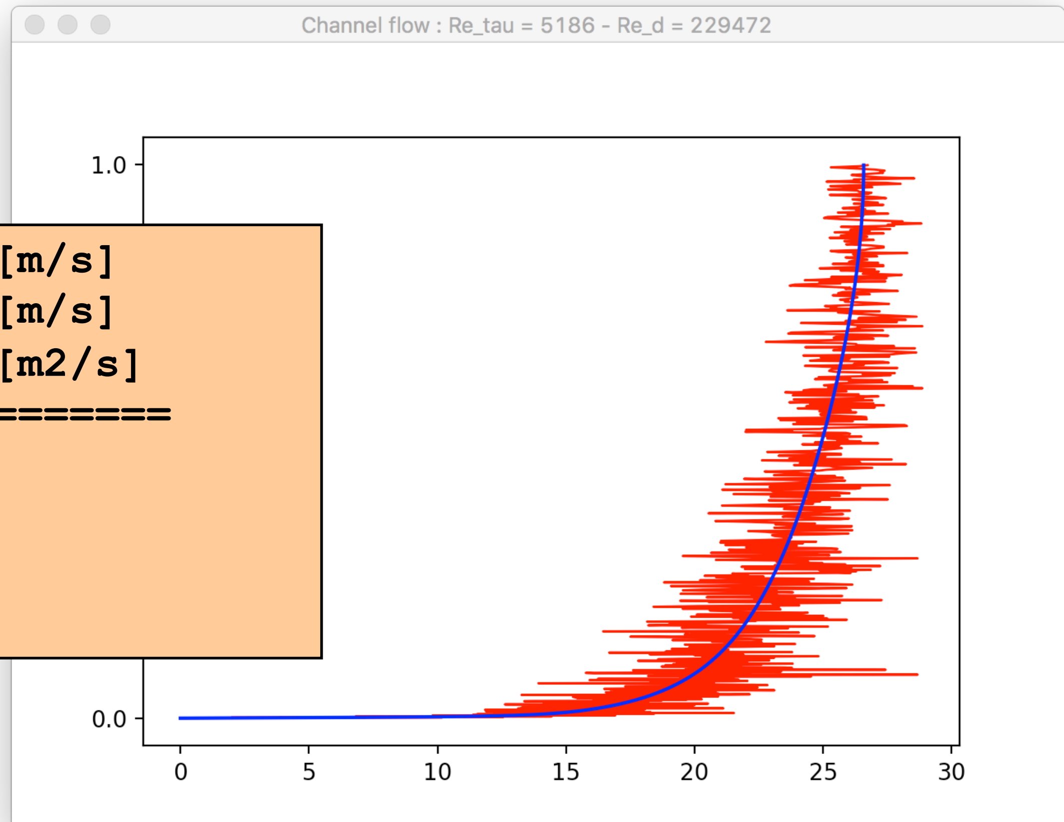
Pertes de charges turbulentes

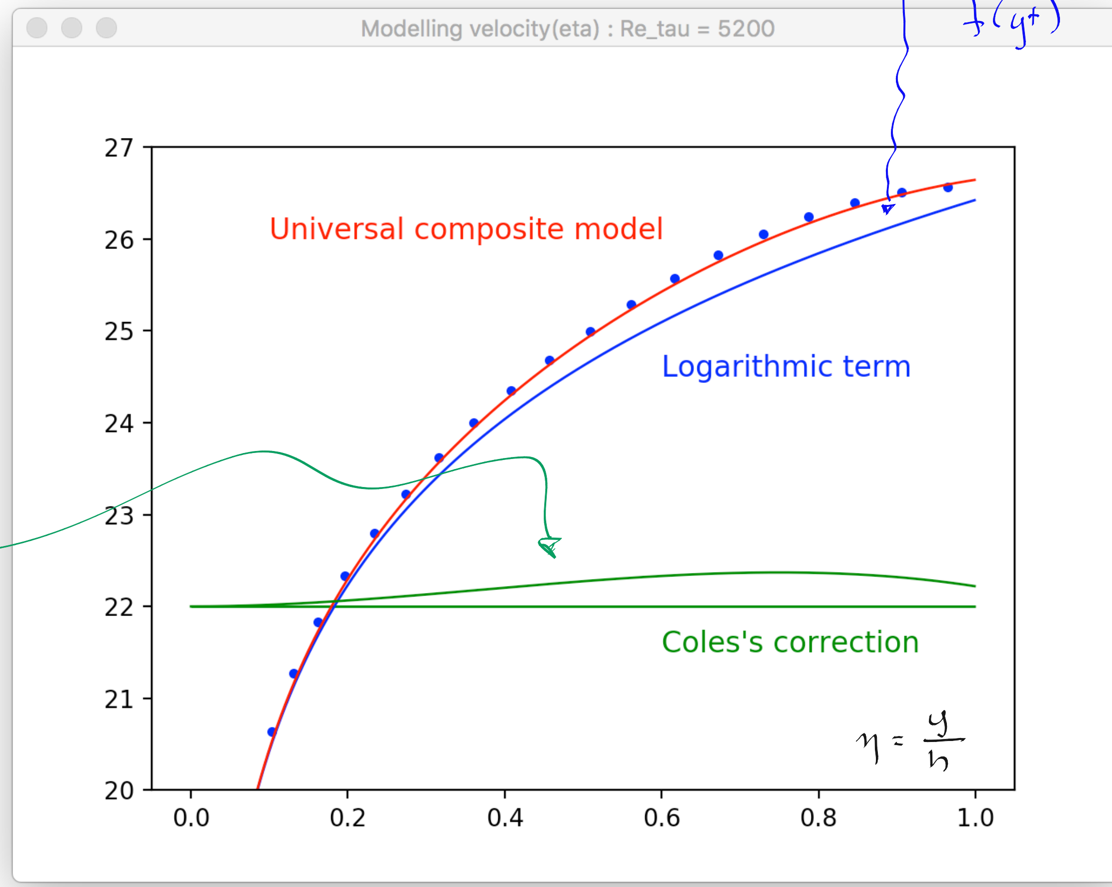
```
u_m    =  9.1788778e-01 [m/s]
u_c    =  1.1025341e+00 [m/s]
nu     =  8.0000000e-06 [m2/s]
===============================
lambda =  8.1716e-03
h+     =    5186
Re_tau =    5186
Re_d   =  229472
```



Ecoulement en canal

$$\frac{1}{\kappa}\log(y^+) + C$$

$$f(y^+)$$



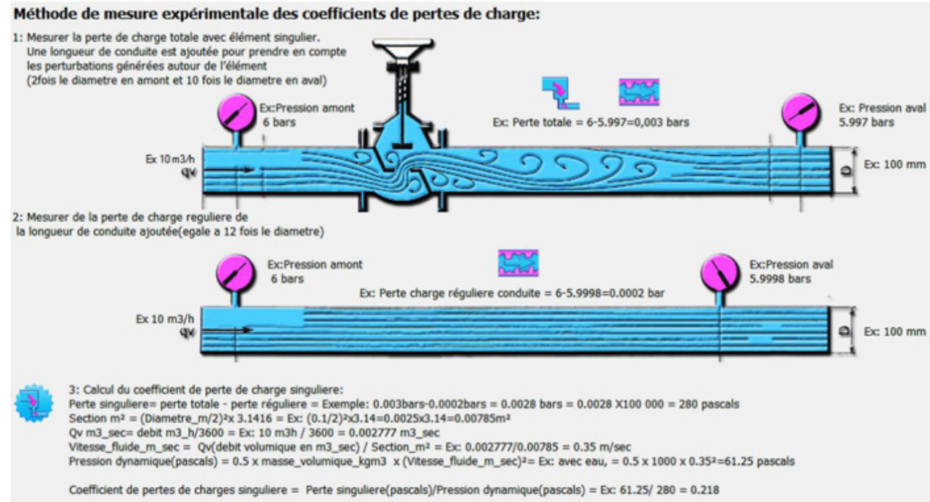
$$\eta = \frac{y}{h}$$

$$D\left[3(\alpha\eta)^2 - 2(\alpha\eta)^3\right]$$

$$G(\eta)$$

```
Profil logarithmique ========= kappa =  0.3840 et C =  4.1500
Correction de Coles ========== alpha =  1.3333 et D =  0.3700
```

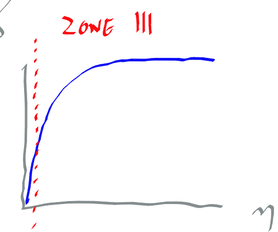
La formule des ingénieurs



$$
\begin{aligned}
\frac{1}{\sqrt{\lambda}} &= -a \log_{10}\left(\frac{b}{Re_d}\,\frac{1}{\sqrt{\lambda}}\right) \\
Re_d &= \frac{2h\overline{u}_m}{\nu} \\
Re_\tau &= \frac{h\overline{u}_\tau}{\nu} = h^+
\end{aligned}
$$

Un peu d'algèbre un brin fastidieuse

$$\frac{2}{\sqrt{\lambda}} = \frac{\bar{U}_m}{\bar{U}_\tau} \simeq \int_0^1 \frac{1}{\varkappa}\log\left(y^+(\eta)\right) + C + G(\eta) \; d\eta$$



$$= \int_0^1 \frac{1}{\varkappa}\underbrace{\log\Bigg[\underbrace{\frac{h\,\bar{U}_\tau}{\nu}}_{h^+}\;\underbrace{\frac{y}{h}}_{\eta}\Bigg]}_{\log(h^+)+\log(\eta)} + C + D\left[3(\alpha\eta)^2 - 2(\alpha\eta)^3\right] d\eta$$

$$= \frac{1}{\varkappa}\log(h^+) + C + \frac{1}{\varkappa}\underbrace{\int_0^1 \log\eta \; d\eta}_{?} + D\underbrace{\left[\frac{3\alpha^2\eta^3}{3} - \frac{2\alpha^3\eta^4}{4}\right]_0^1}_{\alpha^2 - \alpha^3/2}$$

$$(\eta\log\eta)' = \log\eta + 1$$

$$\left[\eta\log\eta - \eta\right]_0^1 = -1$$

$$\frac{2}{\sqrt{\lambda}} = \frac{1}{\varkappa}\log(h^+) + C + \frac{1}{\varkappa}\underbrace{\int_0^1 \log\eta \, d\eta}_{?} + D\underbrace{\left[\frac{3\alpha^2\eta^3}{3} - \frac{2\alpha^3\eta^4}{4}\right]_0^1}_{\alpha^2 - \alpha^3/2}$$

$$(\eta\log\eta)' = \log\eta + 1$$

$$[\eta\log\eta - \eta]_0^1 = -1$$

$$= \frac{1}{\varkappa}\log(\underbrace{h^+}) + \underbrace{\left[C - \frac{1}{\varkappa} + D\left(\alpha^2 - \frac{1}{2}\alpha^3\right)\right]}_{[☺]}$$

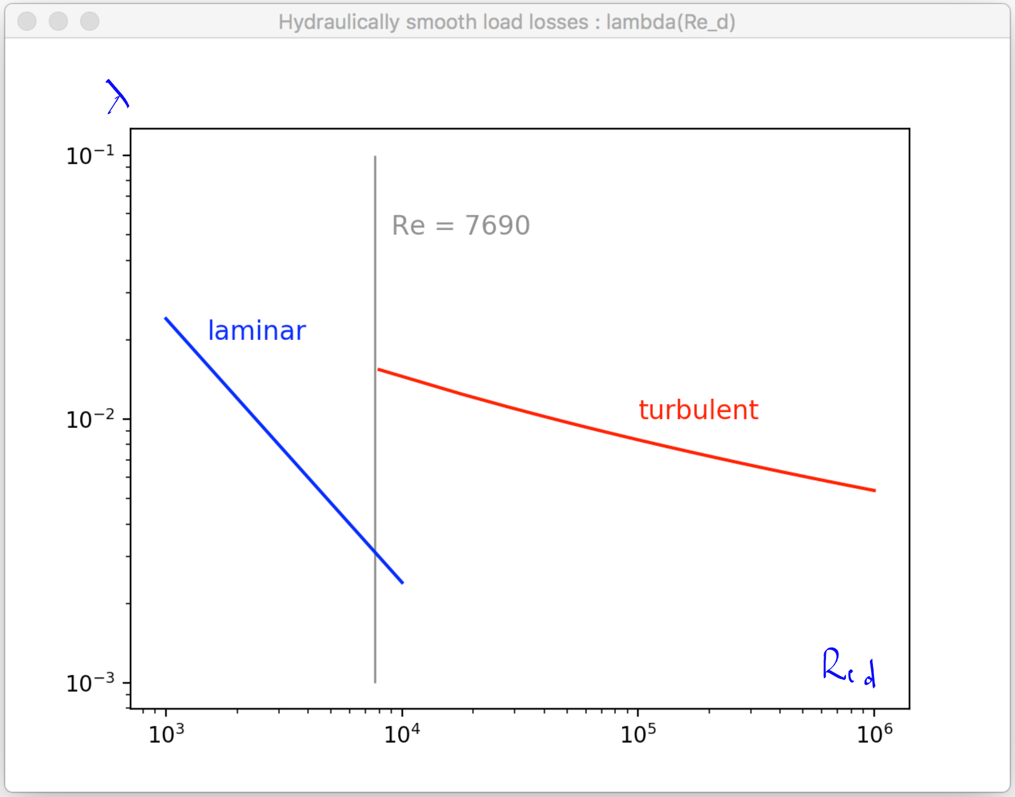
$$\frac{1}{2}\,\frac{2h\,\bar{u}_m}{\nu}\,\frac{\bar{u}_\tau}{\bar{u}_m} = \frac{1}{2}\,Re_d\,\frac{\sqrt{\lambda}}{2}$$

$$\frac{2}{\sqrt{\lambda}} = -\frac{1}{\varkappa}\log\left[\frac{4}{Re_d}\,\frac{1}{\sqrt{\lambda}}\right] + \frac{1}{\varkappa}\log\left[\exp\left(\varkappa\,[☺]\right)\right]$$

$$\frac{1}{\sqrt{\lambda}} = \underbrace{-\frac{2}{\varkappa}\,\frac{1}{\log_{10}(e)}}_{a}\log_{10}\left[\underbrace{\frac{4}{\exp[\varkappa[☺]]}}_{b}\,\frac{1}{\sqrt{\lambda}}\right]$$

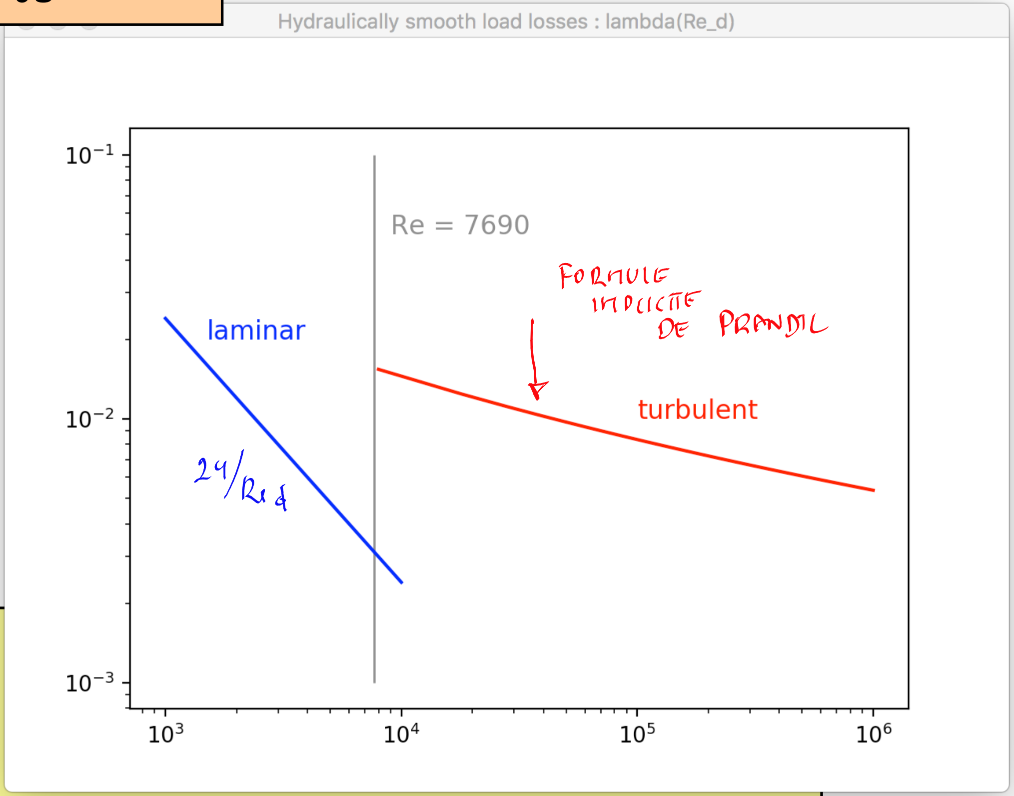
Utilisation de la formule implicite de Prandtl

$$\frac{1}{\sqrt{\lambda}} = -a \log_{10}\left(\frac{b}{Re_d}\frac{1}{\sqrt{\lambda}}\right)$$



```
Re_d =     10000 : lambda =  1.447e-02
Re_d =    100000 : lambda =  8.337e-03
Re_d =   1000000 : lambda =  5.358e-03
```

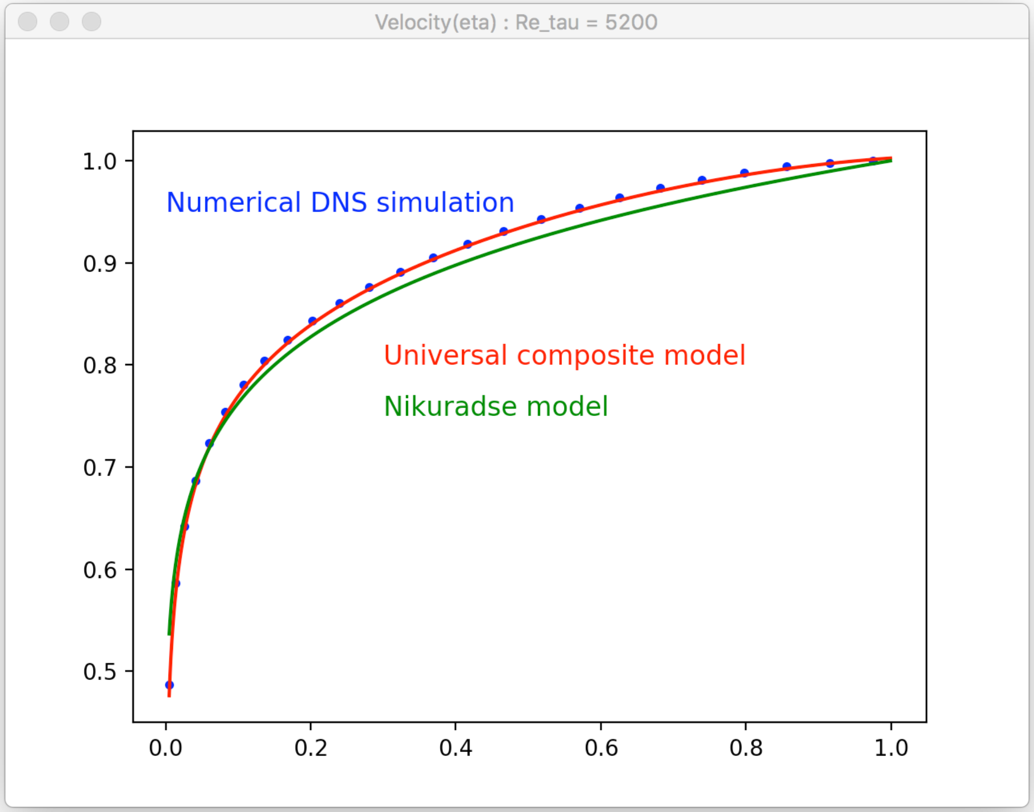
$$\frac{1}{\sqrt{\lambda}} = 1{,}778\ Re^{1/8}$$



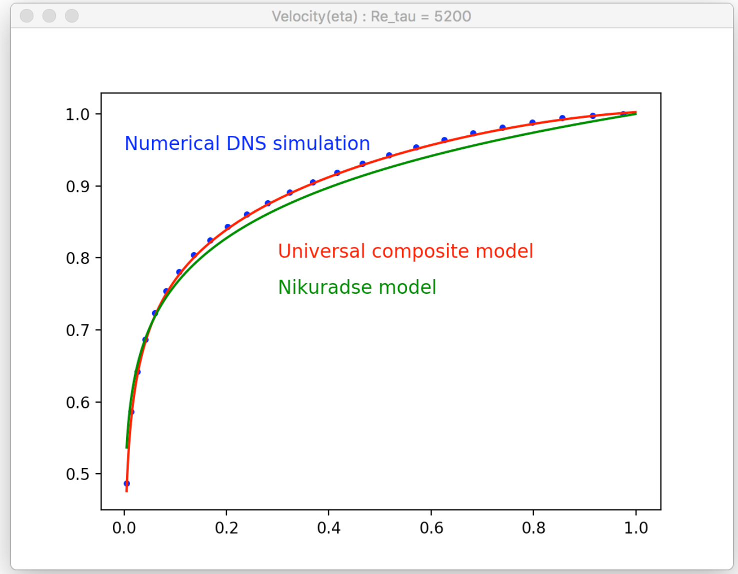

```
def computeLambda(Re) :
  delta = 100; i = 0
  xi = 1.778 * Re**(1/8)
  while delta > 1e-4 and i < 1000:
    old = xi
    xi = a*np.log10(xi*b/Re)
    delta = np.abs(old-xi); i = i+1
  return 1/(xi*xi)

for Re in [1e4,1e5,1e6] :
  print( " Re_d = %8d : lambda = %10.3e " % (Re,computeLambda(Re)))
```

Nikuradse
Un bidouilleur de génie

$$\bar{U}(y) = \bar{U}_c \left[\frac{y}{h}\right]^{1/n}$$



Intégrales du déficit de vitesse



$\boxed{1}$

$\alpha = 1/n$

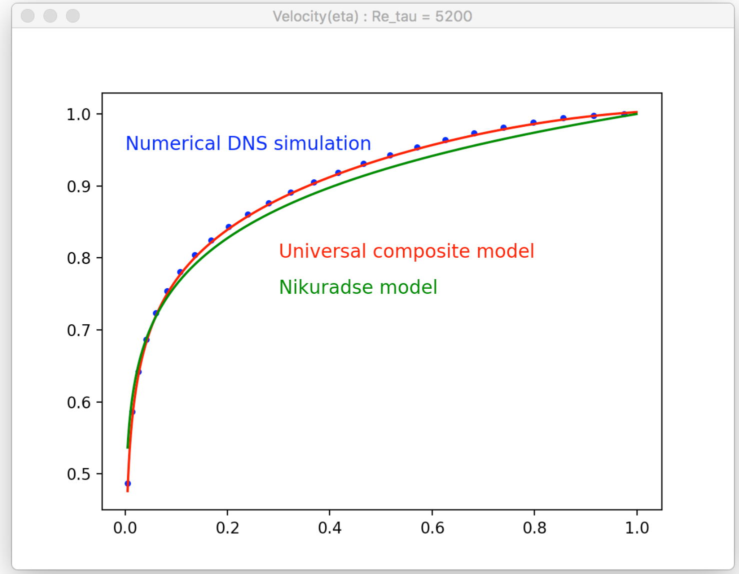
$$\bar U_m = \bar U_c \underbrace{\int_0^1 \eta^{\alpha}\, d\eta}_{\left[\frac{\eta^{\alpha+1}}{\alpha+1}\right]_0^1} = \bar U_c \frac{1}{1/n + 1}$$

$$\boxed{\bar U_m = \bar U_c \frac{n}{n+1}}$$

$$\boxed{\int_0^1 \bar U_c - \bar U(\eta)\, d\eta = \bar U_c - \bar U_c \frac{n}{n+1} = \bar U_c \left[\frac{n+1-n}{n+1}\right] = \bar U_c \frac{1}{n+1}}$$

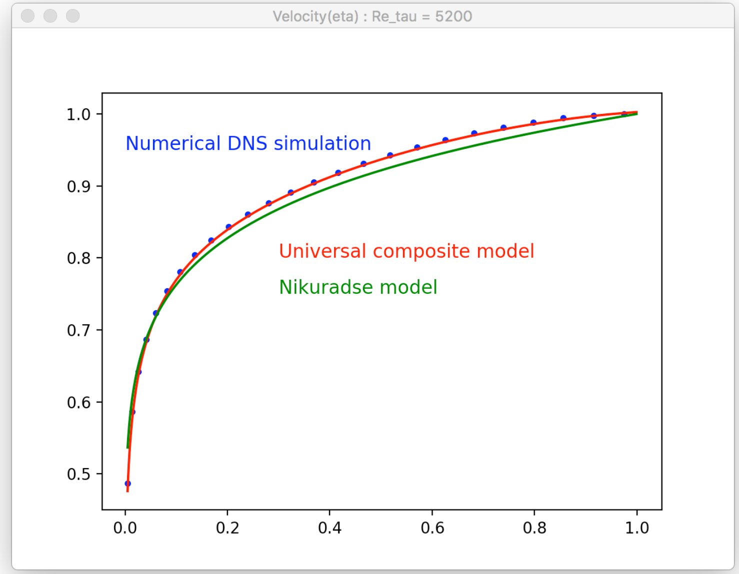
$$I_{Nik} = \int_0^1 \frac{\bar U_c - \bar U(\eta)}{\bar U_\tau}\, d\eta = \underbrace{\frac{1}{\bar U_m}}_{\frac{\cancel{n+1}}{n} \cancel{\frac{1}{\bar U_c}}} \cancel{\bar U_c} \frac{1}{\cancel{(n+1)}} \underbrace{\frac{\bar U_m}{\bar U_\tau}}_{2/\sqrt{\lambda}} = \frac{1}{n} \frac{1}{\sqrt{\lambda}}$$

Nikuradse



$$\boxed{I_{Nik} = \frac{1}{n} \frac{2}{\sqrt{\lambda}}}$$

Composite



$$\frac{\bar U_c - \bar U(\eta)}{\bar U_\tau} = \frac{1}{\varkappa}\log\left[\frac{h\bar U_\tau}{\nu}\right] + \overbrace{G(1)}^{D\left[3\alpha^2-2\alpha^3\right]}$$

$$\underbrace{-\frac{1}{\varkappa}\log\left[\frac{y\bar U_\tau}{\nu}\right]}_{-\frac{1}{\varkappa}\log\left[\underbrace{\frac{y}{h}}_{\eta}\right]} \underbrace{-\,G(\eta)}_{-D\left[3\alpha^2\eta^2-2\alpha^3\eta^3\right]}$$

$$I_{\text{COLES}} = -\frac{1}{\varkappa}\underbrace{\int_0^1 \log(\eta)\,d\eta}_{=-1} + D\left[\underbrace{3\alpha^2\eta - 2\alpha^3\eta}\ \underbrace{-\frac{3\alpha^2\eta^3}{3} - \frac{2\alpha^3\eta^4}{4}}\right]_0^1$$

$$2\alpha^2 - 3\alpha^3/2 = 0$$

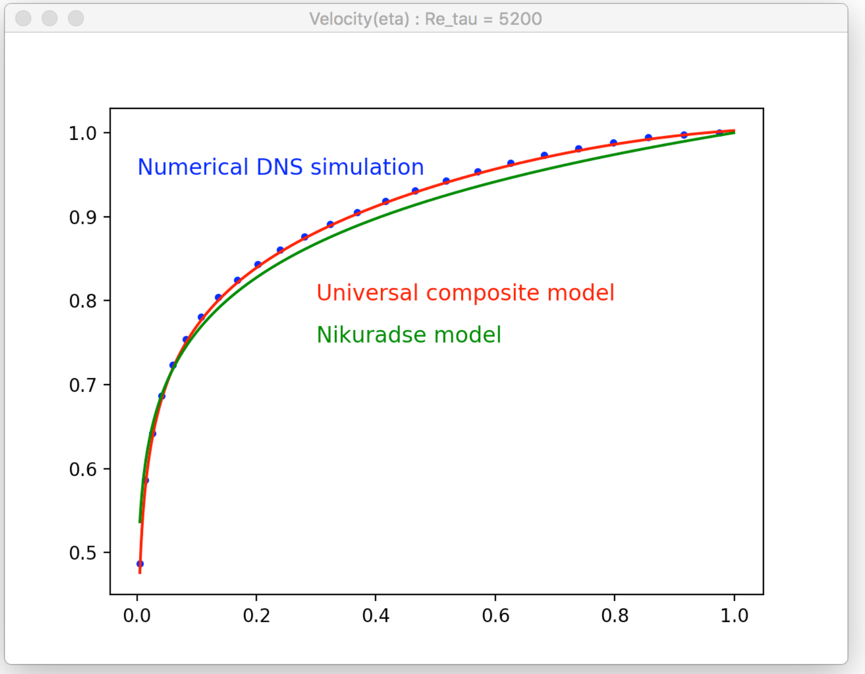
CAR $\alpha = 4/3$

$$\boxed{I_{\text{COLES}} = \frac{1}{\varkappa}}$$

Trouver le bon exposant !

$$n = \frac{2\kappa}{\sqrt{\lambda}}$$

$$\boxed{I_{Nik} = \frac{1}{n}\,\frac{2}{\sqrt{\lambda}}}$$



$$\boxed{I_{Coles} = \frac{1}{\kappa}}$$

```
Profil logarithmique ========= kappa =  0.3840 et C =  4.1500
Correction de Coles ========== alpha =  1.3333 et D =  0.3700
Nikuradse exponent ===========   1/n =  0.1177
Nikuradse exponent ===========     n =  8.4958
Formule implicite de Prandtl =     a = -2.9982 et b =  2.0309
```

Mais, c'est pas aussi joli que le profil universel