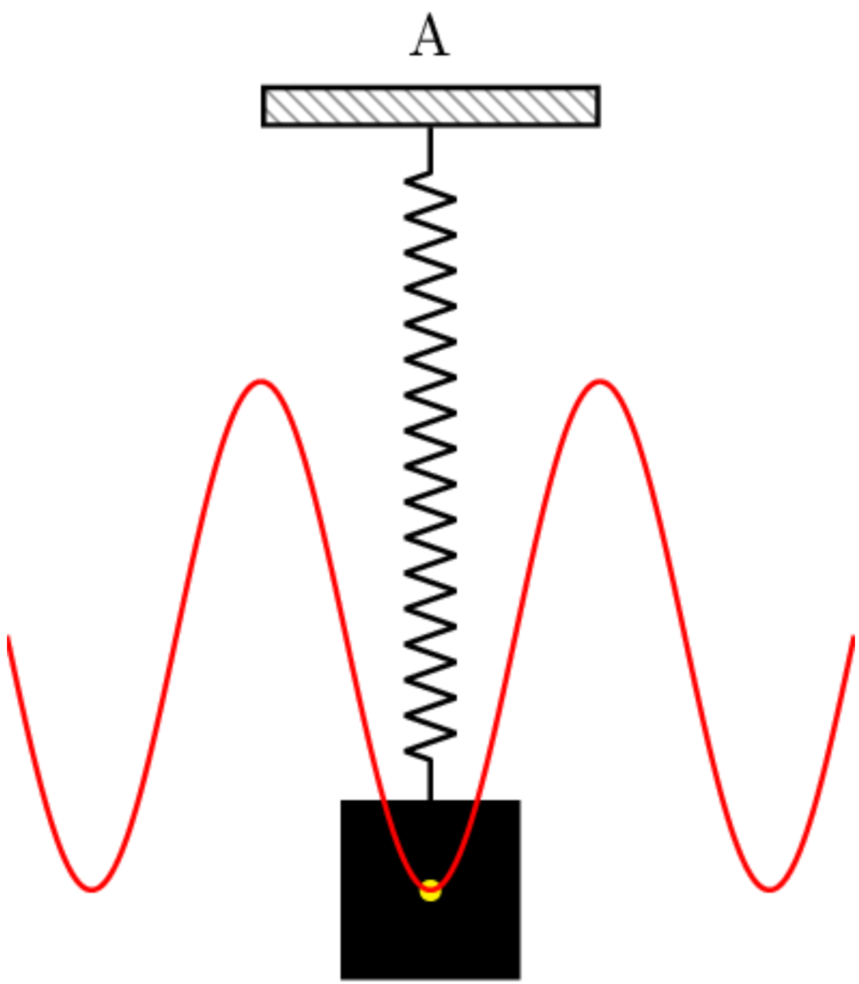
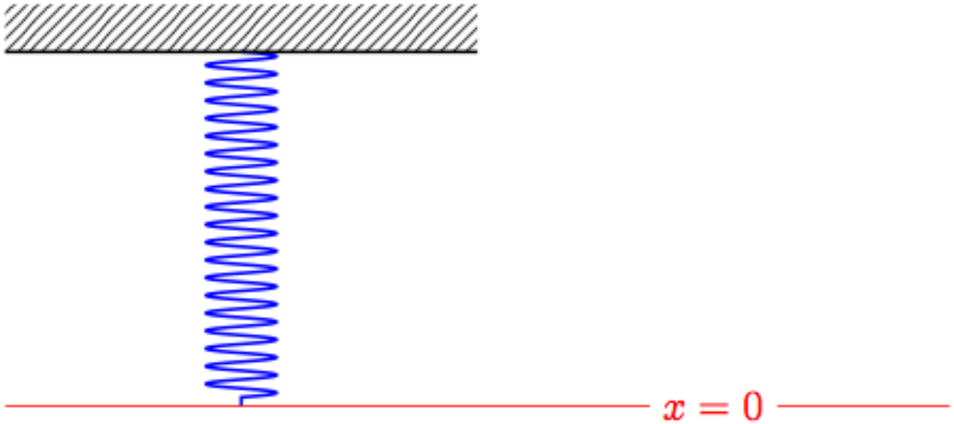
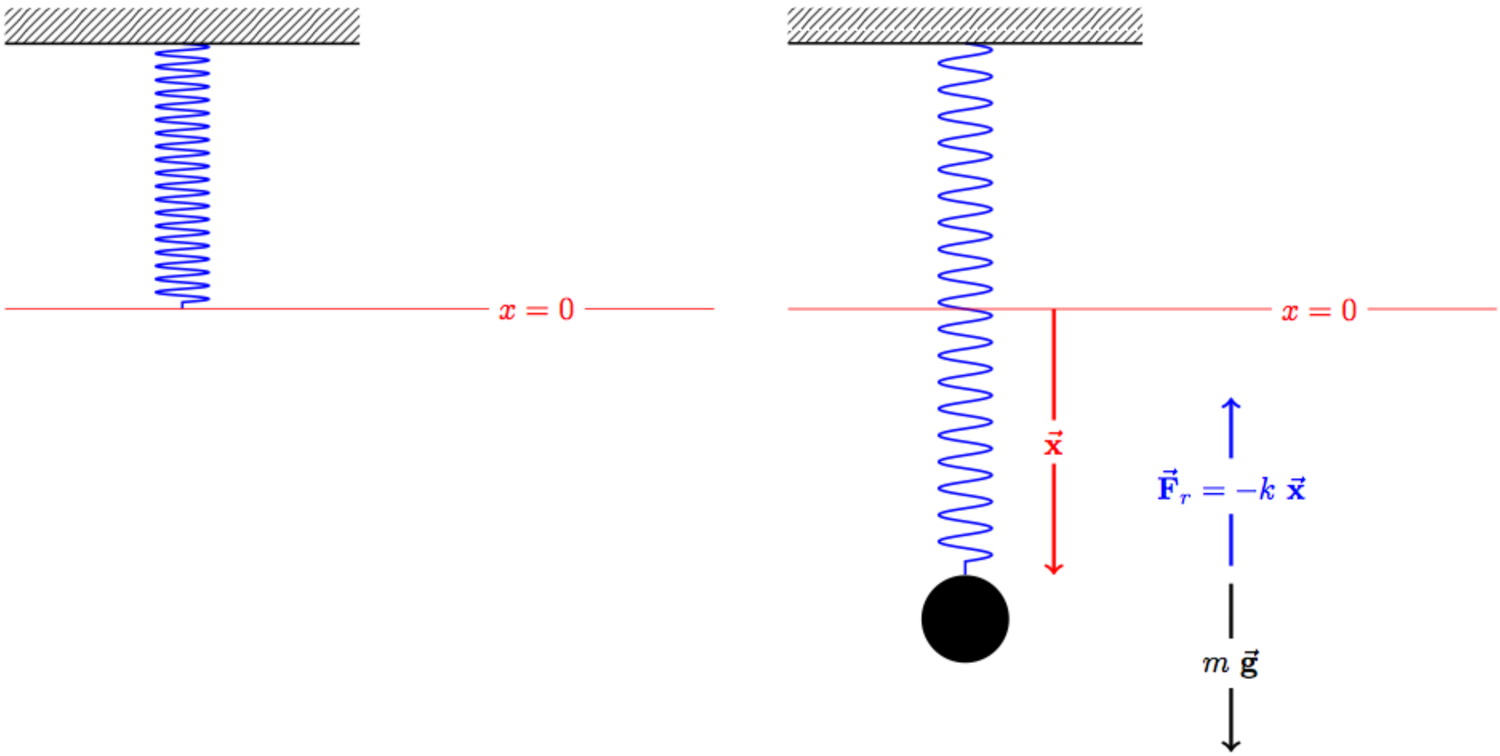


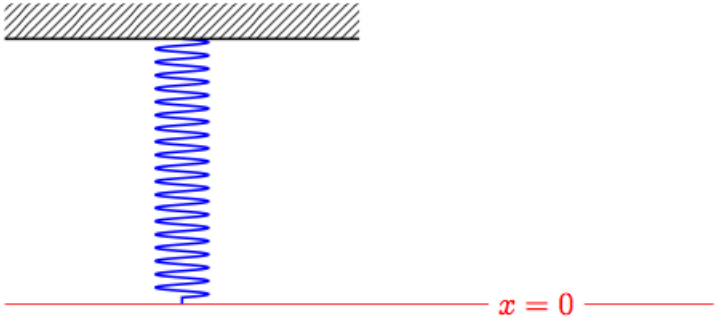

# La force de rappel d'un ressort !




Considérons
un ressort !




Attachons
un poids !




Loi de Hooke

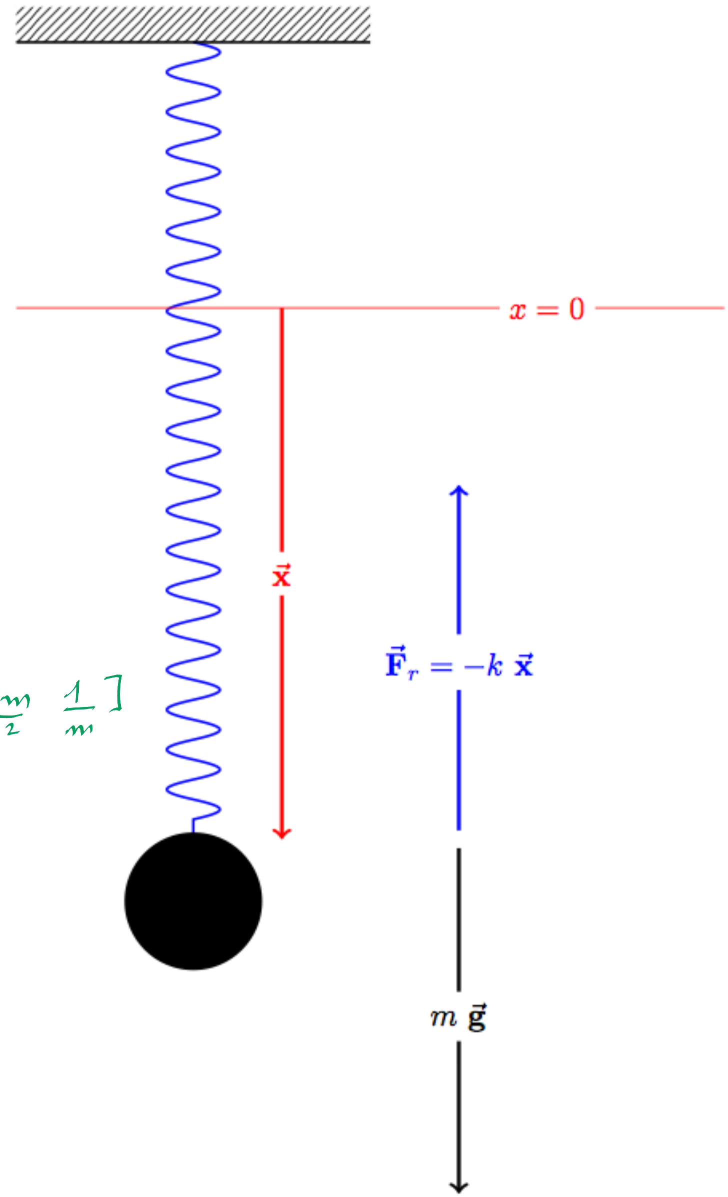
$$\vec{F}_1 = -k\,\vec{x}$$

[N]

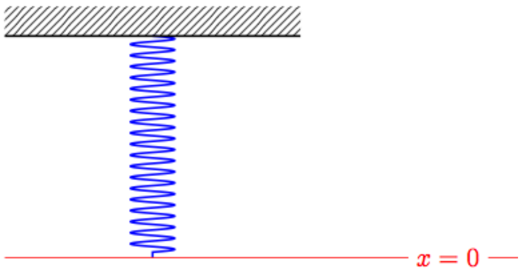
constante du ressort

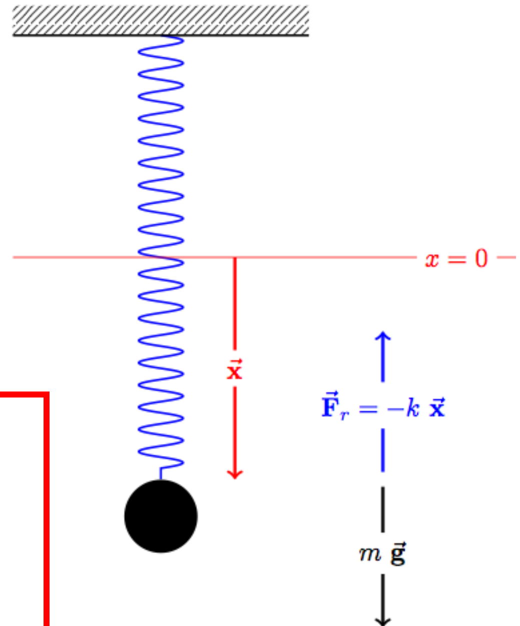
[m]

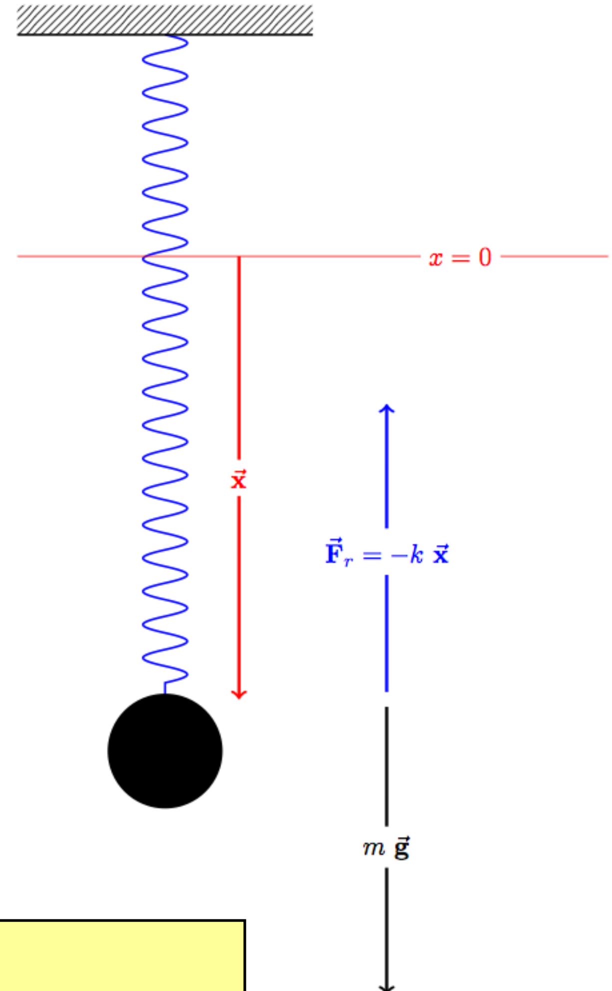
$$[N/m] = \left[\frac{kg\,m}{s^2}\,\frac{1}{m}\right] = \left[\frac{kg}{s^2}\right]$$




# Attachons un truc plus lourd encore !

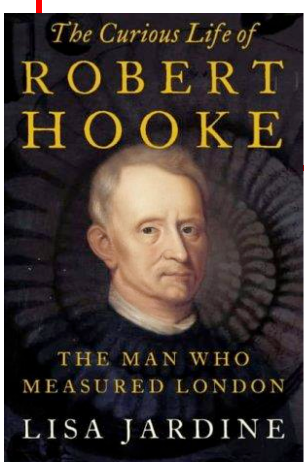


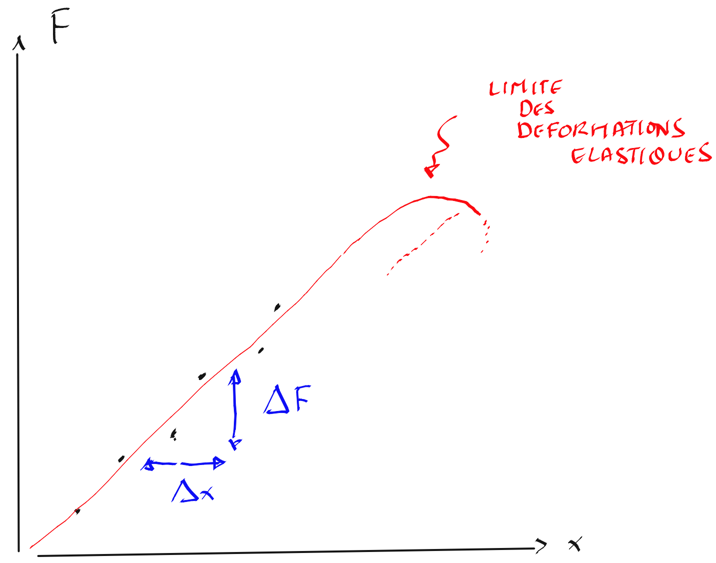




# Loi de Hooke

$$\vec{\mathbf{F}}_r = -k\ \vec{\mathbf{x}}$$



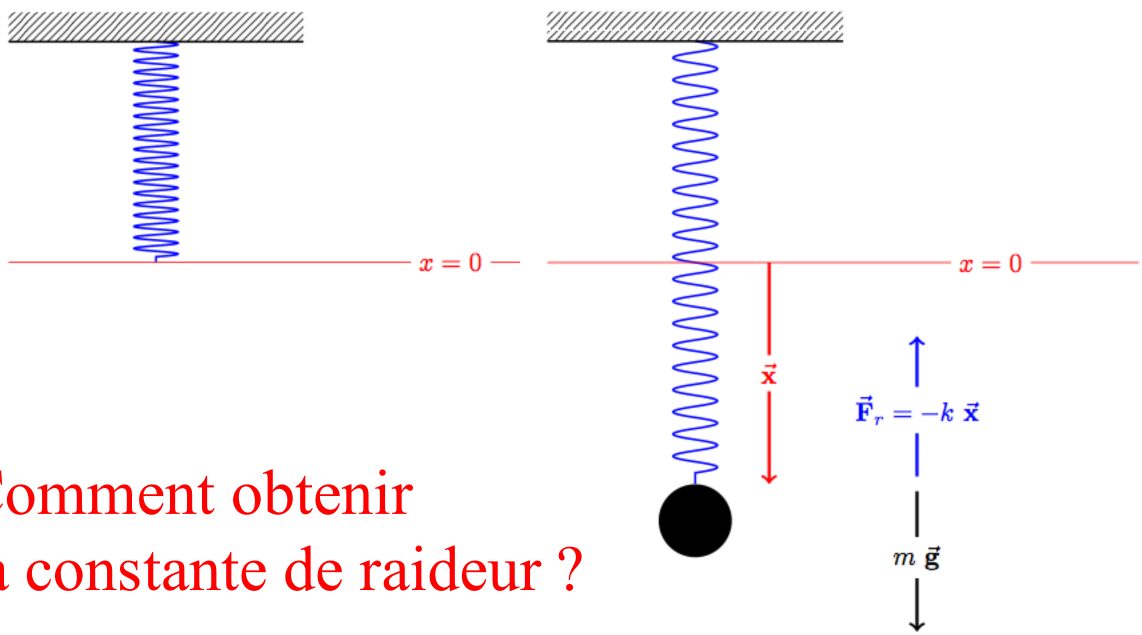

**La force exercée par ressort idéal est donné par la loi de Hooke.**
**La force s'oppose toujours à l'allongement du ressort et est proportionnelle à cet allongement.**
**Le facteur k est appelé constante de raideur du ressort.**




$$k = \frac{\Delta F}{\Delta x}$$

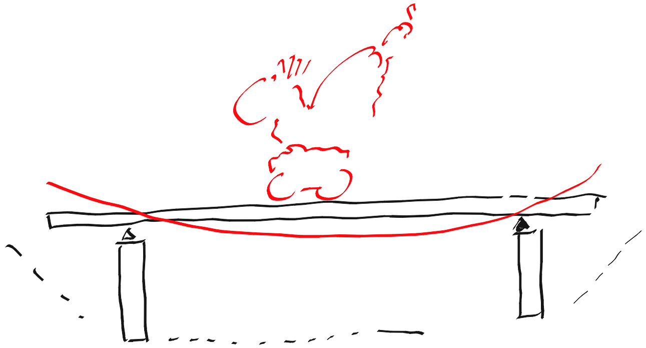
$$\vec{F} = -k\,\vec{x}$$

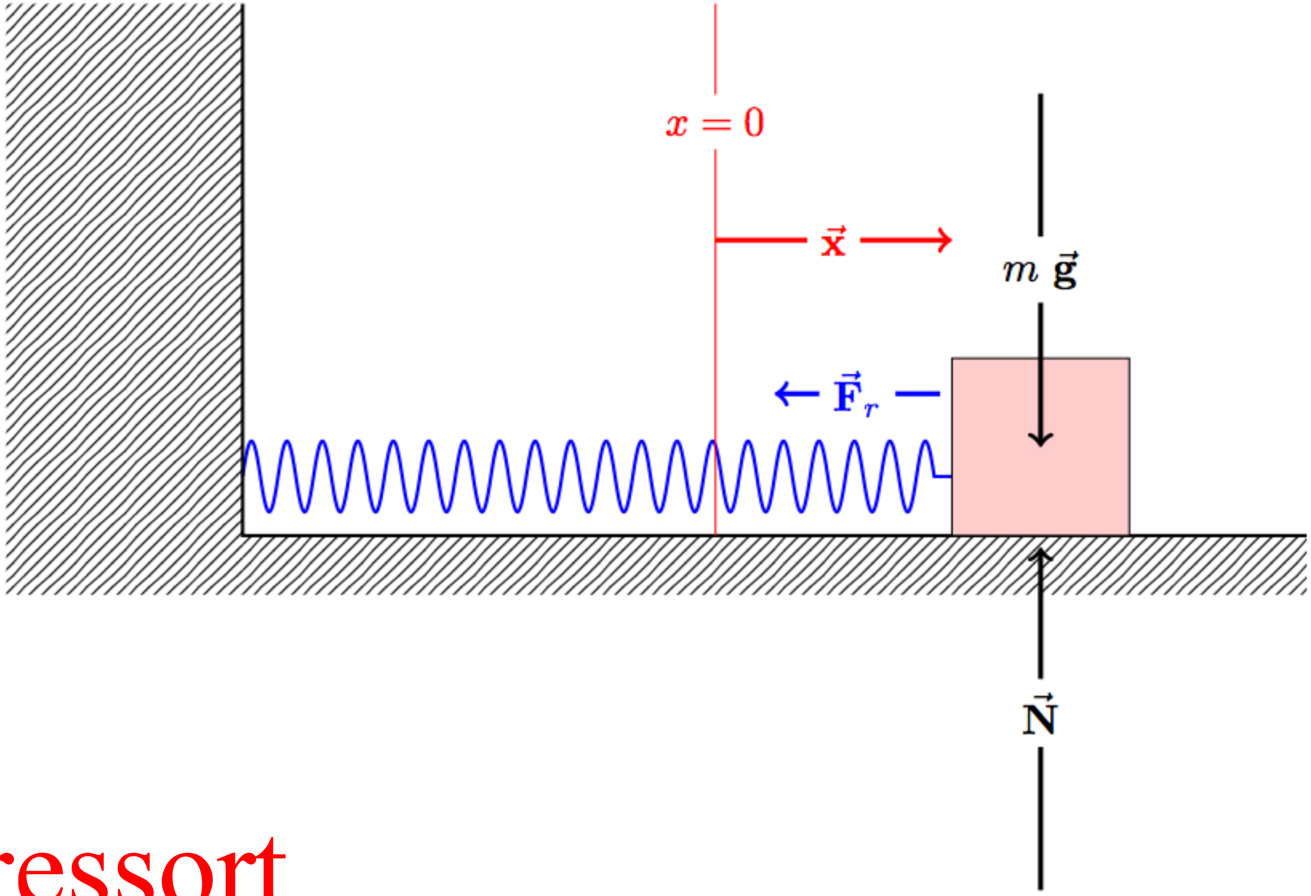
LOI DE COMPORTEMENT




Comment obtenir
la constante de raideur ?

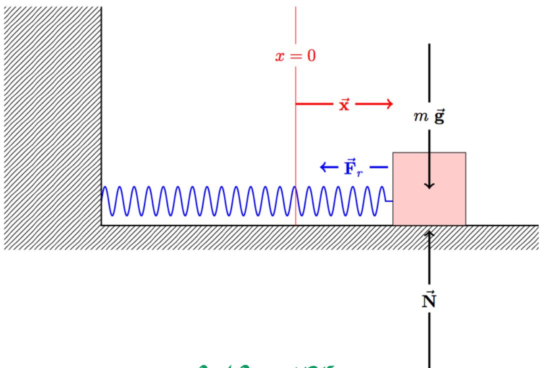
Le bois, l’acier, la brique, les os, le béton armé des ponts autoroutiers sont des matériaux élastiques !






Un ressort
cela oscille !

# Un ressort, cela oscille !



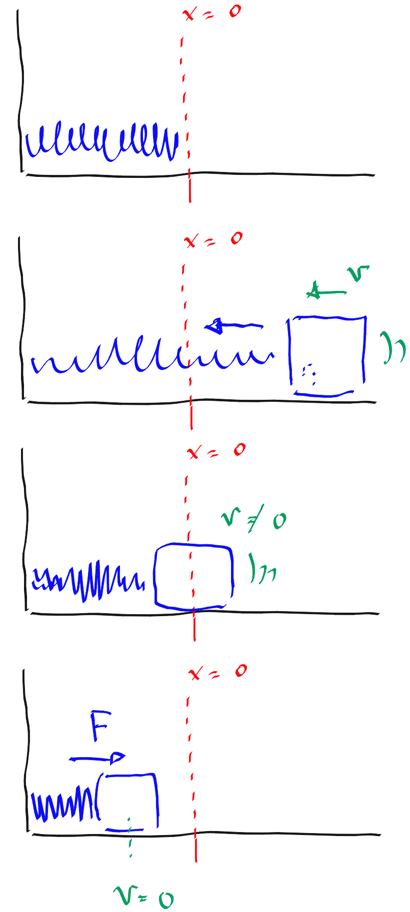

$$m\,a(t) = -k\,x(t)$$

$$m\,x''(t) = -k\,x(t)$$

EQUATION DIFFERENTIELLE ORDINAIRE

$$\boxed{x''(t) = -\frac{k}{m}\,x(t)}$$

sin(....)
cos(...)




AMPLITUDE [m]

$$x(t) = A\cos(\omega t + \varphi)$$

FREQUENCE/VITESSE ANGULAIRE [radian/s]

ANGLE DE PHASE [radian]

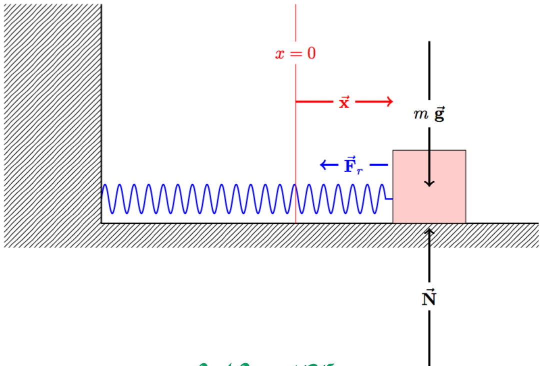
$$x'(t) = -A\,\omega\,\sin(\omega t + \varphi)$$

$$x''(t) = -A\,\omega^2\cos(\omega t + \varphi) = -\omega^2\,x(t)$$

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

FREQUENCE / VITESSE ANGULAIRE

# C’est un mouvement harmonique !




EQUATION DIFFERENTIELLE ORDINAIRE

$$x''(t) = -\frac{k}{m}\, x(t)$$

FREQUENCE

$$f = \frac{1}{T}$$

$$\left[\frac{1}{s}\right] = [\mathrm{Hz}]$$

AMPLITUDE [m]

$$x(t) = A \cos(\omega t + \varphi)$$

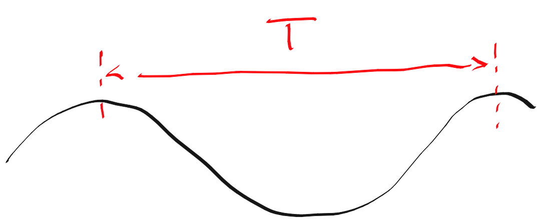
FREQUENCE/VITESSE ANGULAIRE [radian/s]

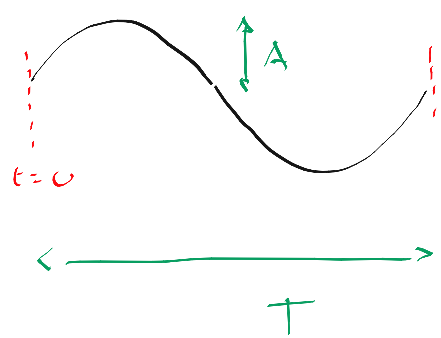
ANGLE DE PHASE [radian]

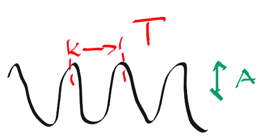
PERIODE

$$T = \frac{2\pi}{\omega} \quad [s]$$

$$= 2\pi \sqrt{\frac{m}{k}}$$



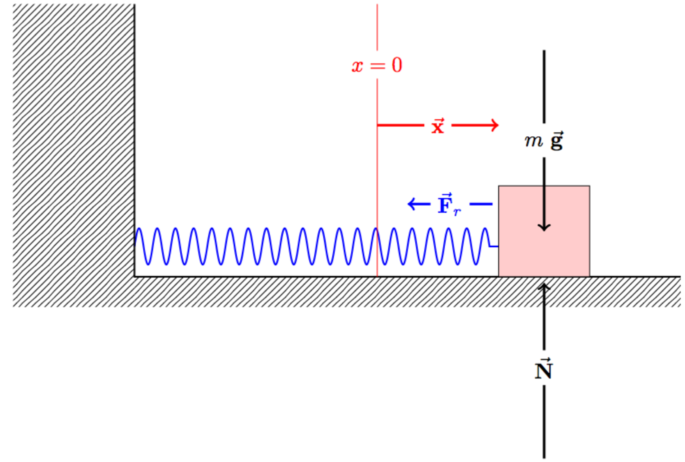



# Le mouvement harmonique !

$$m\ a(t) = -k\ x(t)$$

$$m\ \frac{d^2x}{dt^2}(t) = -k\ x(t)$$

$$\frac{d^2x}{dt^2}(t) = -\frac{k}{m}\ x(t)$$



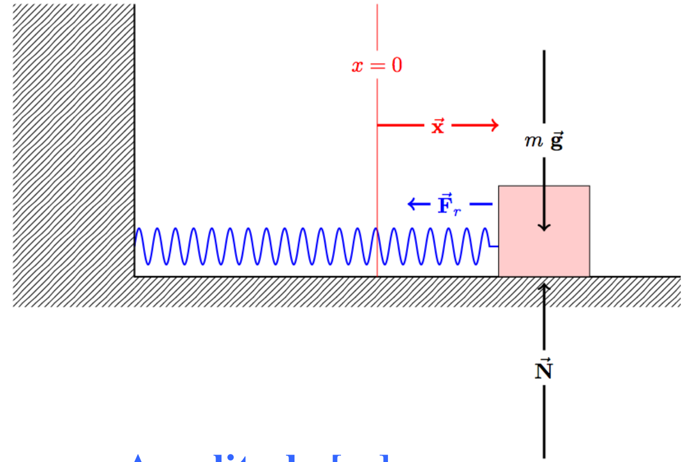

$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

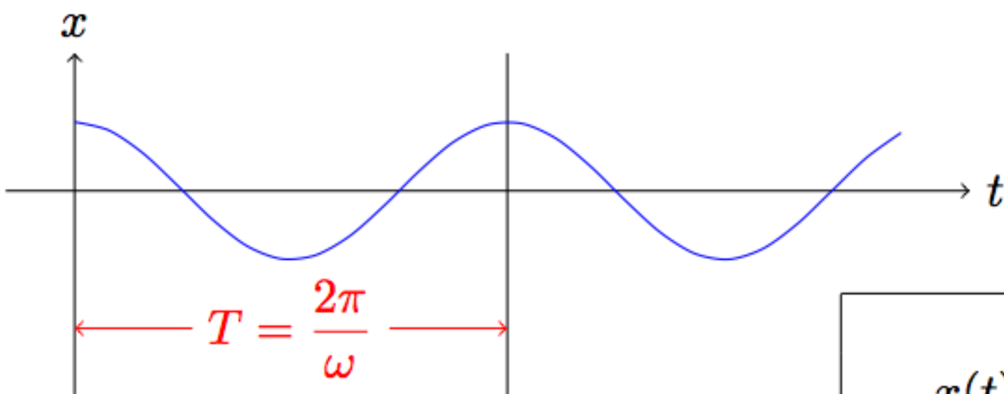
$$x(t) = A\ \cos(\omega t + \phi)$$

$$x'(t) = -\omega\ A\ \sin(\omega t + \phi)$$

$$x''(t) = -\omega^2\ A\ \cos(\omega t + \phi) = -\omega^2\ x(t)$$

# Le mouvement harmonique !




Amplitude [m]

$$x(t) = A \cos ( \omega \, t + \phi )$$

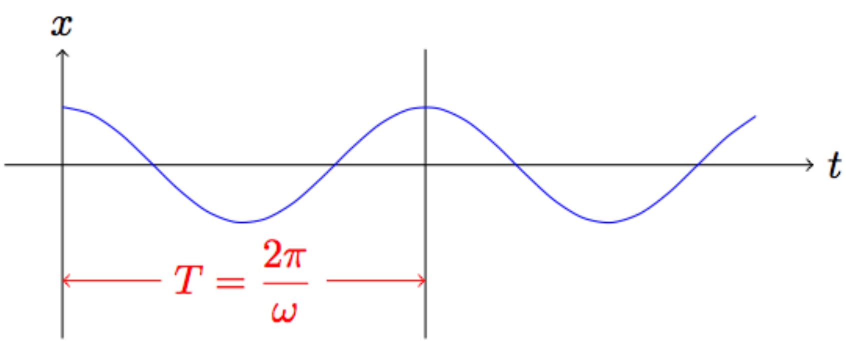
Fréquence angulaire
[radian / sec]

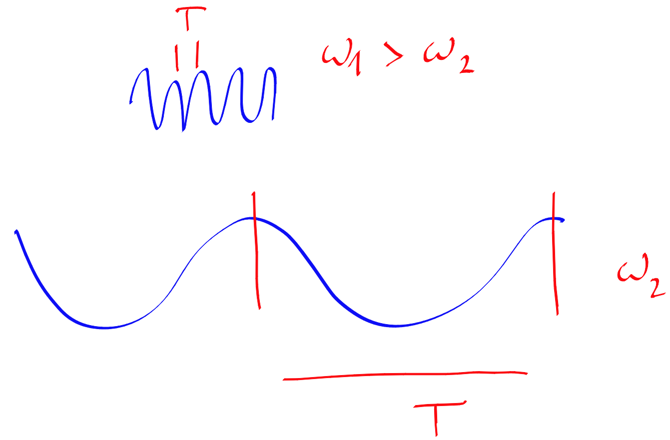
Angle de phase
[radian]

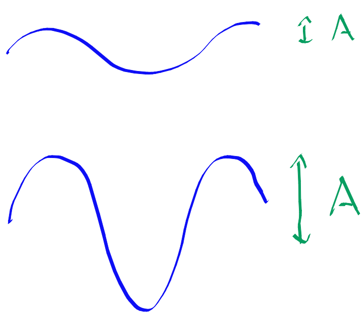
$$\omega = \sqrt{\frac{k}{m}}$$

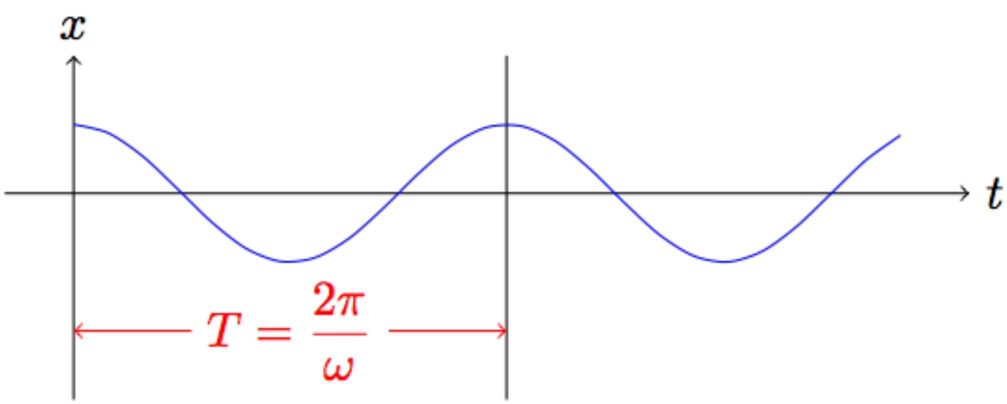
***L'amplitude et l'angle de phase sont déterminés par les conditions initiales de position et de vitesse !***

# Amplitude
# et vitesse angulaire






$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$$

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

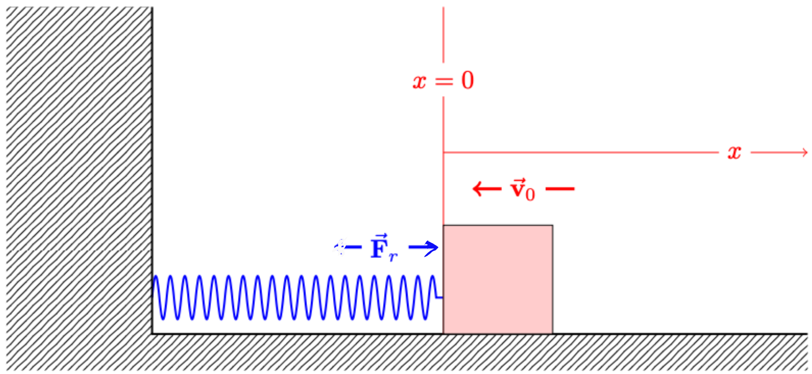
$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

Période : T

Fréquence : f

Fréquence angulaire : ω

# Un petit exemple :-)




$$x(t) = A \cos(\omega t + \varphi)$$

QUESTION QUE VALENT $A$, $\omega$ ET $\varphi$ ?

$t = 0$
$x(0) = 0$
$x'(0) = -3$

$k = 10$ N/m
$m = 0.1$ kg

$\boxed{1}$

$$\omega = \sqrt{\frac{\overset{10}{k}}{\underset{0.1}{m}}} = 10 \text{ rad/s}$$

$$\sqrt{10 \, \frac{10}{1}} = \sqrt{100} = 10$$

$\boxed{2}$

$$x(0) = 0$$

$$\underbrace{\omega t}_{=0} + \varphi = \pi/2$$

$$\hookrightarrow \varphi = \pi/2$$

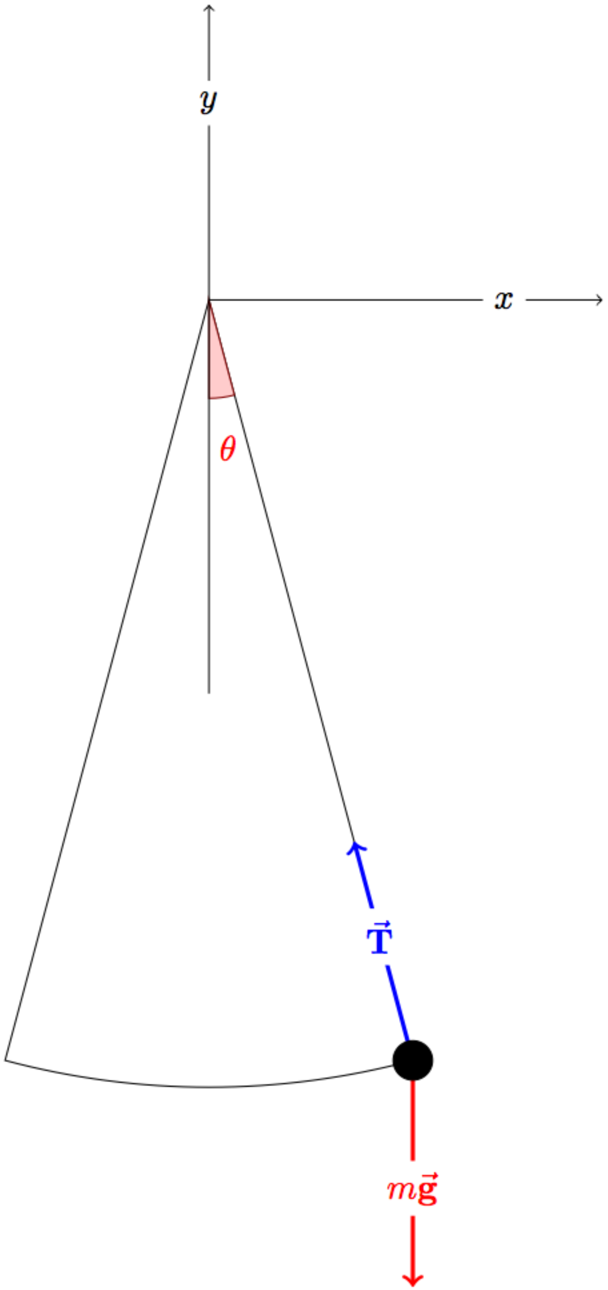
$\boxed{3}$

$$x'(0) = -3$$

$$-A \underset{10}{\omega} \underbrace{\sin(\underbrace{\omega t}_{=0} + \pi/2)}_{=1} = -3$$

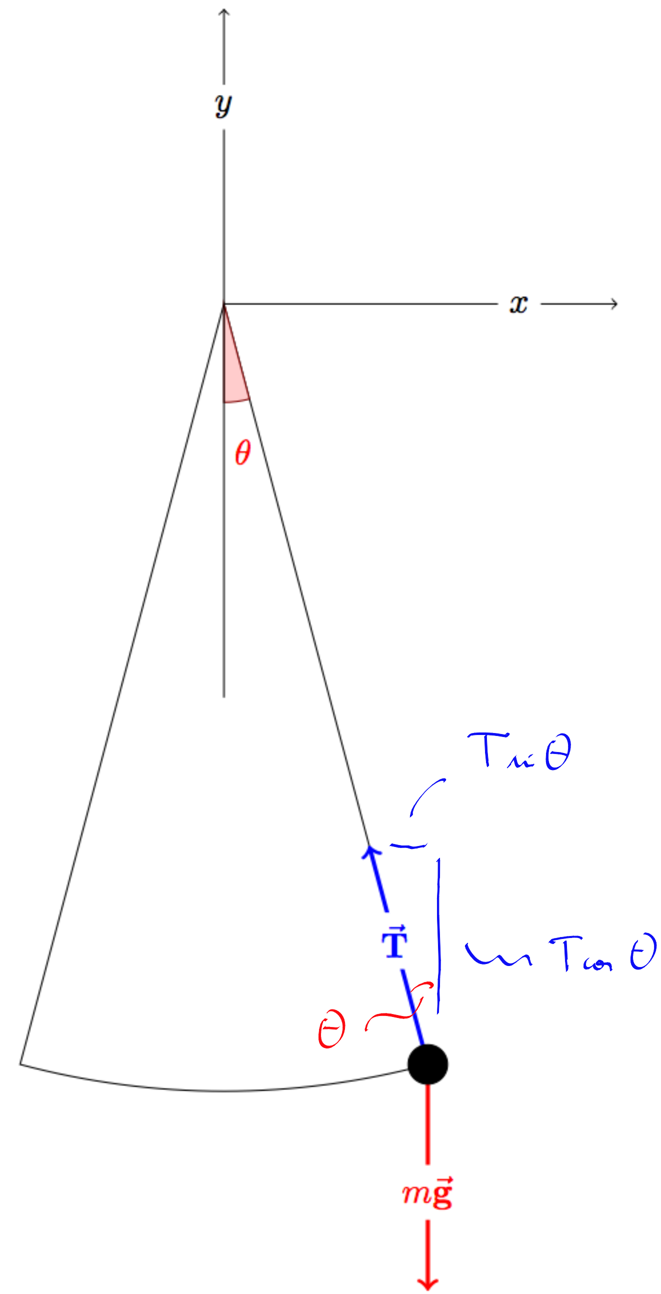
$$\hookrightarrow A = 0.3$$

Un pendule
cela oscille !



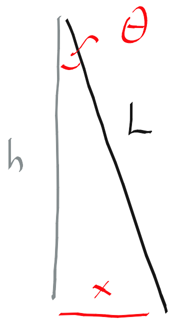
# Dynamique du pendule !

$$\begin{cases} m\,\dfrac{d^2x}{dt^2}(t) & = & -T\sin(\theta(t)) \\[2ex] m\,\dfrac{d^2y}{dt^2}(t) & = & T\cos(\theta(t)) - mg \end{cases}$$



# Small angles approximation

$\theta \ll 1$




$$m\,x''(t) = -T\,\overbrace{\sin(\theta(t))}^{x/L}$$

$$\underbrace{m\,y''(t)}_{=0} = T\,\underbrace{\cos(\theta(t))}_{\simeq 1} - mg$$

$$mg = T$$

$\boxed{1}$

$\cos(\theta) \simeq 1$

| | |
|---|---|
| $5°$ | 0,996 |
| $10°$ | 0,985 |

$\boxed{2}$

$$\sin(\theta) \simeq \frac{x}{L}$$

$\boxed{3}$

$y(t) \simeq L$

$\rightarrow$ $y''(t) = 0$, $y'(t) = 0$

$$\cancel{m}\,x''(t) = -\frac{\cancel{m}\,g}{L}\,x(t)$$

$$\boxed{x''(t) = -\frac{g}{L}\,x(t)}$$

$$\omega = \sqrt{\frac{g}{L}}$$

$g$: $[m/s^2]$

$L$: $[m]$

$\omega$: $[1/s]$

# Pour des petits angles
# - pas si petits d'ailleurs -
# on peut simplifier !

$$\begin{cases} m\,\dfrac{d^2x}{dt^2}(t) & = & -T\,\overbrace{\sin(\theta(t))}^{=\frac{x(t)}{L}} \\ m\,\underbrace{\dfrac{d^2y}{dt^2}(t)}_{\approx 0} & = & T\,\underbrace{\cos(\theta(t))}_{\approx 1} - mg \end{cases}$$

$$\boxed{\frac{d^2x}{dt^2}(t) \quad = \quad -\frac{g}{L}\,x(t)}$$

$$\boxed{T \quad = \quad 2\pi\sqrt{\frac{L}{g}}}$$

$$\frac{d^2x}{dt^2}(t) = -\frac{g}{L}\,x(t)$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Le pendule

$$\frac{d^2x}{dt^2}(t) = -\frac{k}{m}\,x(t)$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Le ressort

# La force de rappel du ressort est conservative !

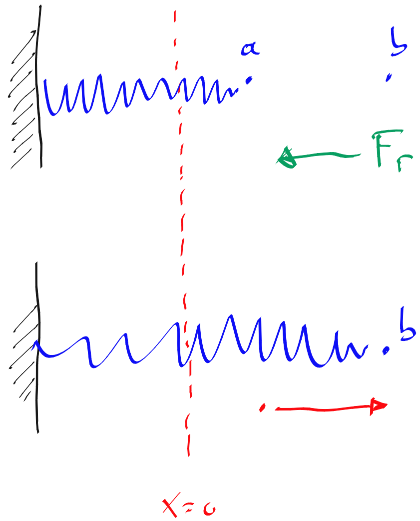
$\frac{d}{dx}\left[\frac{1}{2}kx^2\right] = kx$

$$\underbrace{\left[\frac{1}{2}\,mv^2\right]_a^b}_{\Delta K} = \underbrace{\int_a^b -kx\;dx}_{W_r} = -\underbrace{\left[\frac{1}{2}\,kx^2\right]_a^b}_{\Delta U_r}$$

**Energie cinétique**

**Energie potentielle** $U_r = \frac{1}{2}kx^2$

# Energie potentielle du ressort




$$W = \int_a^b -kx\,dx = \left[-\frac{kx^2}{2}\right]_a^b$$

$$= -\underbrace{\left(\underbrace{\left[\frac{kx_b^2}{2}\right]}_{\text{ENERGIE POTENTIELLE EN } b} - \underbrace{\left[\frac{kx_a^2}{2}\right]}_{\text{ENERGIE POTENTIELLE EN } a}\right)}_{-\Delta U_r}$$

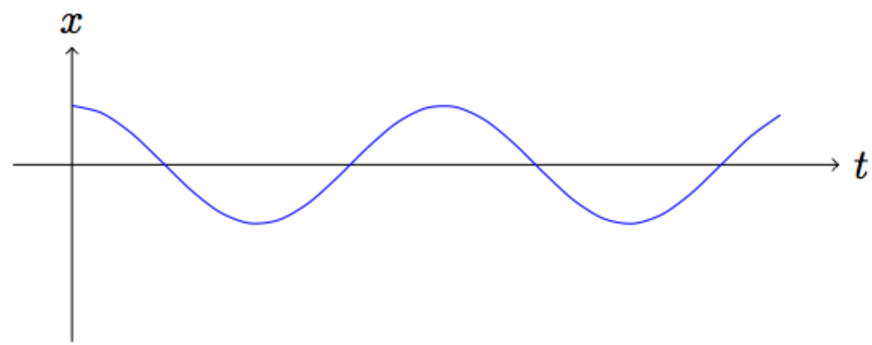
$$U_r = \frac{kx^2}{2}$$

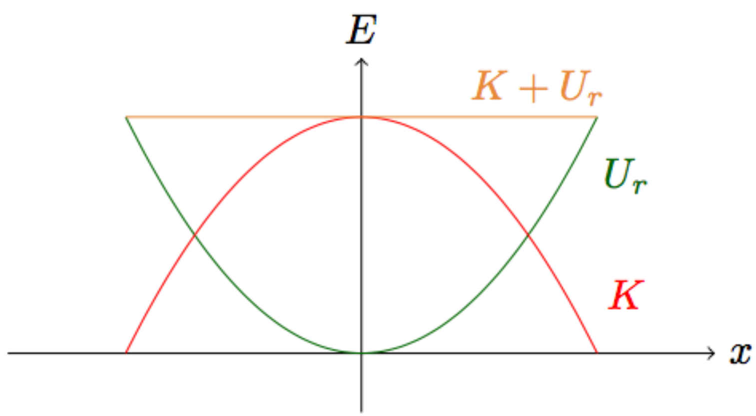
$$\Delta K = W = -\Delta U_r$$

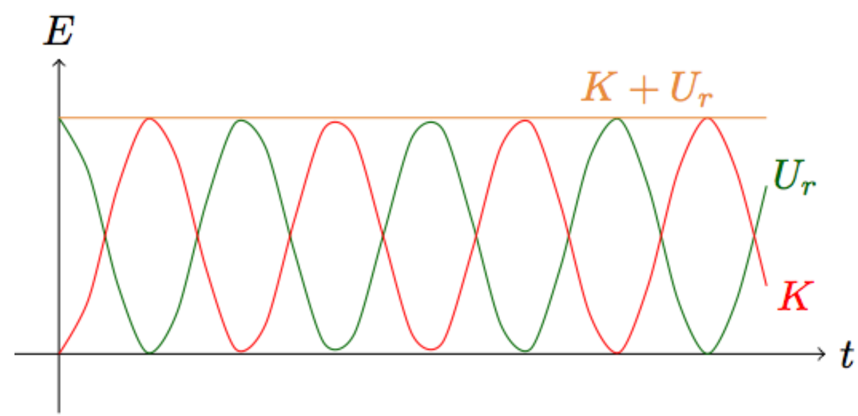
$$\Delta K + \Delta U_r = 0$$

$$\boxed{\frac{mv^2}{2}} \rightleftarrows \boxed{\frac{kx^2}{2}}$$

# On conserve l'énergie mécanique



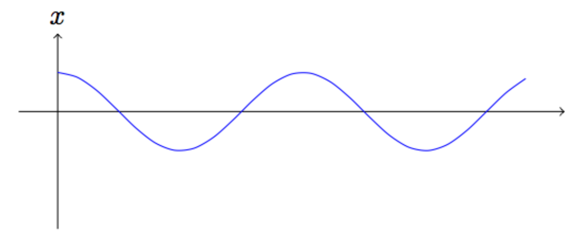

$$\underbrace{\left[\frac{1}{2}\,mv^2\right]_a^b}_{\Delta K} = \underbrace{\int_a^b -kx\;dx}_{W_r} = -\underbrace{\left[\frac{1}{2}\,kx^2\right]_a^b}_{\Delta U_r}$$

**Energie cinétique**

**Energie potentielle**

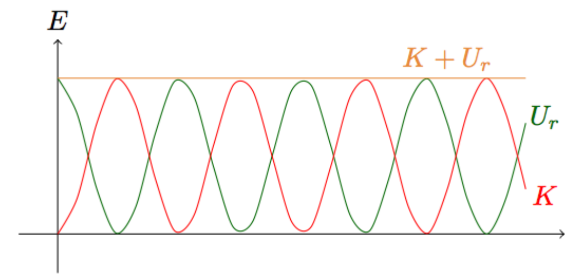
# Conservation de l’énergie mécanique

$$\Delta K + \Delta U_r = 0$$




$$W = \int_{t_a}^{t_b} \underbrace{\underbrace{k A \cos(\dots)}_{F} \underbrace{\omega A \sin(\dots)}_{v} \, dt}$$

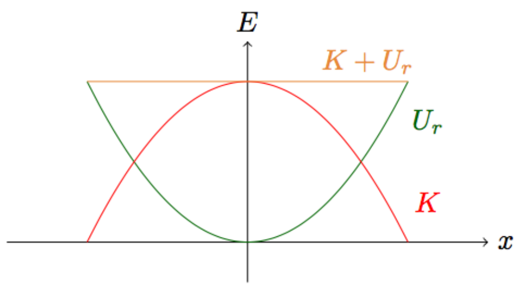
$$\left[ -\frac{k}{2} \underbrace{A^2 \cos^2(\ )}_{x^2} \right]_{t_a}^{t_b}$$




$$K = \frac{1}{2} m \omega^2 A^2 \sin^2(\dots)$$

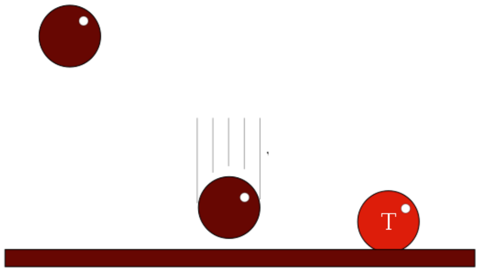
$$U = \frac{1}{2} k A^2 \cos^2(\dots)$$

$$\omega^2 = \frac{k}{m}$$




$$K + U = \frac{1}{2} A^2 k \big( \underbrace{\sin^2(\dots) + \cos^2(\dots)}_{=1} \big) = \text{cst}$$

# La force de gravité est conservative !

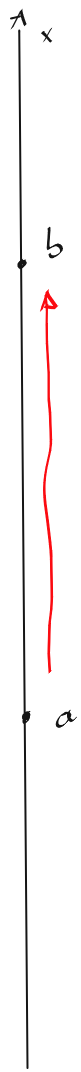



$$\underbrace{\left[\tfrac{1}{2}\; mv^2\right]_a^b}_{\Delta K} = \underbrace{\int_a^b -mg\; dy}_{W_g} = \underbrace{-\left[\; mg\; y\right]_a^b}_{\Delta U_g}$$

**Energie cinétique**

**Energie potentielle**

# Energie potentielle de gravité




$\downarrow mg$

ENERGIE POTENTIELLE DE GRAVITE

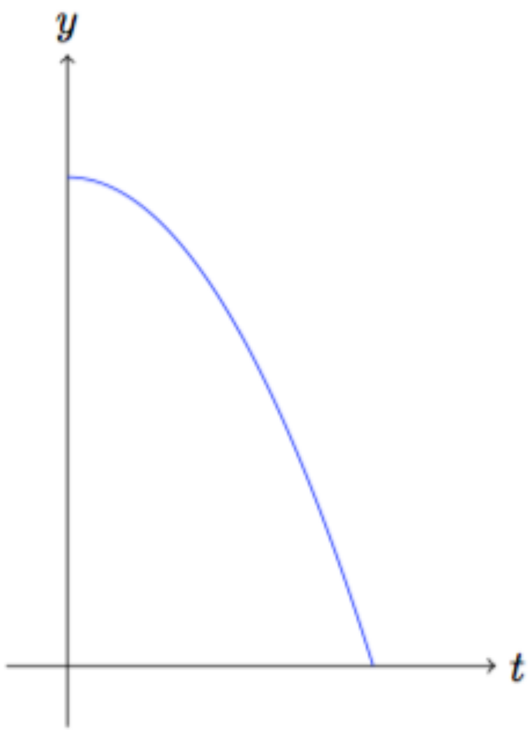
$$U_g = mgx$$

$$W_g = \int_a^b -mg\,dx = -\left[mgx\right]_a^b$$

$$= -\Big(\underbrace{[mgx_b]}_{\text{ENERGIE POTENTIELLE EN } b} - \underbrace{[mgx_a]}_{\text{ENERGIE POTENTIELLE EN } a}\Big)$$

$$\Delta \frac{mv^2}{2} = W_g = -\Delta mgx$$

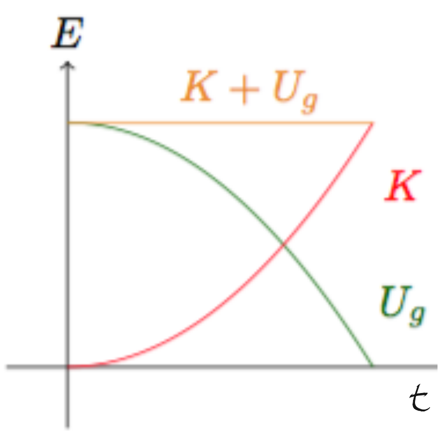
$$\boxed{\frac{1}{2}mv^2} \;\underbrace{\rightleftarrows}_{W_g}\; \boxed{mgx}$$

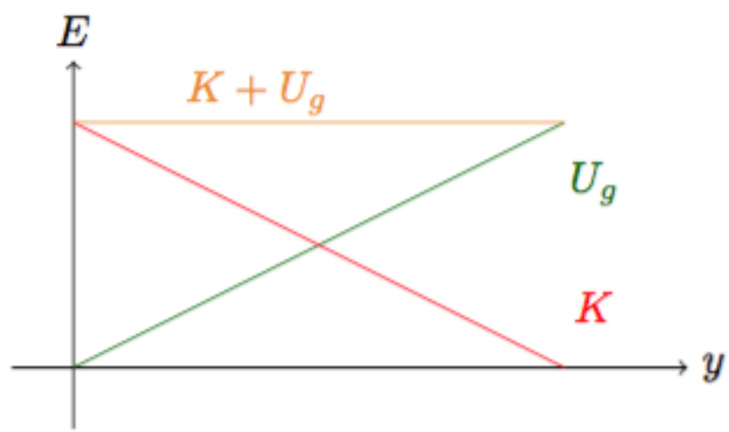



$$\underbrace{\left[\tfrac{1}{2}\, mv^2\right]_a^b}_{\Delta K} = \underbrace{\int_a^b -mg\; dy}_{W_g} = -\underbrace{\left[\, mg\; y\right]_a^b}_{\Delta U_g}$$

**Energie cinétique**

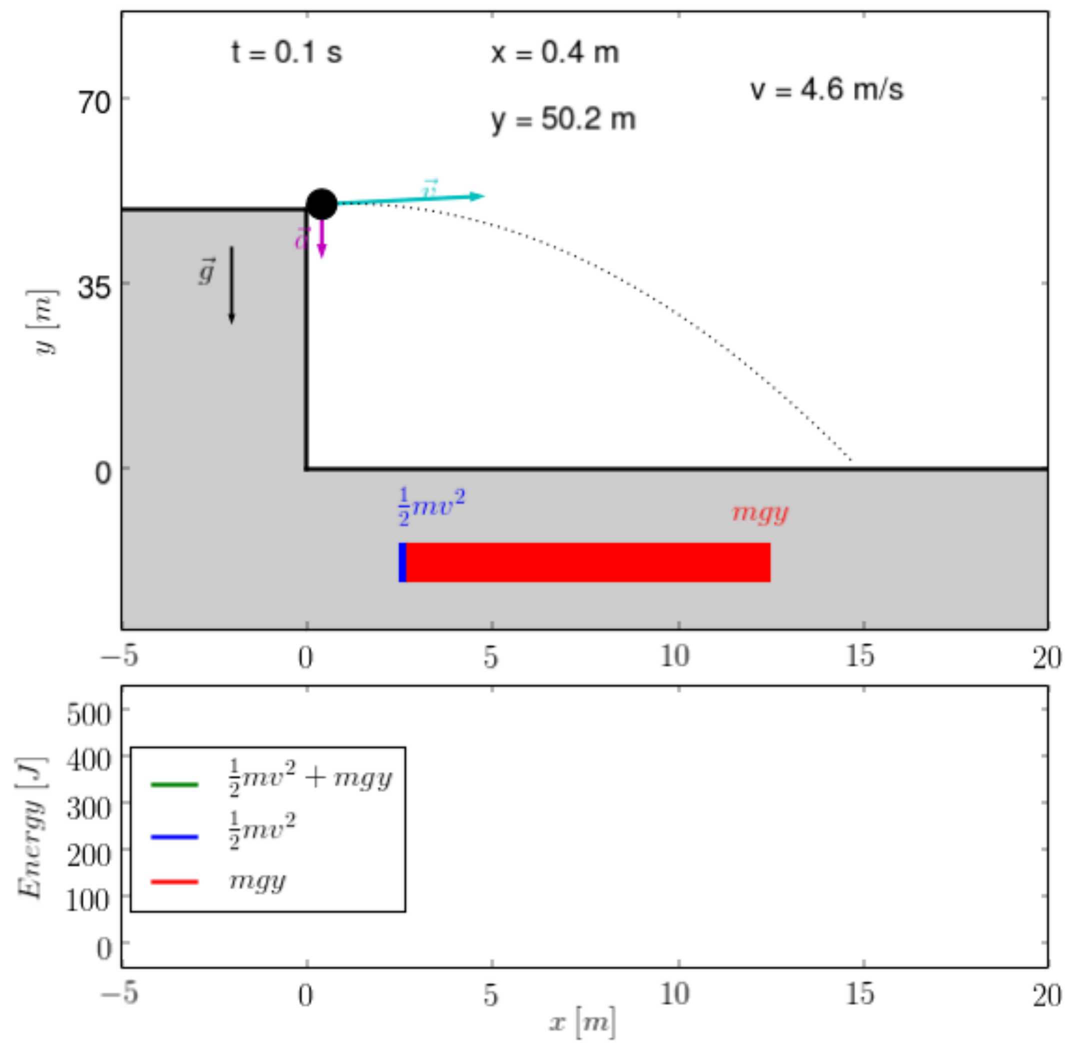
**Energie potentielle**



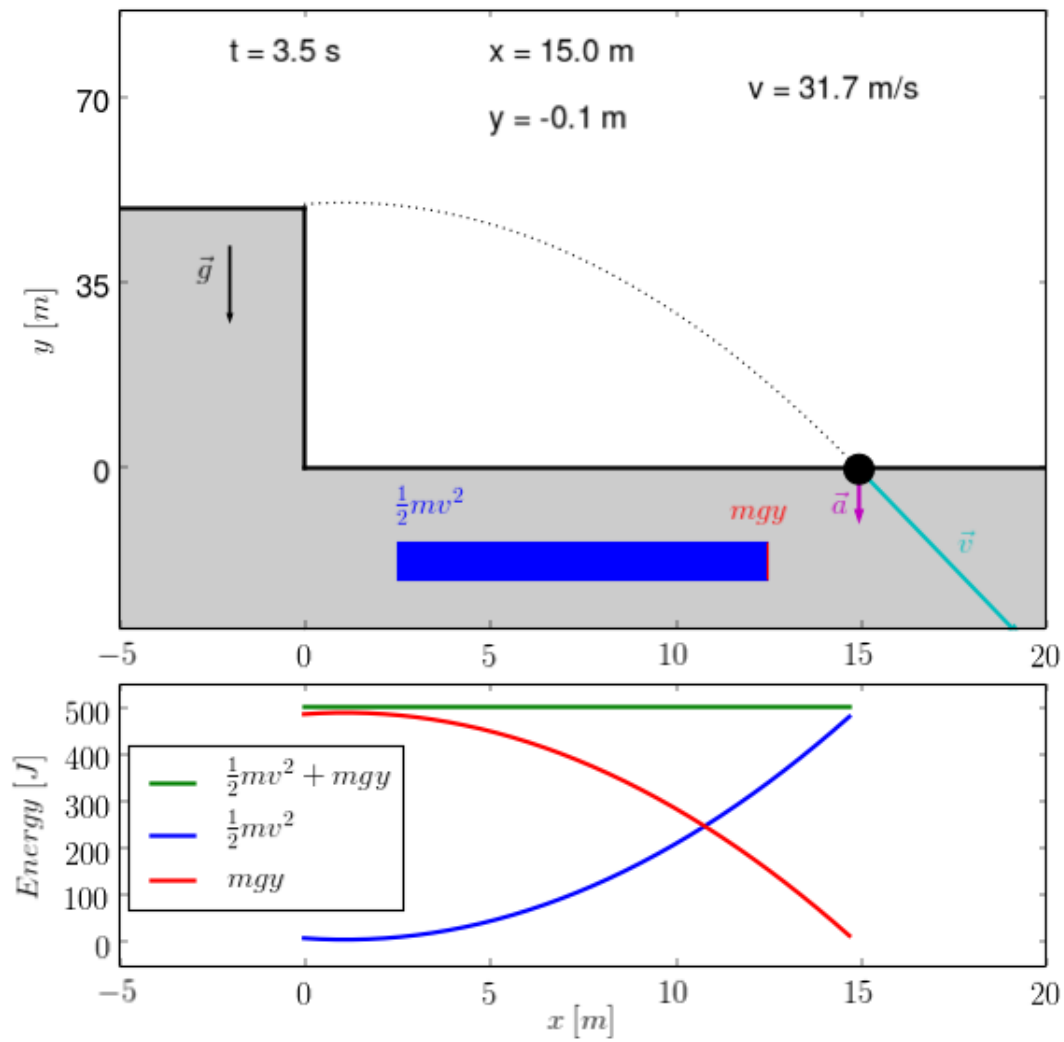




# On conserve l’énergie mécanique

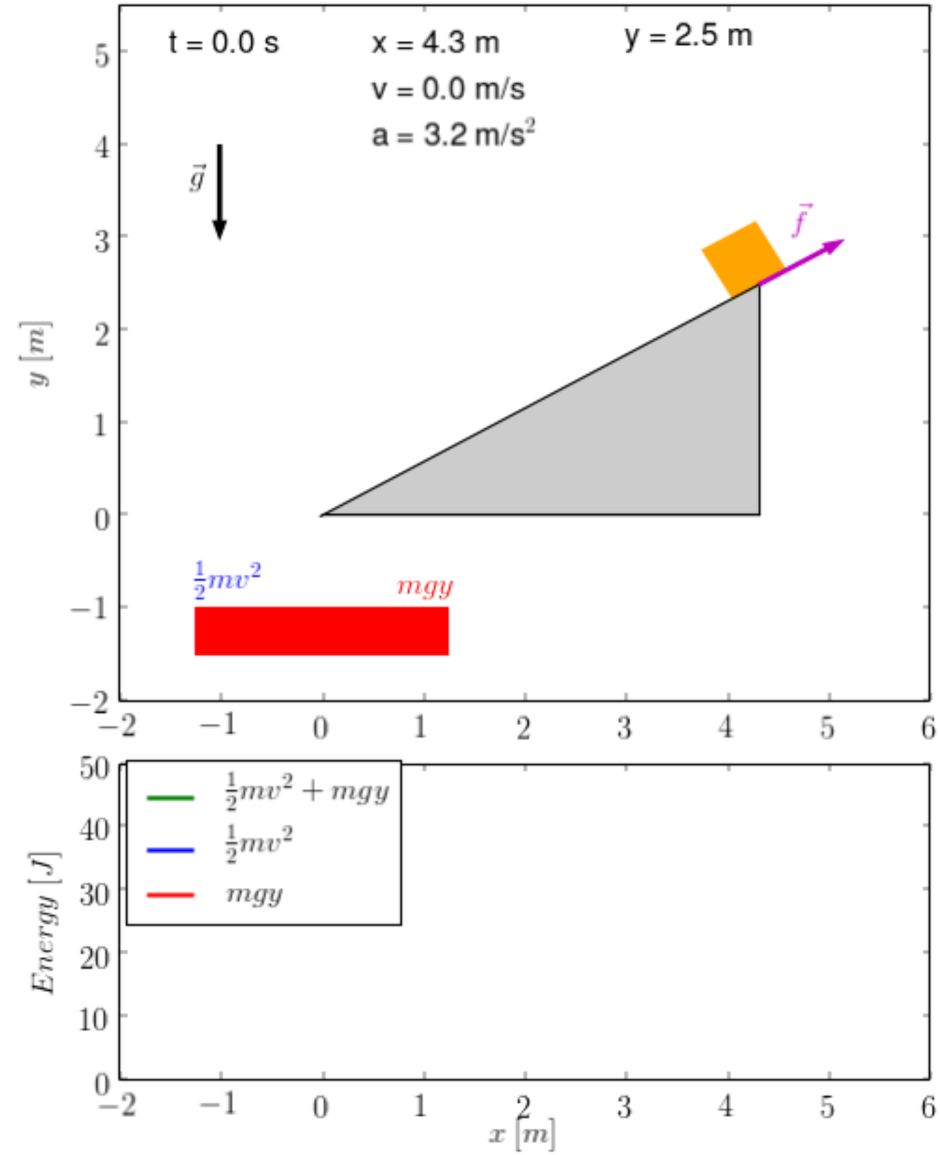
# Conservation de l'énergie mécanique

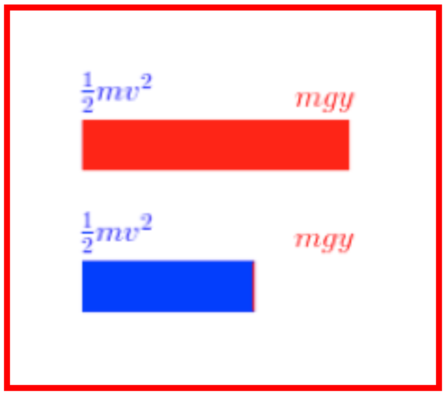


# Conservation de l’énergie mécanique

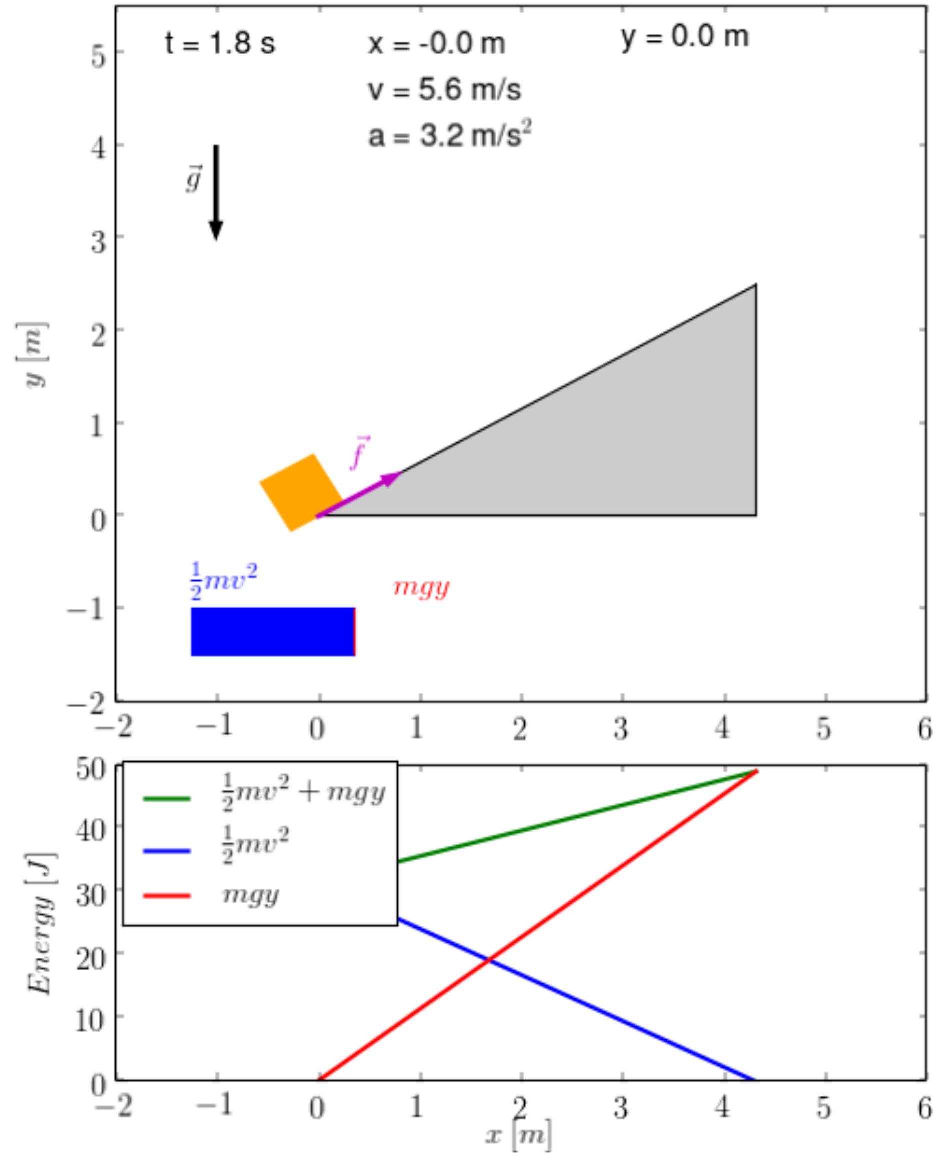


# Quid du frottement ?



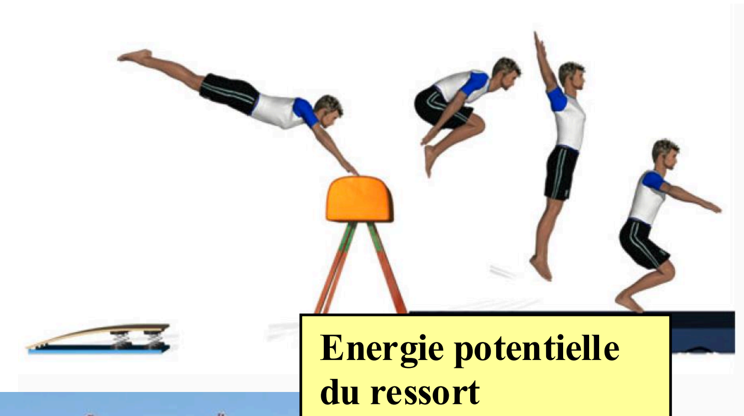



Une partie de l'énergie est dissipée !



# Conservation de l’énergie mécanique

*On conserve l’énergie mécanique dans les phases aériennes !*
*La trainée de l’air est négligeable*



**Energie potentielle du ressort**

**Energie cinétique**

**Energie potentielle de gravité**

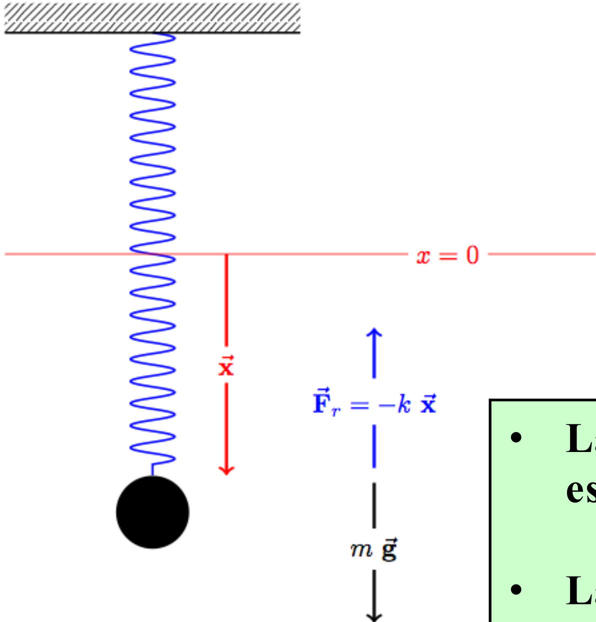
# Ici, la trainée diminue l'énergie mécanique


$$0 = \frac{1}{2}\,\rho\, C_D A\, v^2 - mg$$

$$\Downarrow$$

$$v = \sqrt{\frac{2mg}{\rho\, C_D A}}$$

http://waowen.screaming.net/revision/force&motion/skydiver.htm




- **La force de rappel du ressort s'oppose à sa déformation et est proportionnelle à celle-ci : c'est la loi de Hooke.**

- **La force de gravité et la force du ressort sont des forces conservatives : le travail de ces forces correspondent à un transfert entre énergies potentielle et cinétique.**

- **Une particule soumise uniquement à des forces conservatives conserve son énergie mécanique qui est la somme des énergie cinétique et potentielle !**