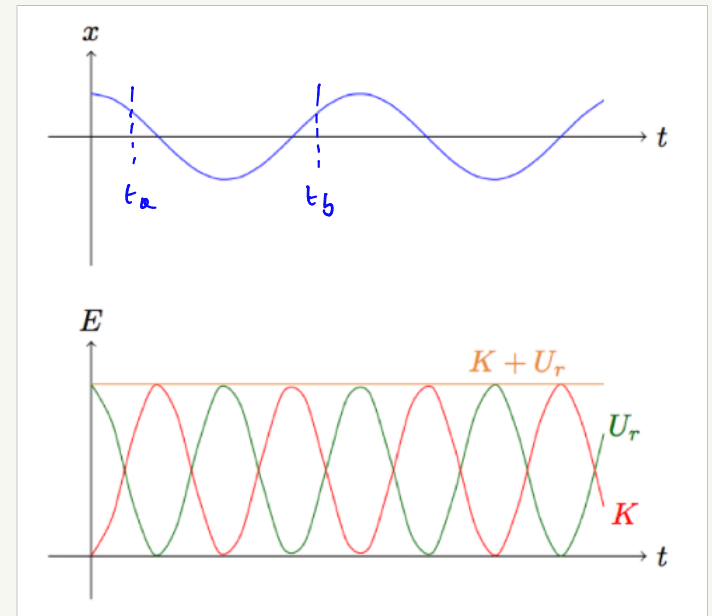
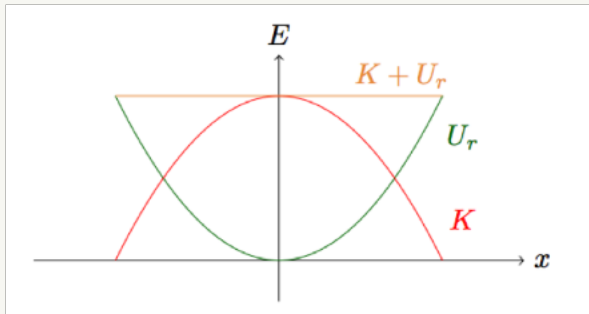
[7]

CONSERVATION DE L'ENERGIE MECANIQUE

$$\Delta K + \Delta U = 0$$

$$W_r = \int_{t_e}^{t_b} \underbrace{-k A \cos(\dots)}_{\left[-\frac{k}{2} A^2 \cos^2(\dots) \right]_{t_e}^{t_b}} \underbrace{\omega A \sin(\dots)}_{x^2} dt$$

$$K + U = \frac{1}{2} k A^2 \underbrace{(\sin^2(\dots) + \cos^2(\dots))}_{=1}$$



$$K = \frac{1}{2} m \omega^2 \sin^2(\dots) A^2$$

$\omega^2 = k/m$

$$= \frac{1}{2} k A^2 \sin^2(\dots)$$

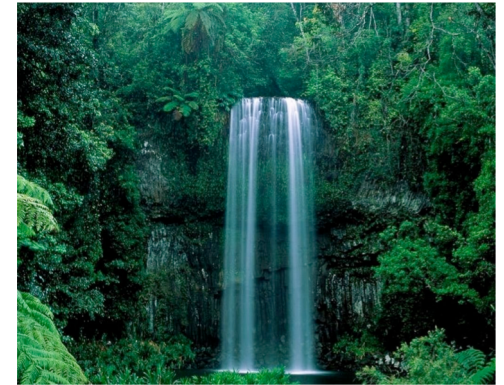
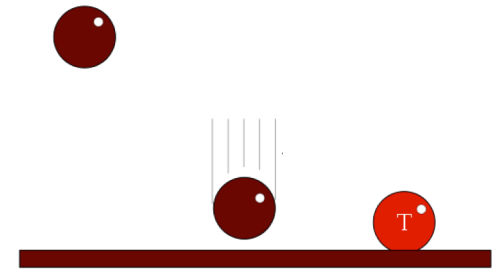
$$U = \frac{1}{2} k \underbrace{A^2 \cos^2(\dots)}_{x^2}$$

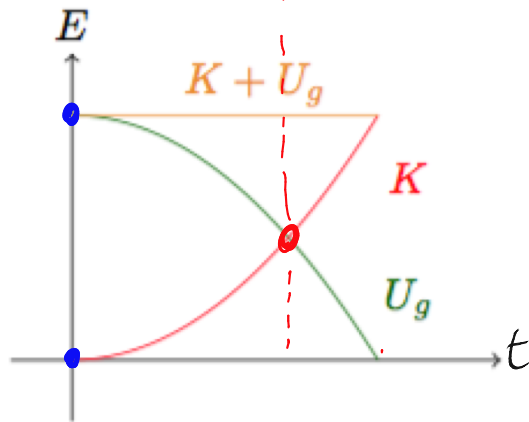
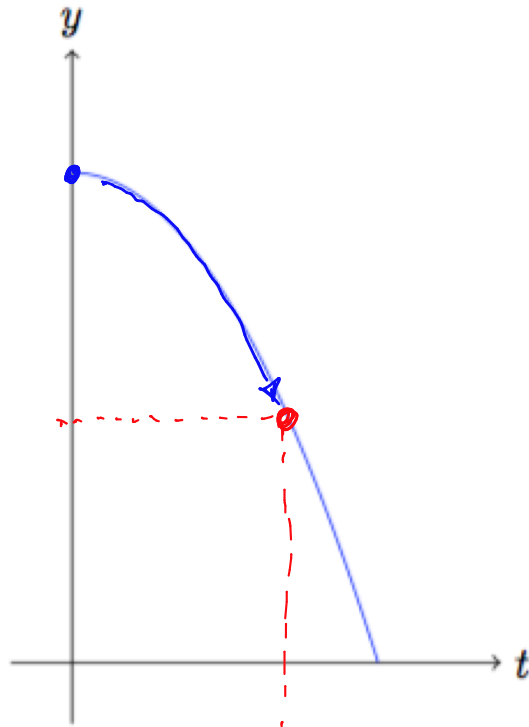
La force de gravité est conservative !

$$\underbrace{\left[\frac{1}{2} m v^2 \right]_a^b}_{\Delta K} = \underbrace{\int_a^b -mg \, dy}_{W_g} = - \underbrace{\left[mg y \right]_a^b}_{\Delta U_g}$$

↑
**Energie
cinétique**

↑
**Energie
potentielle**





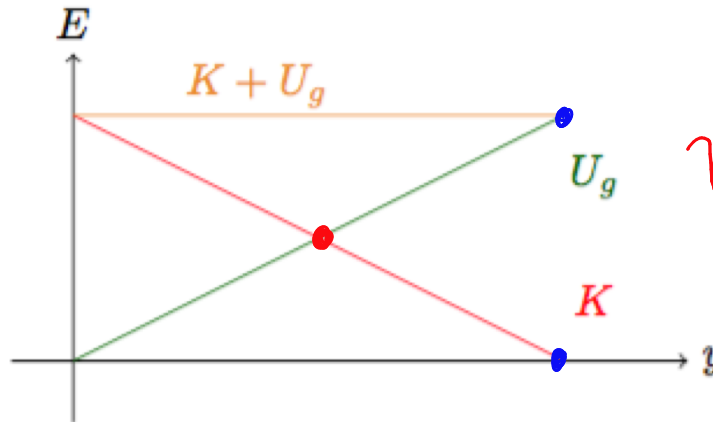
$$\underbrace{\left[\frac{1}{2} m v^2 \right]_a^b}_{\Delta K} = \underbrace{\int_a^b -mg \, dy}_{W_g} = - \underbrace{\left[mg y \right]_a^b}_{\Delta U_g}$$

**Energie
cinétique**

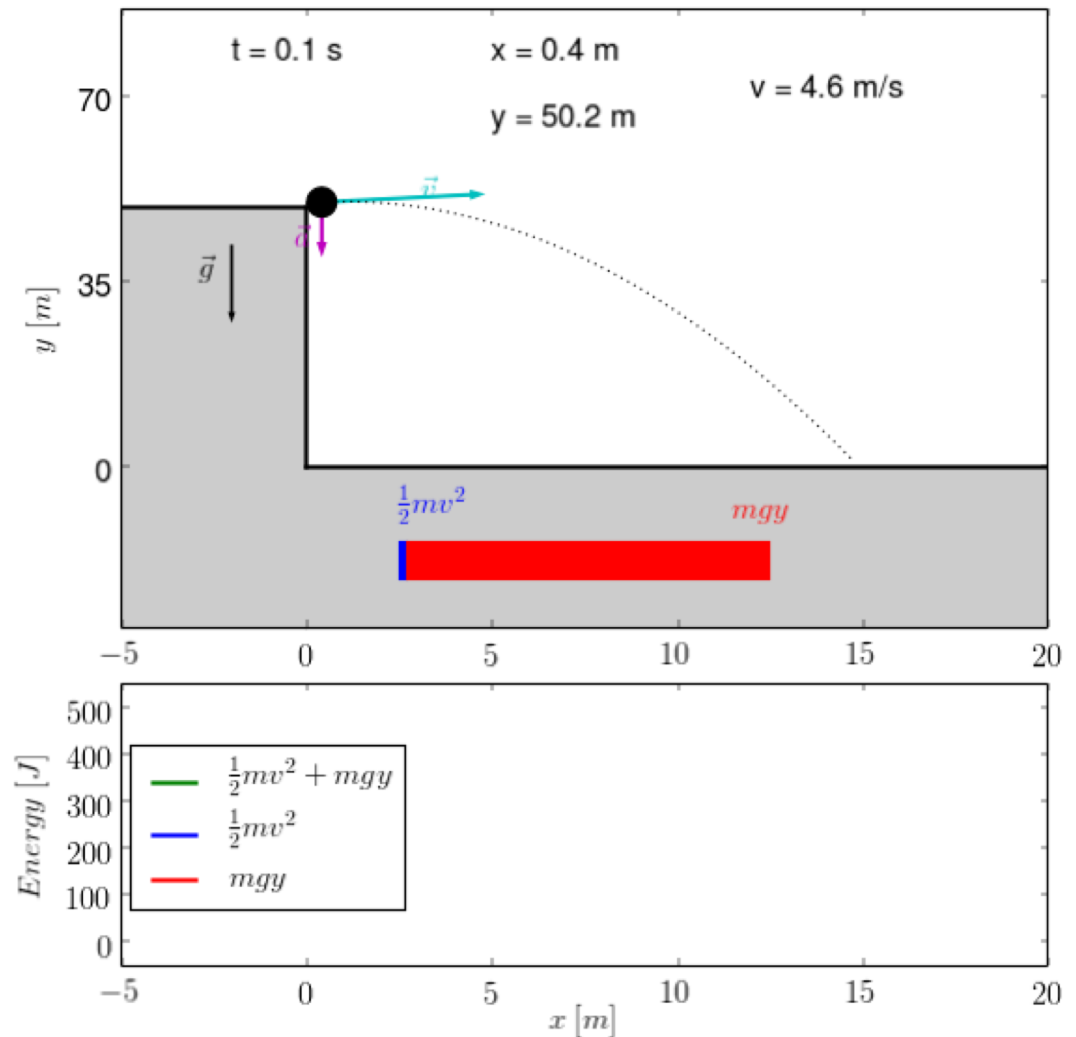
**Energie
potentielle**



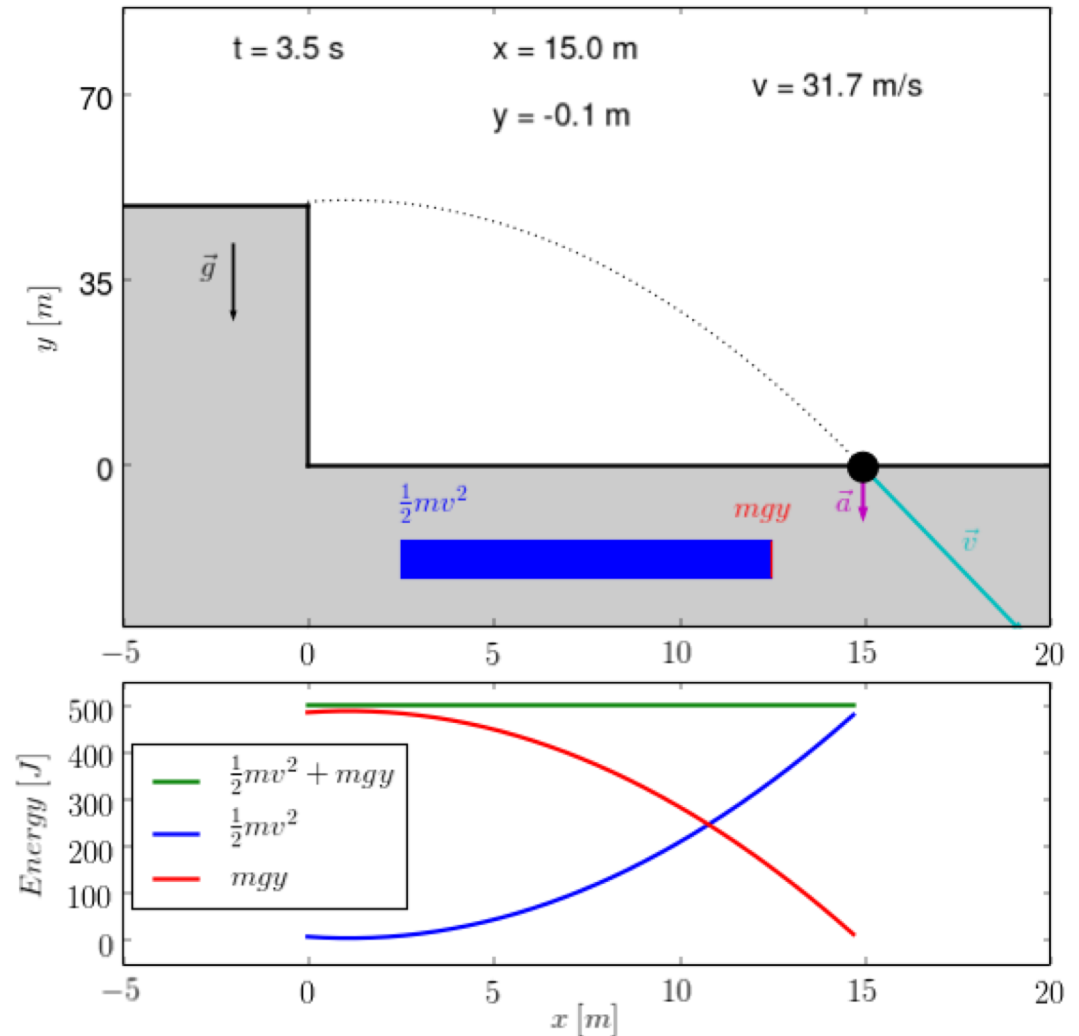
**On conserve
l'énergie
mécanique**



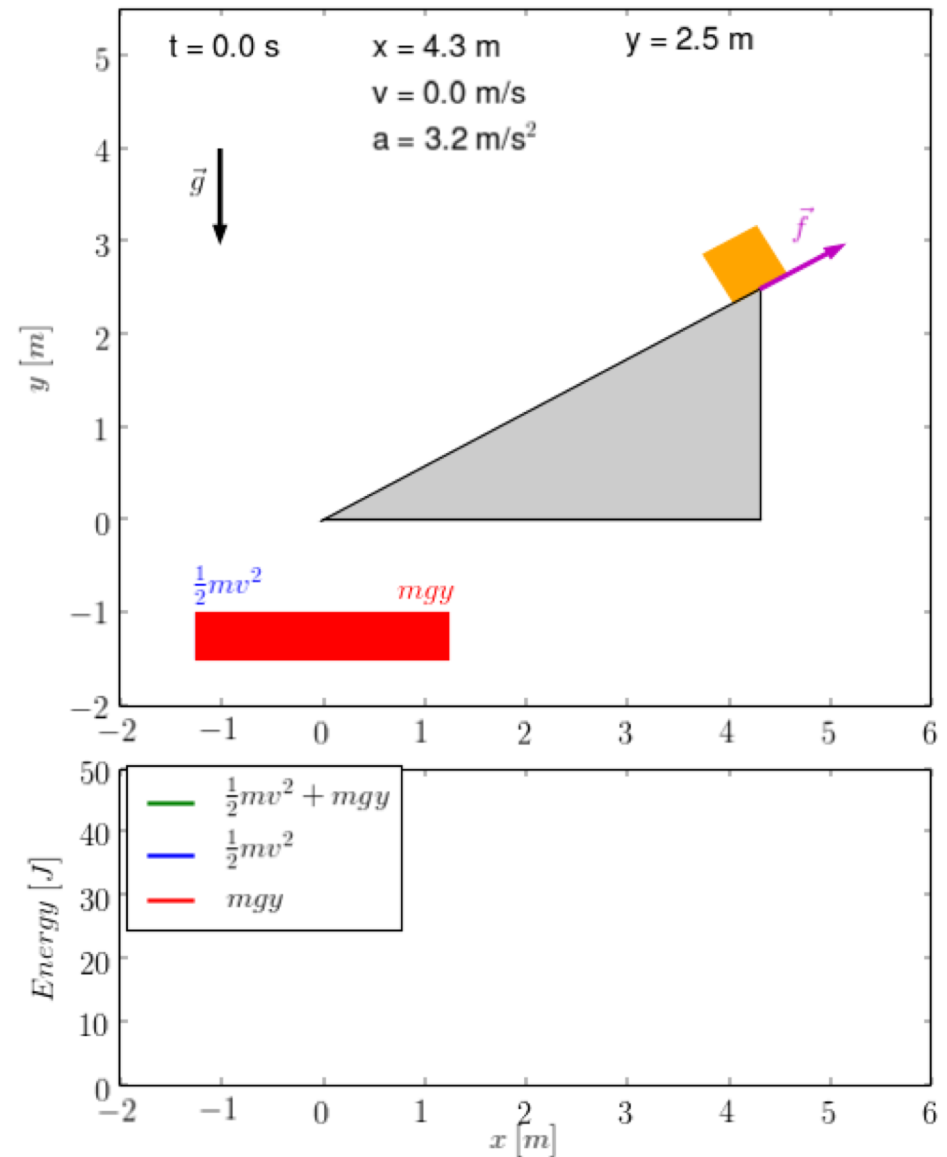
Conservation de l'énergie mécanique

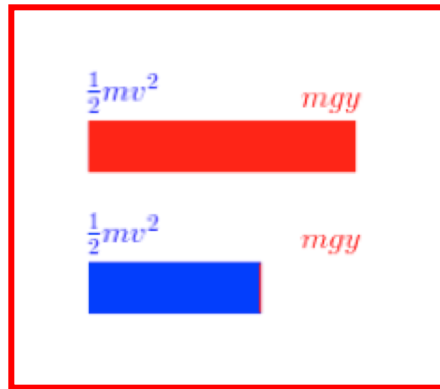


Conservation de l'énergie mécanique

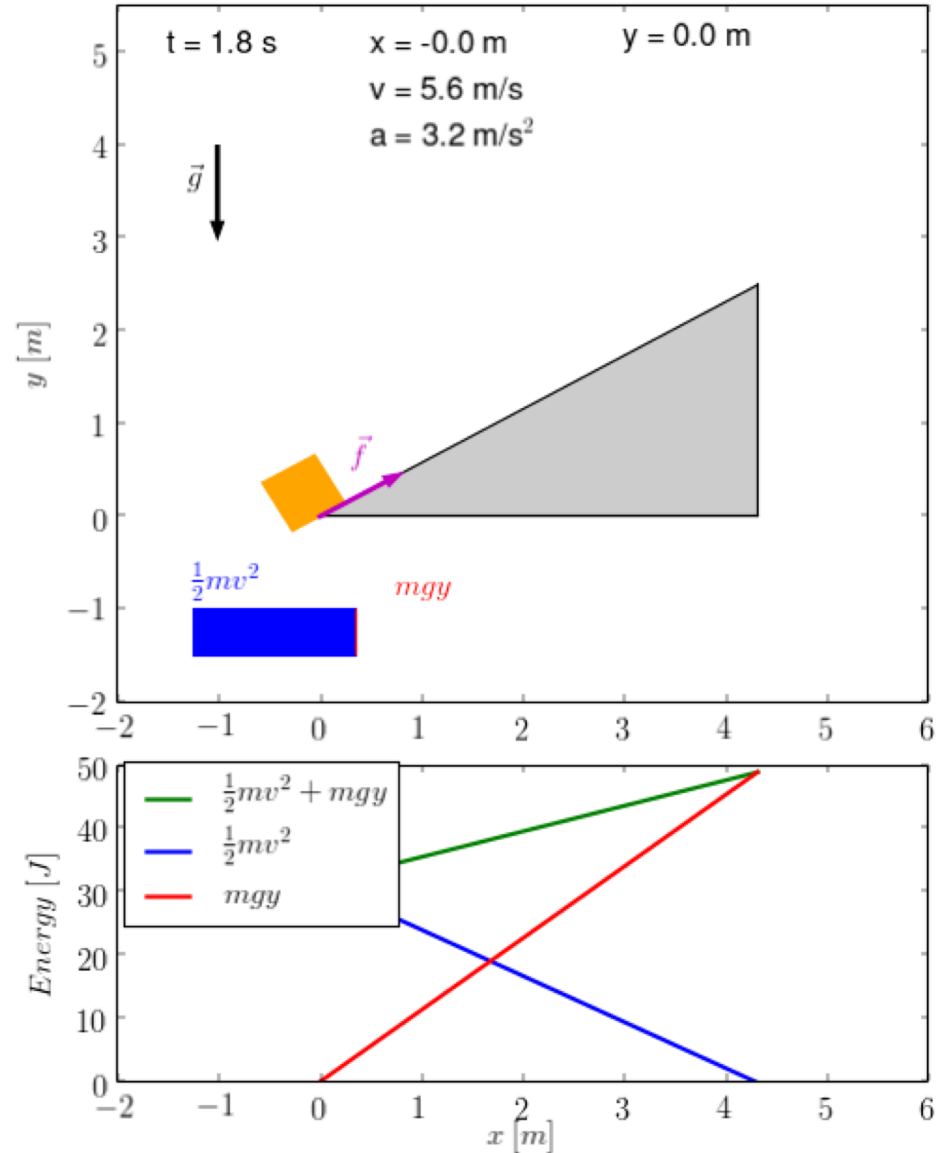


Quid du
frottement ?





Une partie
de l'énergie
est dissipée !



Conservation de l'énergie mécanique



*On conserve l'énergie
mécanique dans les phases aériennes !
La traînée de l'air est négligeable*



**Energie potentielle
du ressort**

**Energie
cinétique**

**Energie potentielle
de gravité**

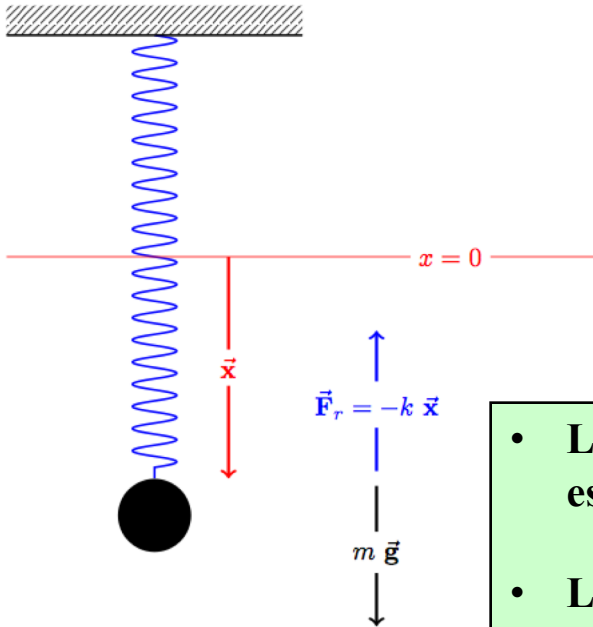


Ici, la trainée
diminue
l'énergie
mécanique



$$0 = \frac{1}{2} \rho C_D A v^2 - mg$$

$$v = \sqrt{\frac{2mg}{\rho C_D A}}$$



- La force de rappel du ressort s'oppose à sa déformation et est proportionnelle à celle-ci : c'est la **loi de Hooke**.
- La force de gravité et la force du ressort sont des forces conservatives : le travail de ces forces correspondent à un **transfert entre énergies potentielle et cinétique**.
- Une particule soumise uniquement à des forces conservatives **conserve son énergie mécanique** qui est la somme des énergie cinétique et potentielle !



Ne pas
oublier !