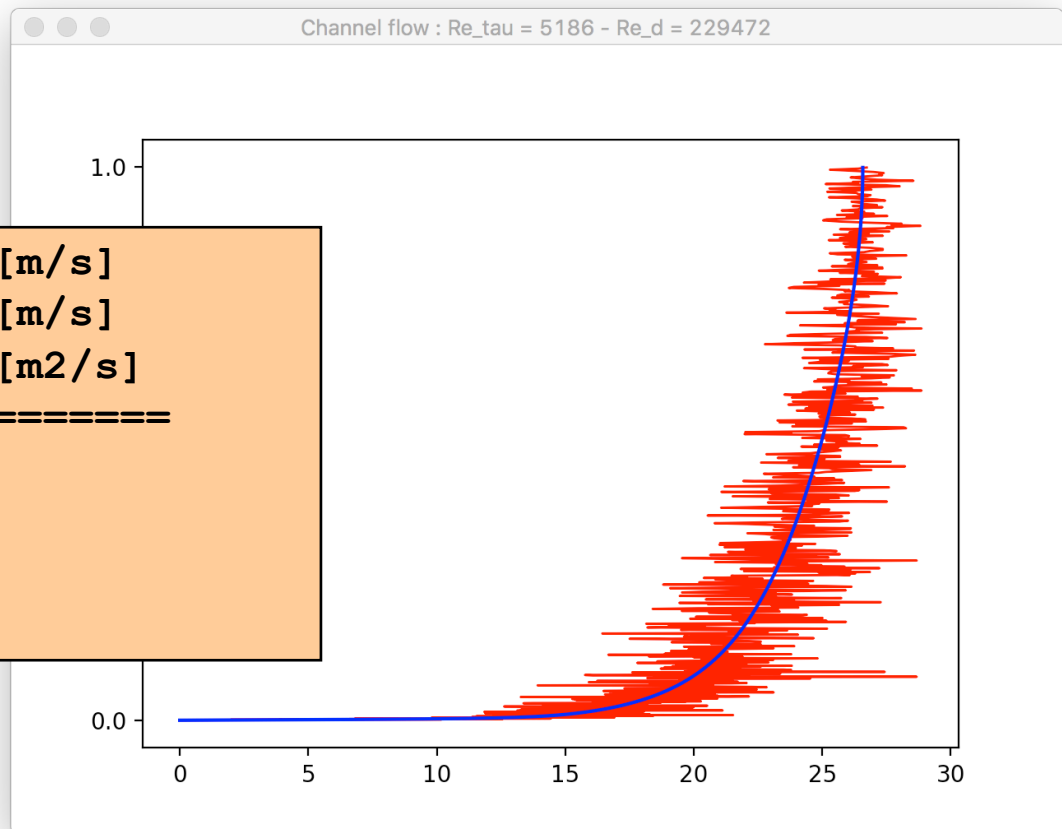


# Pertes de charges turbulentes

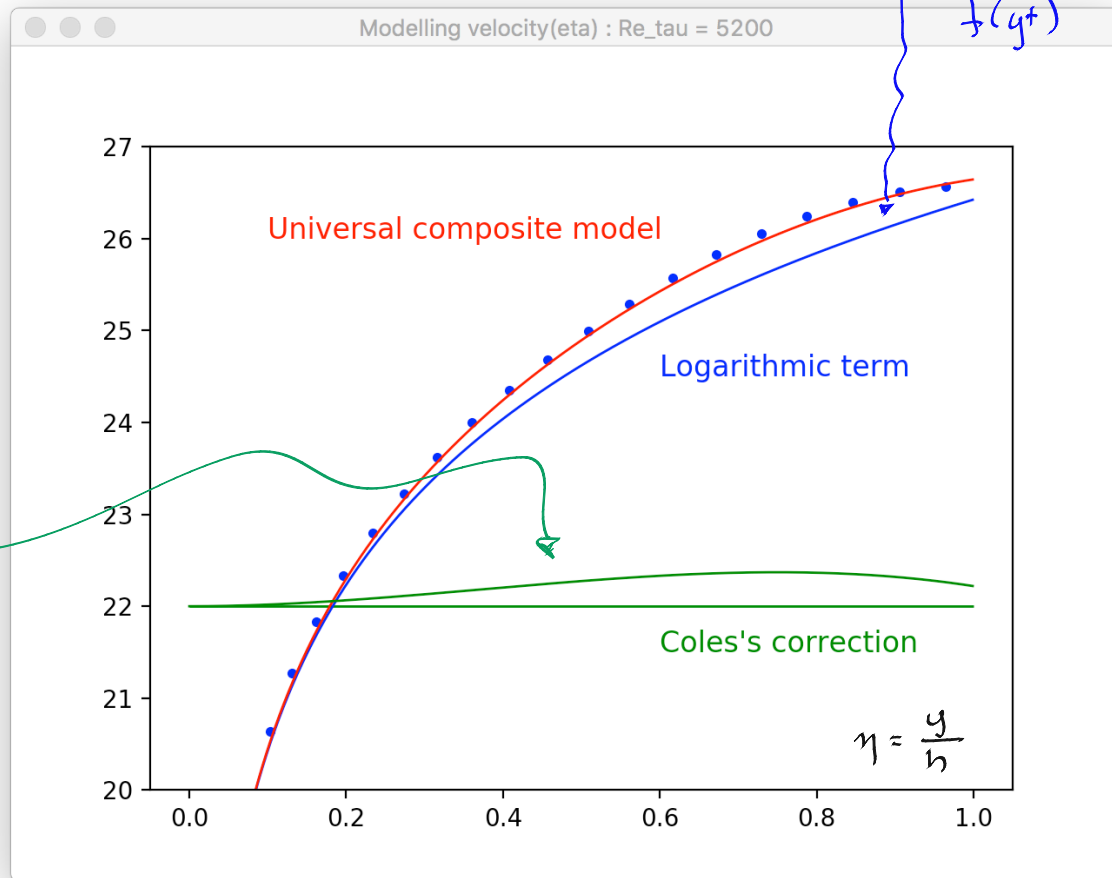
```
u_m      = 9.1788778e-01 [m/s]
u_c      = 1.1025341e+00 [m/s]
nu       = 8.0000000e-06 [m2/s]
```

```
=====
lambda   = 8.1716e-03
h+       = 5186
Re_tau   = 5186
Re_d     = 229472
```



# Écoulement en canal

$$D [3(\alpha\eta)^2 - 2(\alpha\eta)^3]$$

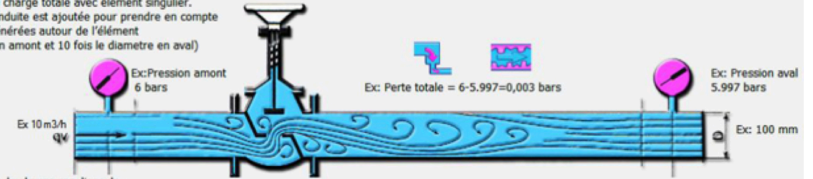


Profil logarithmique ===== kappa = 0.3840 et C = 4.1500  
 Correction de Coles ===== alpha = 1.3333 et D = 0.3700

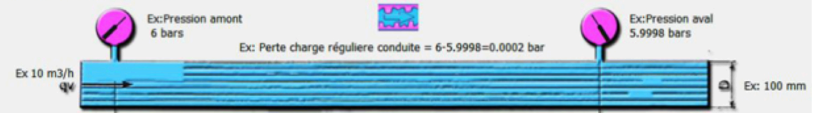
# La formule des ingénieurs

## Méthode de mesure expérimentale des coefficients de pertes de charge:

1: Mesurer la perte de charge totale avec élément singulier.  
Une longueur de conduite est ajoutée pour prendre en compte les perturbations générées autour de l'élément (2fois le diamètre en amont et 10 fois le diamètre en aval)



2: Mesurer de la perte de charge régulière de la longueur de conduite ajoutée (égale à 12 fois le diamètre)



3: Calcul du coefficient de perte de charge singulière:

Perte singulière = perte totale - perte régulière = Exemple: 0.003bars - 0.0002bars = 0.0028 bars = 0.0028 X 100 000 = 280 pascals

Section m² = (Diamètre\_m/2)² x 3.1416 = Ex: (0.1/2)² x 3.14 = 0.0025 x 3.14 = 0.00785m²

Qv m³\_sec = débit m³\_h/3600 = Ex: 10 m³/h / 3600 = 0.002777 m³\_sec

Vitesse\_fluide\_m\_sec = Qv(débit volumique en m³\_sec) / Section\_m² = Ex: 0.002777/0.00785 = 0.35 m/sec

Pression\_dynamique(pascals) = 0.5 x masse\_volumique\_kgm³ x (Vitesse\_fluide\_m\_sec)² = Ex: avec eau, = 0.5 x 1000 x 0.35² = 61.25 pascals

Coefficient de pertes de charges singulière = Perte\_singulière(pascals)/Pression\_dynamique(pascals) = Ex: 61.25/ 280 = 0.218

$$\frac{1}{\sqrt{\lambda}} = -a \log_{10} \left( \frac{b}{Re_d} \frac{1}{\sqrt{\lambda}} \right)$$

$$Re_d = \frac{2h\bar{u}_m}{\nu}$$

$$Re_\tau = \frac{h\bar{u}_\tau}{\nu} = h^+$$

# Un peu d'algèbre un brin fastidieuse

$$\begin{aligned}
 & \frac{2}{\sqrt{\lambda}} = \frac{\bar{u}_m}{2\bar{u}_\tau} = \int_0^1 \frac{1}{\beta} \log(y^+(\eta)) + C + G(\eta) \, d\eta \\
 & \qquad \qquad \qquad \text{ZONE III} \\
 & \qquad \qquad \qquad \eta \\
 & = \int_0^1 \frac{1}{\beta} \log \left[ \underbrace{\frac{h \bar{u}_\tau}{\beta}}_{h^+} \underbrace{\frac{y}{h}}_{\eta} \right] + C + D \left[ 3(\alpha\eta)^2 - 2(\alpha\eta)^3 \right] \, d\eta \\
 & \qquad \qquad \qquad \log(h^+) + \log(\eta) \\
 & = \frac{1}{\beta} \log(h^+) + C + \frac{1}{\beta} \int_0^1 \log \eta \, d\eta + D \left[ \frac{3\alpha^2 \eta^3}{3} - \frac{2\alpha^3 \eta^4}{4} \right]_0^1 \\
 & \qquad \qquad \qquad \underbrace{\qquad \qquad \qquad}_{\alpha^2 - \alpha^3/2} \\
 & \qquad \qquad \qquad (\eta \log \eta)' = \log \eta + 1 \\
 & \qquad \qquad \qquad \underbrace{\qquad \qquad \qquad}_{\int} \\
 & \qquad \qquad \qquad \left[ \eta \log \eta - \eta \right]_0^1 = -1
 \end{aligned}$$

$$\frac{2}{\sqrt{x}} = \frac{1}{\partial x} \log(h^+) + C + \frac{1}{\partial x} \int_0^1 \log \eta \, d\eta + D \left[ \frac{3\alpha^2 \eta^3}{3} - \frac{2\alpha^3 \eta^4}{4} \right]_0^1$$

$(\eta \log \eta)' = \log \eta + 1$        $\int_0^1 \log \eta \, d\eta = -1$   
 $[\eta \log \eta - \eta]_0^1 = -1$

$\alpha^2 - \alpha^3/2$

$$= \frac{1}{\partial x} \log(h^+) + \left[ C - \frac{1}{\partial x} + D(\alpha^2 - \frac{1}{2}\alpha^3) \right]$$

[?]

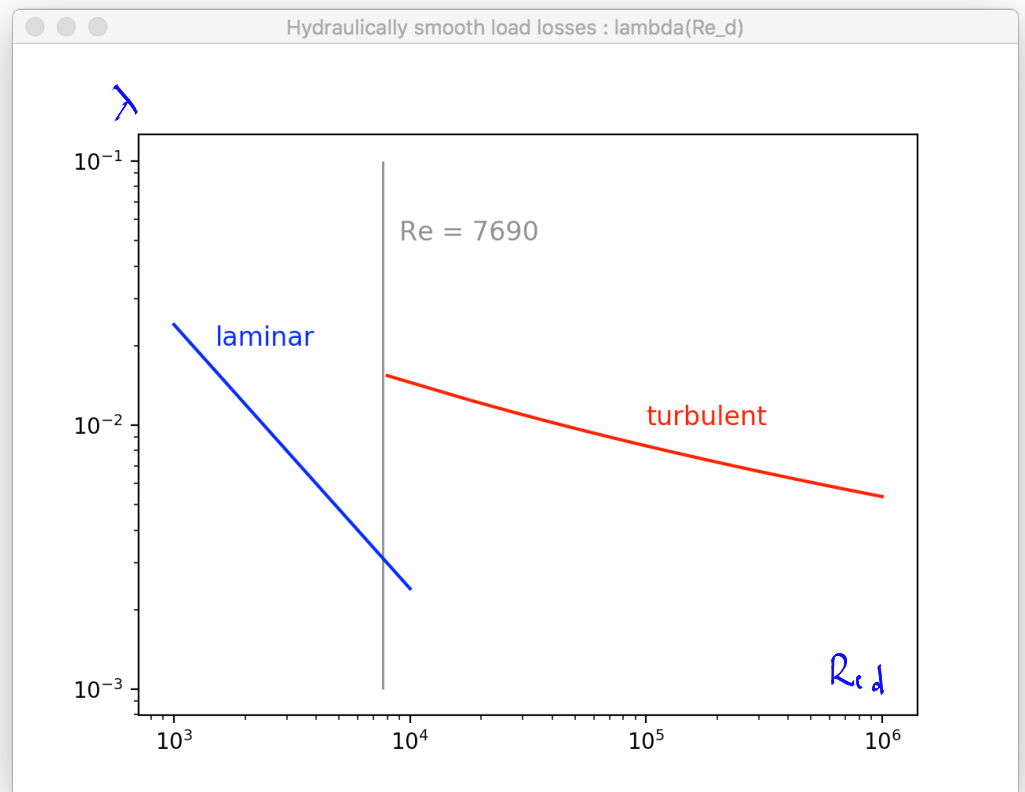
$$\frac{1}{2} \frac{2h \bar{u}_m}{\bar{u}_m} = \frac{1}{2} \text{Re}_d \frac{\sqrt{x}}{2}$$

$$\frac{2}{\sqrt{x}} = -\frac{1}{\partial x} \log \left[ \frac{4}{\text{Re}_d} \frac{1}{\sqrt{x}} \right] + \frac{1}{\partial x} \log \left[ \exp(x [\text{?}]) \right]$$

$$\frac{1}{\sqrt{x}} = \underbrace{-\frac{2}{\partial x} \frac{1}{\log_{10}(e)}}_a \log_{10} \left[ \underbrace{\frac{4}{\exp[\kappa [\text{?}]]}}_b \frac{1}{\sqrt{x}} \right]$$

# Utilisation de la formule implicite de Prandtl

$$\frac{1}{\sqrt{\lambda}} = -a \log_{10} \left( \frac{b}{Re_d} \frac{1}{\sqrt{\lambda}} \right)$$



```

Re_d = 10000 : lambda = 1.447e-02
Re_d = 100000 : lambda = 8.337e-03
Re_d = 1000000 : lambda = 5.358e-03

```

$$\frac{1}{\sqrt{x}} = 1,778 \operatorname{Re}^{1/8}$$

```

def computeLambda(Re) :
    delta = 100; i = 0
    xi = 1.778 * Re**(1/8)
    while delta > 1e-4 and i < 1000:
        old = xi
        xi = a*np.log10(xi*b/Re)
        delta = np.abs(old-xi); i = i+1
    return 1/(xi*xi)

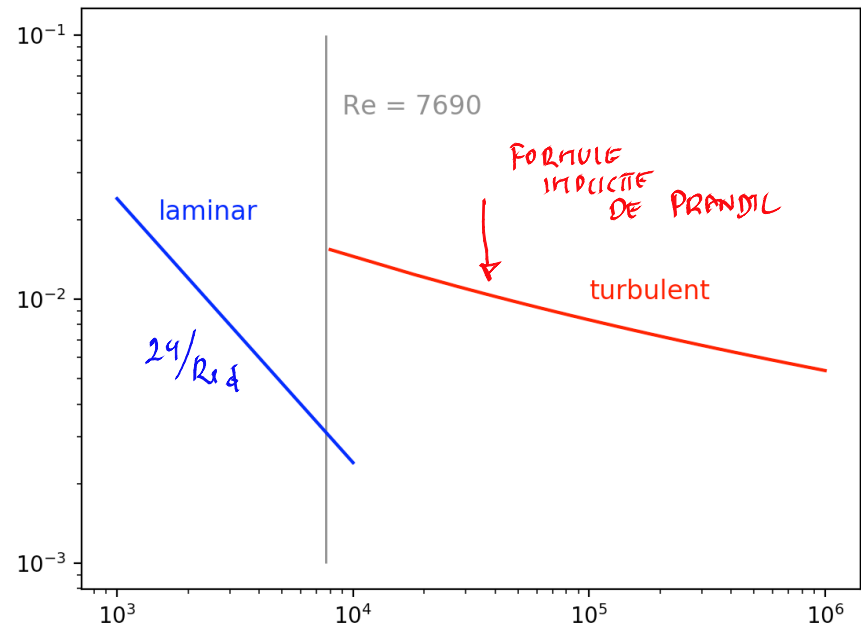
```

```

for Re in [1e4,1e5,1e6] :
    print( " Re_d = %8d : lambda = %10.3e " % (Re,computeLambda(Re)) )

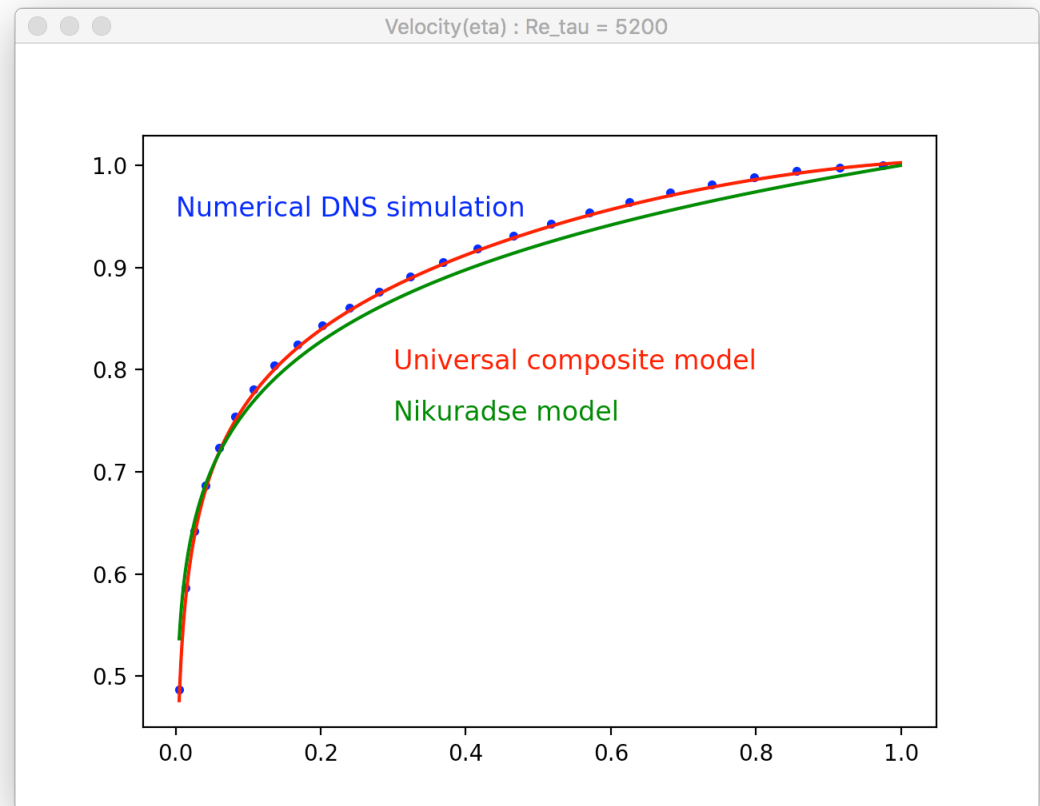
```

Hydraulically smooth load losses :  $\lambda(\operatorname{Re}_d)$



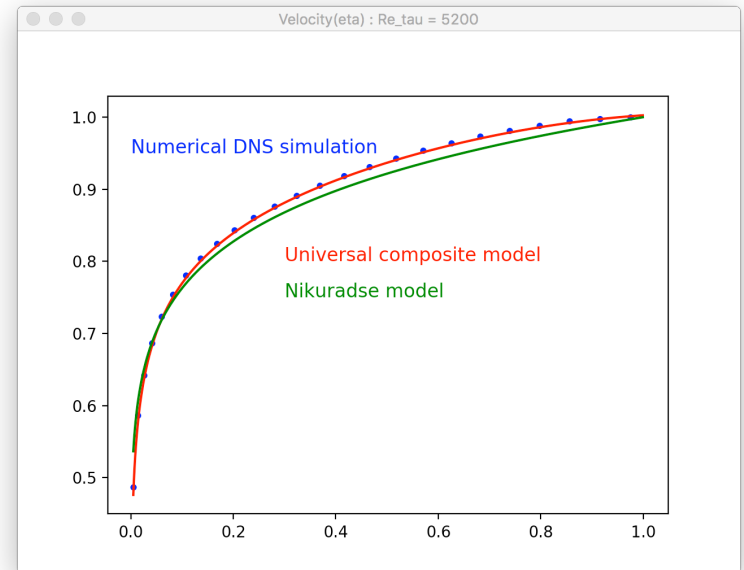
# Nikuradse Un bidouilleur de génie

$$\bar{u}(y) = \bar{u}_c \left[ \frac{y}{h} \right]^{1/n}$$





# Intégrales du déficit de vitesse



$$\bar{U}_m = \bar{U}_c \int_0^1 \eta^\alpha d\eta = \bar{U}_c \frac{1}{\frac{1}{m} + 1}$$

$\alpha = 1/m$

$$\left[ \frac{\eta^{\alpha+1}}{\alpha+1} \right]_0^1$$

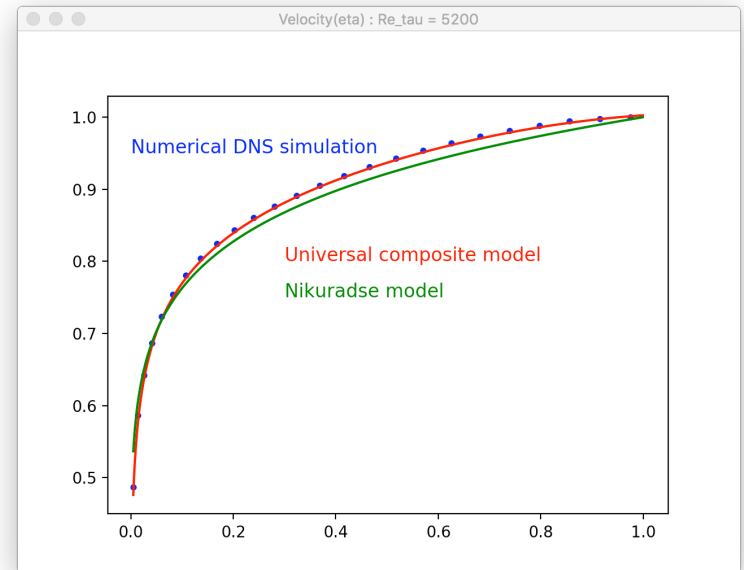
$$\bar{U}_m = \bar{U}_c \frac{m}{m+1}$$

$$\int_0^1 \bar{U}_c - \bar{U}(\eta) d\eta = \bar{U}_c - \bar{U}_c \frac{m}{m+1} = \bar{U}_c \left[ \frac{m+1-m}{m+1} \right] = \bar{U}_c \frac{1}{m+1}$$

$$I_{Nik} = \int_0^1 \frac{\bar{U}_c - \bar{U}(\eta)}{\bar{U}_c} d\eta = \frac{1}{\bar{U}_m} \frac{1}{(m+1)} \frac{\bar{U}_m}{\bar{U}_c} = \frac{1}{m} \frac{1}{\sqrt{\lambda}}$$

$\frac{1}{\bar{U}_m} \frac{1}{(m+1)} \frac{\bar{U}_m}{\bar{U}_c} = \frac{1}{m} \frac{1}{\sqrt{\lambda}}$

# Nikuradse



$$I_{Nik} = \frac{1}{n} \frac{2}{\sqrt{\lambda}}$$



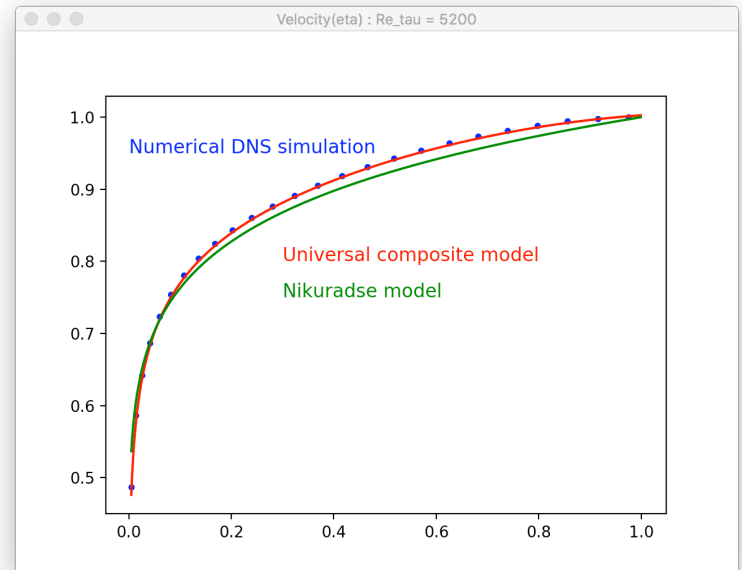
# Composite

$$D[3\alpha^2 - 2\alpha^3]$$

$$\frac{\bar{u}_c - \bar{u}(\eta)}{\bar{u}_\tau} = \frac{1}{\alpha} \log \left[ \frac{h \bar{u}_\tau}{15} \right] + G(\eta) - \frac{1}{\alpha} \log \left[ \frac{y \bar{u}_\tau}{15} \right] - G(\eta) - \frac{1}{\alpha} \log \left[ \frac{y}{h} \right]$$

$\eta$

$$-D[3\alpha^2 \eta^2 - 2\alpha^3 \eta^3]$$



$$I_{COLES} = -\frac{1}{\alpha} \int_0^1 \log(\eta) d\eta + D \left[ 3\alpha^2 \eta - 2\alpha^3 \eta - \frac{3\alpha^2 \eta^3}{3} - \frac{2\alpha^3 \eta^4}{4} \right]_0^1$$

$= -1$   $2\alpha^2 - 3\alpha^3/2 = 0$   $CAR \alpha = 4/3$

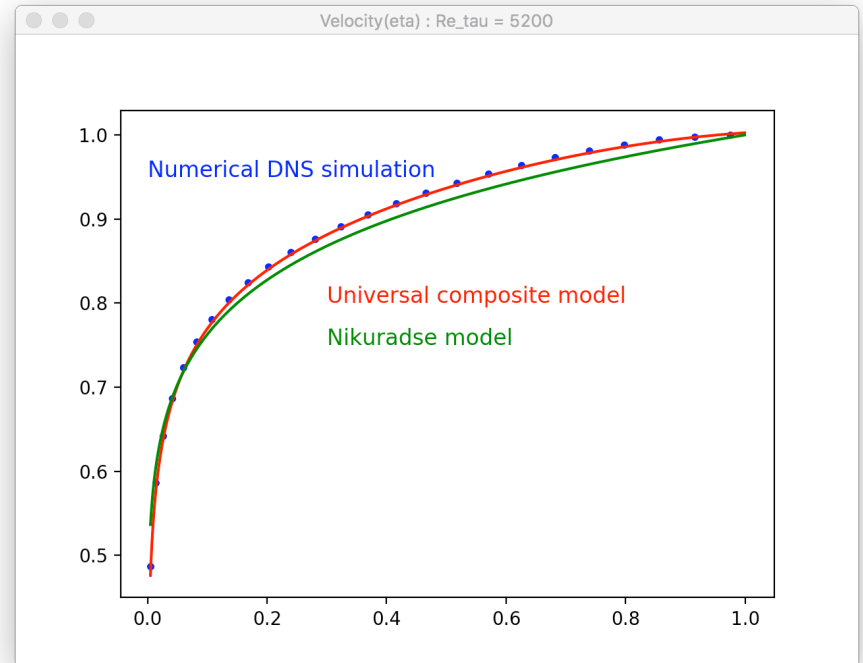
$$I_{COLES} = \frac{1}{\alpha}$$

# Trouver le bon exposant !

$$\eta = \frac{2 \delta^*}{\sqrt{\lambda}}$$

$$\bar{I}_{Nik} = \frac{1}{n} \frac{2}{\sqrt{\lambda}}$$

$$I_{COLES} = \frac{1}{\delta^*}$$



Profil logarithmique =====	kappa = 0.3840 et C = 4.1500
Correction de Coles =====	alpha = 1.3333 et D = 0.3700
Nikuradse exponent =====	1/n = 0.1177
Nikuradse exponent =====	n = 8.4958
Formule implicite de Prandtl =	a = -2.9982 et b = 2.0309

Mais, c'est pas aussi joli  
que le profil universel

