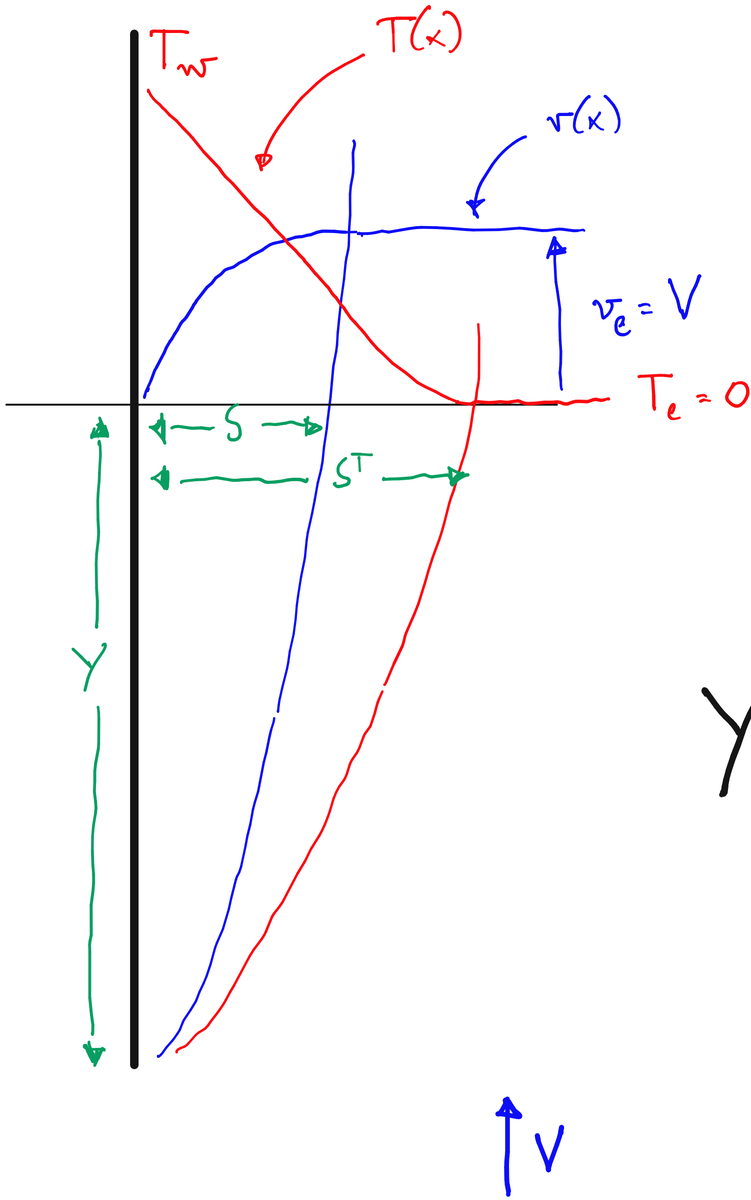


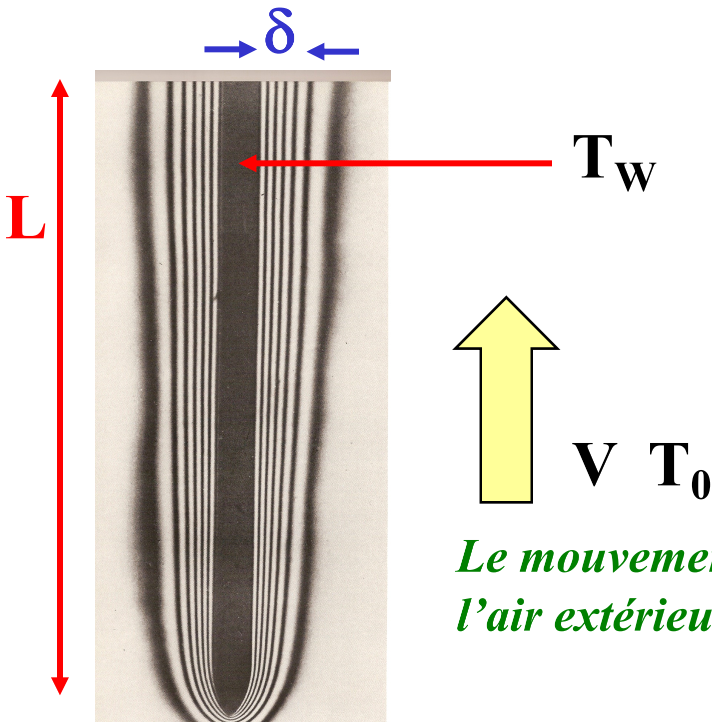
Mais, tout d'abord, un peu de convection forcée

$$Y \gg \delta, \ \delta_T$$

EPAISSEUR DE LA COUCHE LIMITE DE VITESSE

EPAISSEUR DE LA COUCHE LIMITE THERMIQUE

Mais, tout d’abord, un peu de convection forcée



Le mouvement vertical de l’air extérieur est <u>forcé</u>

Problème de l’écoulement

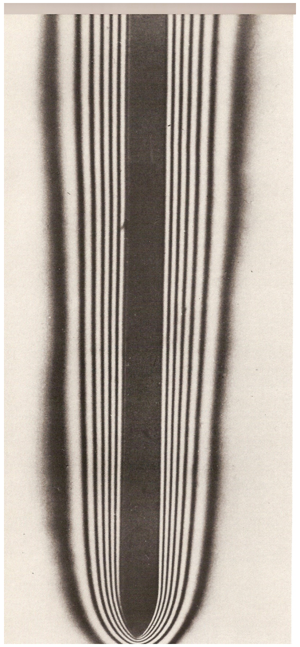
$$\nabla \cdot \mathbf{v} = 0,$$

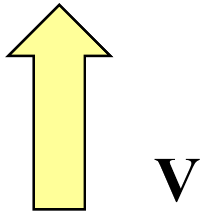
$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g},$$

$$\rho c \frac{DT}{Dt} = 2\mu \, \mathbf{d} : \mathbf{d} + r + \nabla \cdot (k \nabla T).$$

Problème thermique

Plus facile car, il est ici possible de <u>découpler</u> le problème de l’écoulement et le problème thermique !



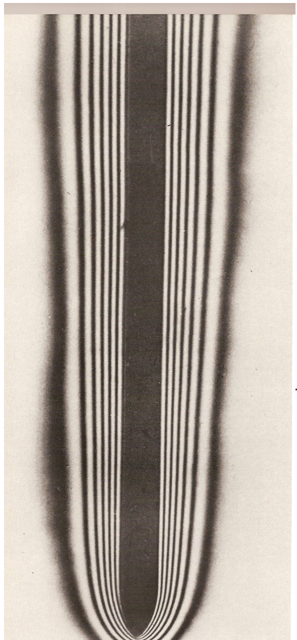


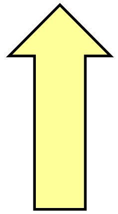
-i- problème de l’écoulement

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} &= \nu\,\frac{\partial^2 v}{\partial x^2}
\end{aligned}
$$

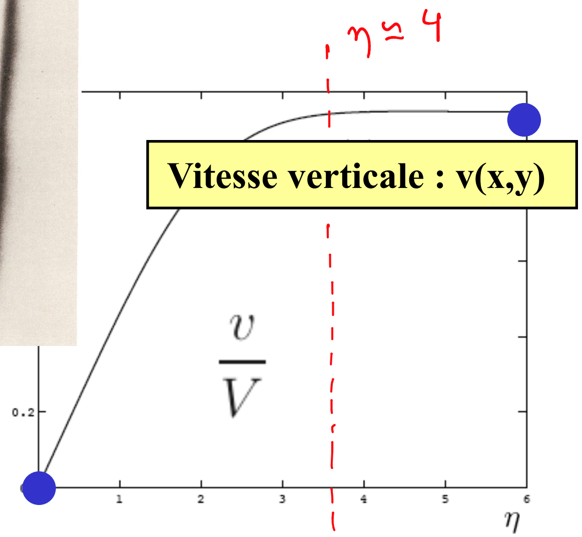
On introduit
une variable de similitude basée
une estimation de la couche limite

$$\eta(x,y) = \frac{x}{\delta(y)} = \frac{x}{\sqrt{\dfrac{2\nu y}{V}}}$$

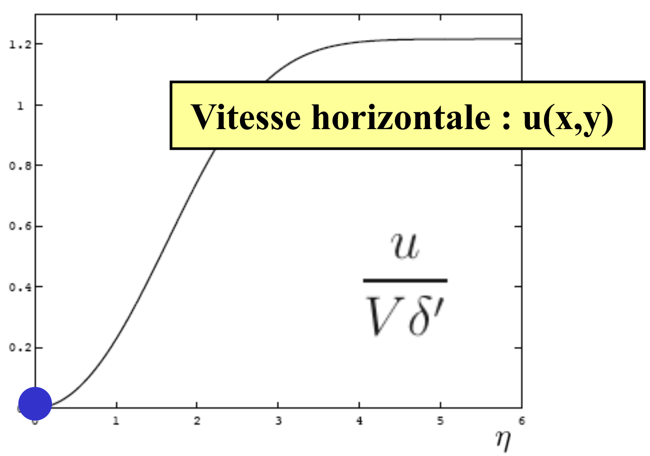




Solution de Blasius



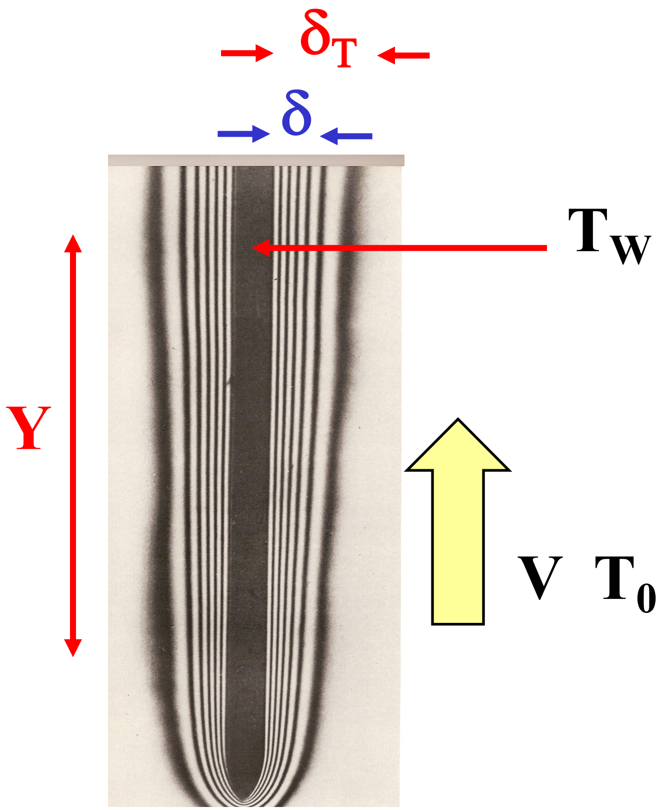
$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} &= \nu\,\frac{\partial^2 v}{\partial x^2}
\end{aligned}
$$



L'estimation de l'ordre de grandeur de la couche limite était bien adéquat !

$$\eta(x,y) = \frac{x}{\delta(y)} = \frac{x}{\sqrt{\frac{2\nu y}{V}}}$$

-ii- problème thermique



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = \nu\,\frac{\partial^2 v}{\partial x^2}$$

$\delta \ll Y$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial x^2}$$

$\delta_T \ll Y$

Près de la plaque, les effets conductifs sont dominants…
C’est la zone dite de couche limite thermique
Loin de la plaque, la conduction est négligeable

On définit l’épaisseur thermique comme le lieu géométrique où la conduction et la convection sont du même ordre de grandeur.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$$

$\frac{U}{\delta} = \frac{V}{Y}$; $\partial\left(\frac{V\Delta T}{Y}\right)$

$\partial\left(\frac{V\Delta T}{Y}\right)$

$\frac{V\Delta T}{Y}$

$\frac{\alpha\Delta T}{\delta_T^2}$

$\frac{\delta}{Y} = \sqrt{\frac{1}{Re}}$

$\frac{\delta_T}{Y} = \sqrt{\frac{1}{Pe}} = \sqrt{\frac{1}{Re\,Pr}}$

$\frac{\blacksquare}{\blacksquare} = \frac{V\Delta T}{Y}\frac{\delta_T^2}{\Delta T\,\alpha} = 1$

$\frac{\delta_T^2}{Y^2} = \frac{\alpha}{VY}$

Et δ_T ?

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2}$$

Lieu où l'ordre de la convection et de la conduction sont identiques

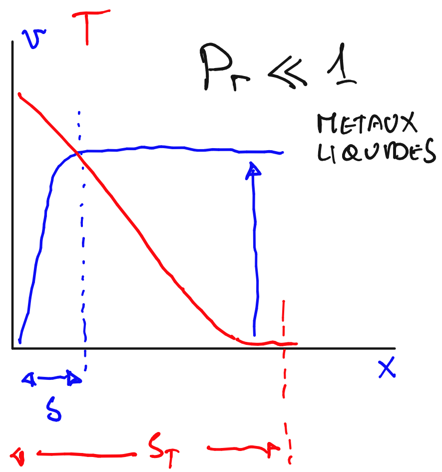
$$\frac{\text{Convection}}{\text{Conduction}} = \frac{V\Delta T/Y}{\alpha\Delta T/\delta_T^2} = \underbrace{\frac{VY}{\alpha}}_{Pe_Y}\frac{\delta_T^2}{Y^2} = 1$$

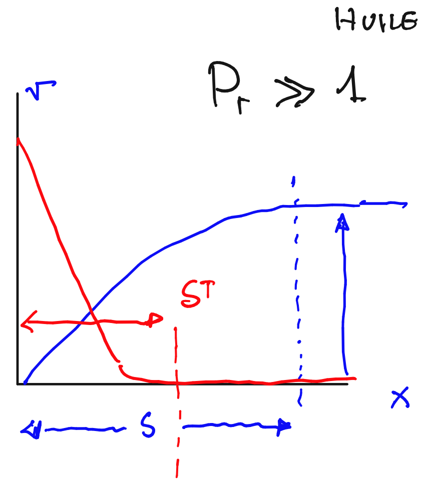
Et δ_T ?

$$\frac{\delta_T}{Y} = \sqrt{\frac{1}{Pe_Y}} = \sqrt{\frac{1}{Pr\,Re_Y}}$$

Epaisseurs de couches limites et le nombre de Prandtl

$$\frac{\delta_T}{\delta} = \sqrt{\frac{1}{Pr}}$$





En fait, c'est uniquement l'ordre de grandeur…
Lorsqu'on calcule la vraie épaisseur pour Blasius, c'est plutôt un exposant 1/3 !
Car l'ordre de grandeur de vitesse est un peu surestimé en prenant U !

Métaux liquides	**Pr << 1**
Gaz	**Pr = 0.7**
Eau	**Pr = 2… 7**
Huiles	**Pr >> 1**

Attention : Prantl est une fonction de la température (davantage pour les liquides que pour les gaz) !

Soyons un peu plus général : et la dissipation visqueuse ?

Problème de l'écoulement

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

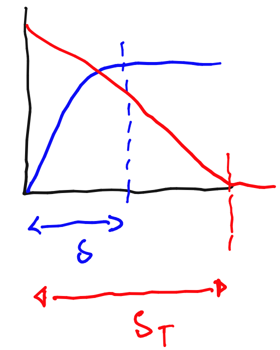
$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{c}\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\frac{\partial^2 T}{\partial x^2}$$

Problème thermique

Eckert : dissipation visqueuse ?

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{c}\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\frac{\partial^2 T}{\partial x^2}$$



$\frac{\nu}{c}\frac{V^2}{\delta^2}$

DISSIPATION VISQUEUSE

$\alpha\frac{\Delta T}{\delta_T^2}$

DIFFUSION/CONDUCTION THERMIQUE

$$\frac{\blacksquare}{\blacksquare} = \frac{\nu}{c}\frac{V^2}{\delta^2}\frac{\delta_T^2}{\alpha\Delta T} = \underbrace{\frac{V^2}{c\Delta T}}_{Ec}$$

$\frac{\nu}{\alpha}\frac{\delta_T^2}{\delta^2}\frac{V^2}{c\Delta T}$

P_r $1/P_r$

Eckert : dissipation visqueuse ?

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{c}\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\frac{\partial^2 T}{\partial x^2}$$

Dissipation visqueuse: $\mathcal{O}(\frac{\nu V^2}{c\delta^2})$

Conduction: $\mathcal{O}(\frac{\alpha \Delta T}{\delta_T^2})$

$$\frac{\color{blue}\blacksquare}{\color{green}\blacksquare} = \frac{\nu V^2}{\alpha c \Delta T}\underbrace{\left(\frac{\delta_T}{\delta}\right)^2}_{Pr^{-1}} = \frac{V^2}{c\Delta T} = Ec$$

Nombre d'Eckert

$$Ec = \frac{u_e^2}{c(T_w - T_e)}$$

Picture was taken on August 22, 2000

caractérise un écoulement d'un fluide !

$$\frac{\text{Energie cinétique}}{\text{Energie interne}}$$

$$Pr = 1$$
$$Ec \ll 1$$

$$Pr = 1$$
$$Ec \not\ll 1$$

Couches limites identiques

$$Pr \neq 1$$
$$Ec \ll 1$$

$$Pr \neq 1$$
$$Ec \not\ll 1$$

Dissipation visqueuse négligeable

Quatre cas possibles !

$$Pr = 1 \qquad Ec \ll 1$$

$$Pr = 1 \qquad Ec \not\ll 1$$

$$Pr \neq 1 \qquad Ec \ll 1$$

$$Pr \neq 1 \qquad Ec \not\ll 1$$

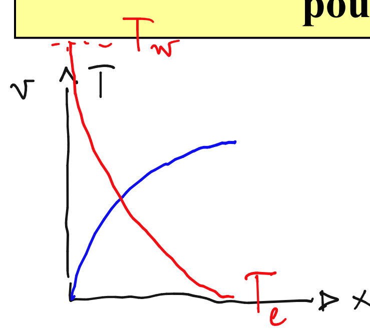
$$u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = \nu\,\frac{\partial^2 v}{\partial x^2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \nu\,\frac{\partial^2 T}{\partial x^2}$$

$$Pr = 1$$

$$Ec \ll 1$$

Les équations d'énergie et de quantité de mouvement expriment les mêmes opérateurs différentiels pour la température et la vitesse verticale !



$$T = Av + B$$

Si, si, c'est aussi simple !

Relation de Crocco

$$cT + \frac{v^2}{2} = Av + B$$

$Pr = 1$, $Ec \ll 1$	$Pr = 1$, $Ec \not\ll 1$
$Pr \neq 1$, $Ec \ll 1$	$Pr \neq 1$, $Ec \not\ll 1$

On a les mêmes opérateurs différentiels pour l'énergie interne totale et la vitesse verticale !

$$Pr = 1$$
$$Ec \not\ll 1$$

$$v\left[u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right] = v\left[\nu\,\frac{\partial^2 v}{\partial x^2}\right]$$

$$c\left[u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y}\right] = c\left[\frac{\nu}{c}\left(\frac{\partial v}{\partial x}\right)^2 + \nu\,\frac{\partial^2 T}{\partial x^2}\right]$$

Relation de Crocco

$$cT + \frac{v^2}{2} = Av + B$$

$$\frac{\partial}{\partial x}\left(\frac{v^2}{2}\right) = v\frac{\partial v}{\partial x}$$

$$u\frac{\partial}{\partial x}\left[cT + \frac{v^2}{2}\right] + v\frac{\partial}{\partial y}\left[cT + \frac{v^2}{2}\right] = \nu\frac{\partial^2}{\partial x^2}\left[cT + \frac{v^2}{2}\right]$$

$$v\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = v\left[\nu\frac{\partial^2 v}{\partial x^2}\right]$$

$$\nu\frac{\partial}{\partial x}\left[v\frac{\partial v}{\partial x}\right] = \nu\left(\frac{\partial v}{\partial x}\right)^2 + \nu v\frac{\partial^2 v}{\partial x^2}$$

$$c\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = c\left[\frac{\nu}{c}\left(\frac{\partial v}{\partial x}\right)^2 + \nu\frac{\partial^2 T}{\partial x^2}\right]$$

$Pr = 1$ $Ec \ll 1$	$Pr = 1$ $Ec \not\ll 1$
$Pr \neq 1$ $Ec \ll 1$	$Pr \neq 1$ $Ec \not\ll 1$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

$$Pr \neq 1$$
$$Ec \ll 1$$

$$\theta(\eta) = \frac{T - T_w}{T_e - T_w}$$

On procède comme pour Blasius !

$$Pr\ f\theta' + \theta'' = 0$$

Solution de similitude

$$\exp(g)\, Pr\, f\, \theta' + \exp(g)\, \theta'' = 0$$

$$Pr\, f = g'(\eta)$$

$$\exp(g)\, Pr\, f = (\exp(g))'$$

$$\left(\exp(g)\, \theta'\right)' = 0$$

$$\exp(g)\, \theta' = A$$

$$\theta' = A \exp(-g)$$

$$\theta = A \int_0^\eta \exp(-g) + B$$

$$Pr\ f\theta' + \theta'' = 0$$

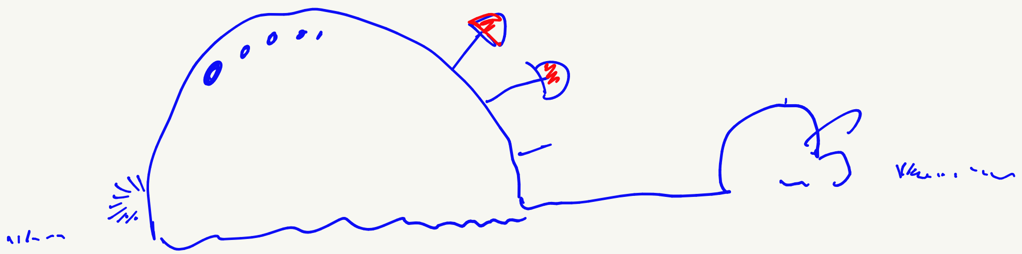
Calcul de la solution de similitude

$$\Theta(\eta) = A \int_0^{\eta} \exp(-g(\xi))\, d\xi + B$$

$$\Theta(\eta) = \frac{\int_0^{\eta} \exp\left[-P_r \int_0^{\xi} f(\zeta)\, d\zeta\right] d\xi}{\int_0^{\infty} \cdots\cdots \, d\xi}$$

$$\Theta(\eta) = \frac{T - T_w}{T_e - T_w}$$

- $\Theta(0) = 0$
- $\Theta(\eta \to \infty) = 1$



$$g'(\eta) = P_r f(\eta)$$

$Pr = 1$, $Ec \ll 1$	$Pr = 1$, $Ec \not\ll 1$
$Pr \neq 1$, $Ec \ll 1$	$Pr \neq 1$, $Ec \not\ll 1$

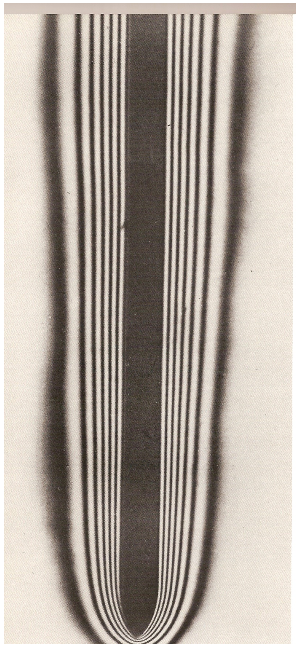
$$Pr \neq 1$$
$$Ec \not\ll 1$$

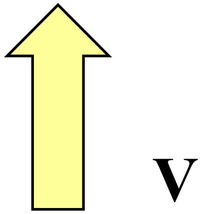
Le cas le plus général
(et donc aussi le plus probable !)

$$u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} = \nu\,\frac{\partial^2 v}{\partial x^2}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \frac{\nu}{c}\left(\frac{\partial v}{\partial x}\right)^2 + \alpha\,\frac{\partial^2 T}{\partial x^2}$$

Pas de solution analytique…





$$\tau_w = \mu \left.\frac{\partial v}{\partial x}\right|_{x=0}$$

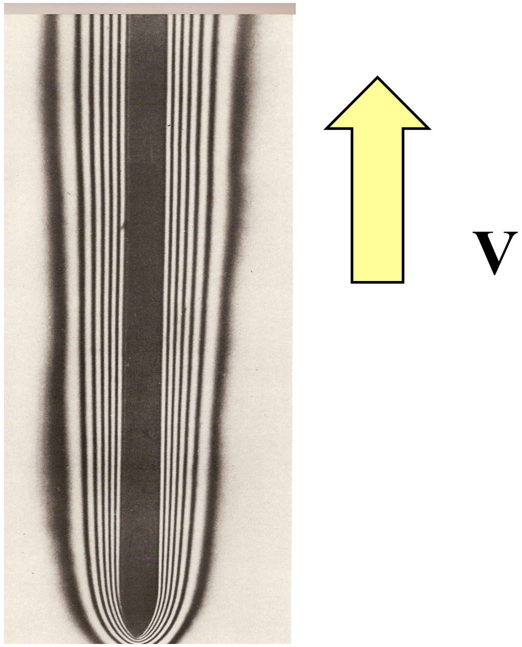
$$= \mu V \underbrace{f''|_{\eta=0}}_{0,4696} \; \frac{1}{\delta} \qquad \frac{1}{\delta} = \sqrt{\frac{V}{2\nu y}}$$

$$= 0,332 \sqrt{\frac{\mu^2 V^3}{\nu y}} \qquad \sqrt{\frac{\nu^2 \rho^2 V^4}{\nu y \; V}}$$

$$= \rho \frac{V^2}{2} \; 0,664 \underbrace{\sqrt{\frac{\nu}{V y}}}_{Re^{-1/2}}$$

$$C_f = \frac{\tau_w}{\rho V^2/2} = 0,664 \; Re^{-1/2}$$

Calcul de la force de trainée



$$q_w = -k \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

$$= -k\,(T_e - T_w)\,\Theta'\big|_{\eta=0}\,\frac{1}{\delta}$$

$$\frac{1}{\delta} = \sqrt{\frac{V}{2\,\nu\,y}}$$

$$\Theta'\big|_{\eta=0} \simeq 0{,}332\,\sqrt{2}\,P_r^{1/3}$$

VALABLE si $P_r > 0.01$

$$= -k\,(T_e - T_w)\;0{,}332\,\sqrt{\frac{V}{\nu\,y}}\;P_r^{1/3}$$

$$= -\rho c V (T_e - T_w)\;0{,}332\;\frac{k}{\rho c}\,\sqrt{\frac{V}{V^2\,\nu\,y}}\;P_r^{1/3}$$

$$\frac{k}{\rho c} = \alpha, \qquad \sqrt{\frac{V}{V^2\,\nu\,y}} = \frac{1}{\nu}\sqrt{\frac{\nu}{V y}}$$

$$\alpha\,\frac{1}{\nu}\sqrt{\frac{\nu}{V y}} = P_r^{-1}\,R_e^{-1/2}$$

$$= -\rho c V (T_e - T_w)\;0{,}332\;P_r^{-2/3}\,R_e^{-1/2}$$

$$S_t = \frac{-q_w}{\rho c V (T_e - T_w)} = 0{,}332\;P_r^{-2/3}\,R_e^{-1/2}$$

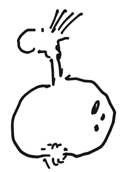
Calcul du flux de chaleur

$$C_f = 2\ P_r^{2/3}\ S_t$$

FORCE DE FROTTEMENT ADIMENSIONNELLE

FLUX DE CHALEUR ADIMENSIONNEL

$$C_f = \frac{\tau_w}{\rho V^2/2} = 0{,}664\ Re^{-1/2}$$



$$S_t = \frac{-q_w}{\rho c V (T_e - T_w)} = 0{,}332\ P_r^{-2/3}\ Re^{-1/2}$$

Analogie de Reynolds