# Le plus grand paquebot prend le large !

$\nu = 10^{-6}\ m^2/s$

$$Re = \frac{10 \ 100}{10^{-6}} = 10^9$$


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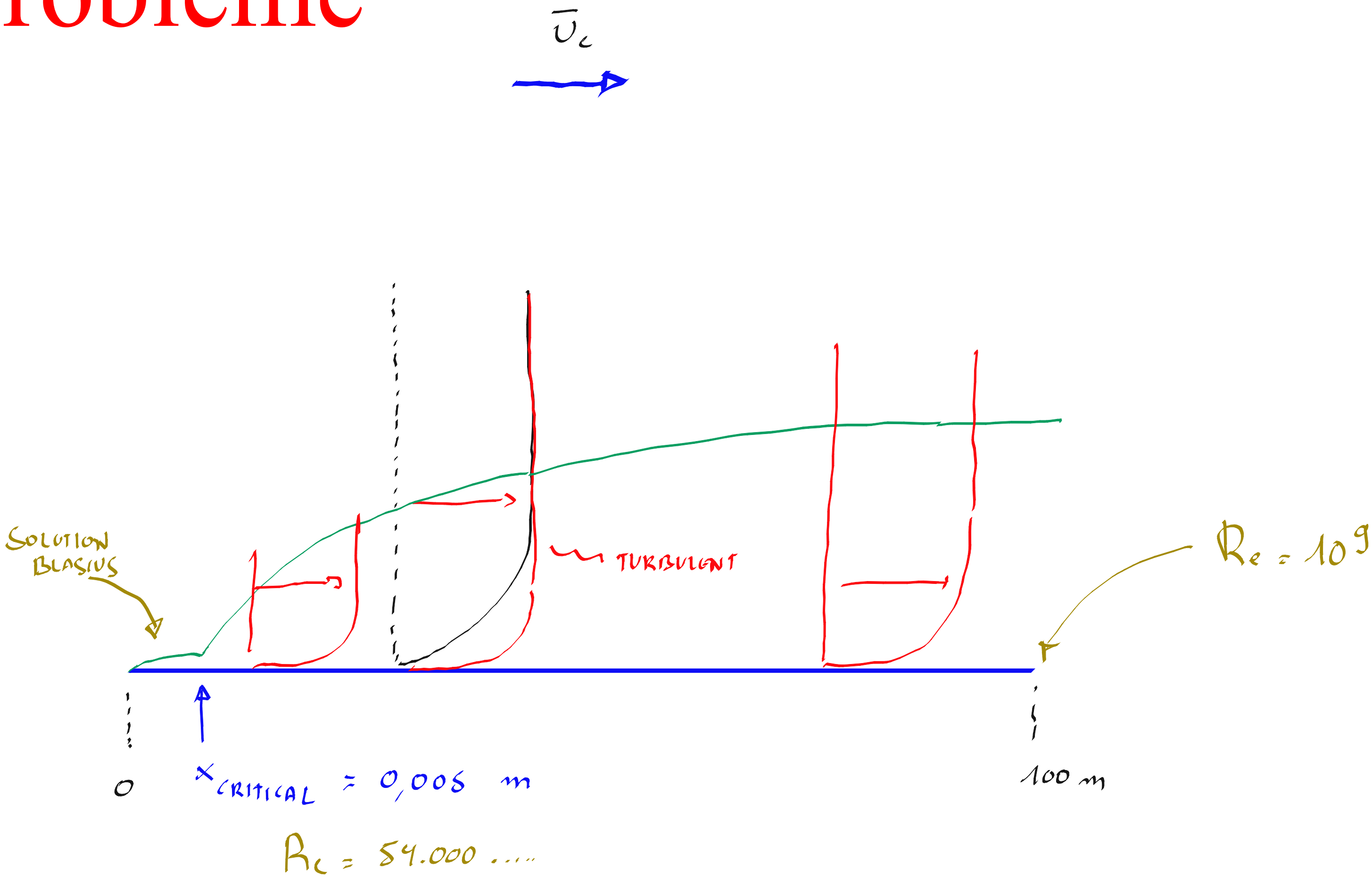
Re         =  1.00e+09
x_critical =  5.40e-03
```

# Dessiner le problème



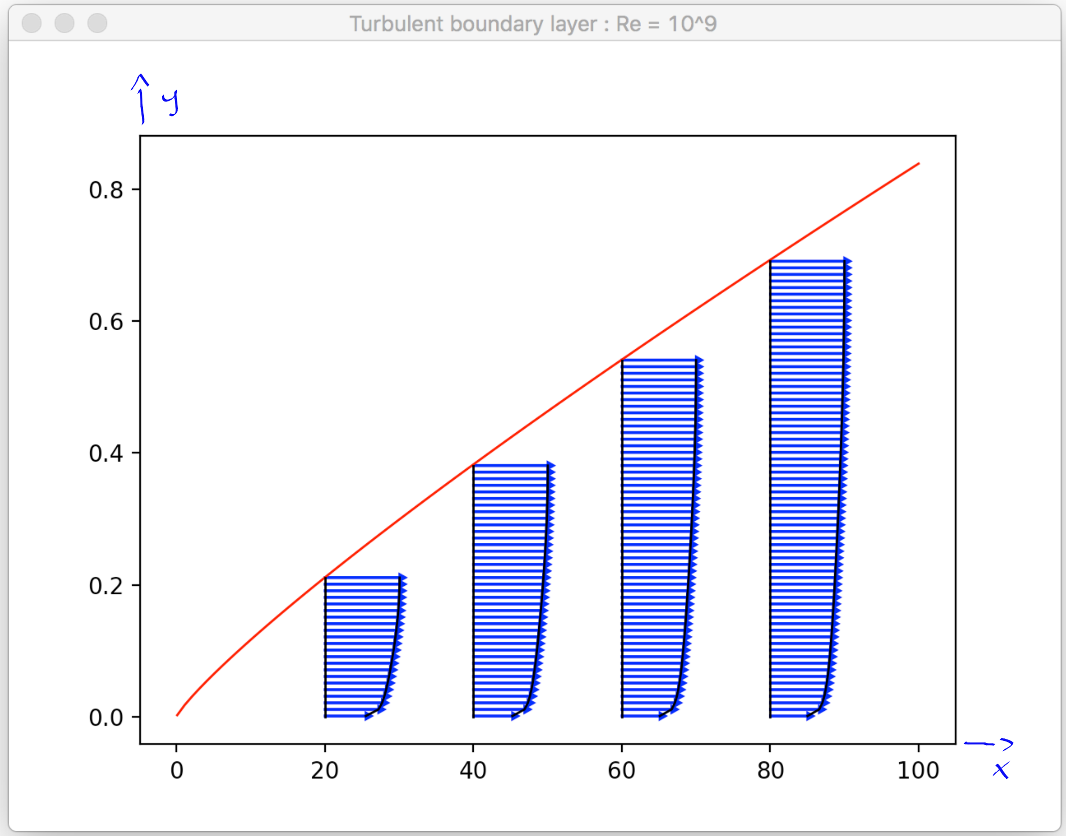
# Dessiner le problème

$$C_f = \frac{\overline{\tau}_w}{\rho\, \overline{v}_e^2/2}$$

$$\overline{v}_\tau^2 = \frac{\overline{\tau}_w}{\rho}$$

$$= \frac{\cancel{\rho}\, \overline{v}_\tau^2 \; 2}{\cancel{\rho}\, \overline{v}_e^2}$$

$$\sqrt{\frac{C_f}{2}} = \frac{\overline{v}_\tau}{\overline{v}_e} \qquad \square$$

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===== Profil logarithmique ======== kappa =  0.4100 et C =  5.0000
===== Correction de Coles ========= alpha =  1.1650 et D =  2.6829
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# Obtenir la formule des ingénieurs

$$\sqrt{\frac{2}{C_f}} = -a \log\left(\frac{b}{Re_\delta}\sqrt{\frac{2}{C_f}}\right)$$

$$Re_\delta = \frac{\delta \overline{u}_e}{\nu}$$

$$Re_x = \frac{x \overline{u}_e}{\nu}$$

$$\sqrt{\frac{2}{C_f}} = \underbrace{\frac{\overline{U}_e}{\overline{U}_\tau}}_{\overline{U}^+(\eta=1)\ (y^+=\delta^+)} = \frac{1}{\varkappa}\log\Big[\overbrace{\underbrace{\frac{\delta\,\overline{U}_\tau}{\nu}}}^{\delta^+}\Big] + C + G(1)$$

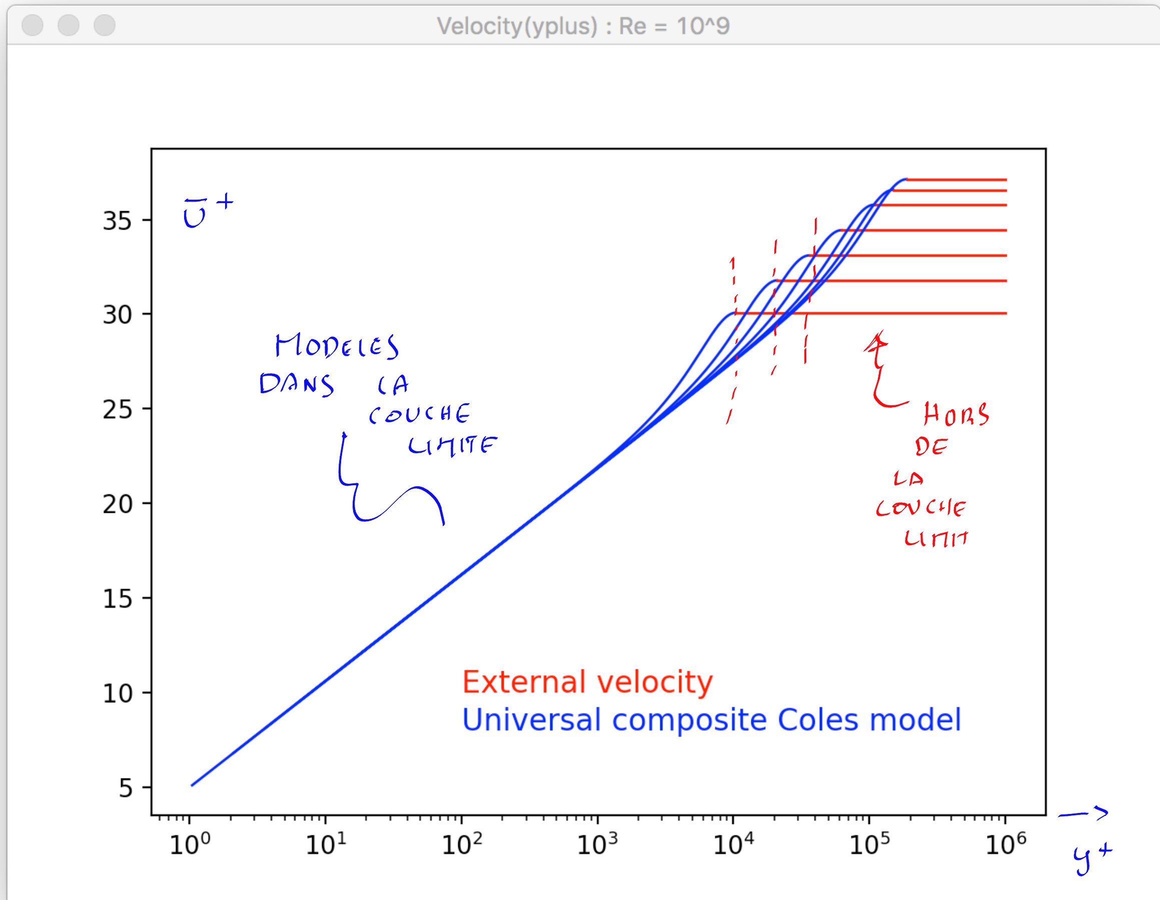
$$\underbrace{\frac{\delta\,\overline{U}_e}{\nu}}_{Re_\delta}\ \underbrace{\frac{\overline{U}_\tau}{\overline{U}_e}}_{\sqrt{\frac{C_f}{2}}}$$

$$\boxed{\begin{aligned} a &= 1/\varkappa \\ b &= 1/22 \end{aligned}}$$

$$= -\frac{1}{\varkappa}\log\left[\frac{1}{Re_\delta}\sqrt{\frac{2}{C_f}}\right] + \frac{1}{\varkappa}\log\left[\overbrace{\exp(\varkappa(C+G(1)))}\right]$$

$$= -\frac{1}{\varkappa}\log\left[\underbrace{\exp[-\varkappa(C+G(1))]}_{b}\ \frac{1}{Re_\delta}\sqrt{\frac{2}{C_f}}\right]$$

# Profils universels qui ne sont plus universels !



# Un profil simplifié

$$\overline{u}(\eta) = U\eta^{1/n}$$

$$\eta = \frac{y}{\delta}$$

$$\frac{\Theta}{\delta} = \frac{n}{(n+1)(n+2)}$$

$n = 7$

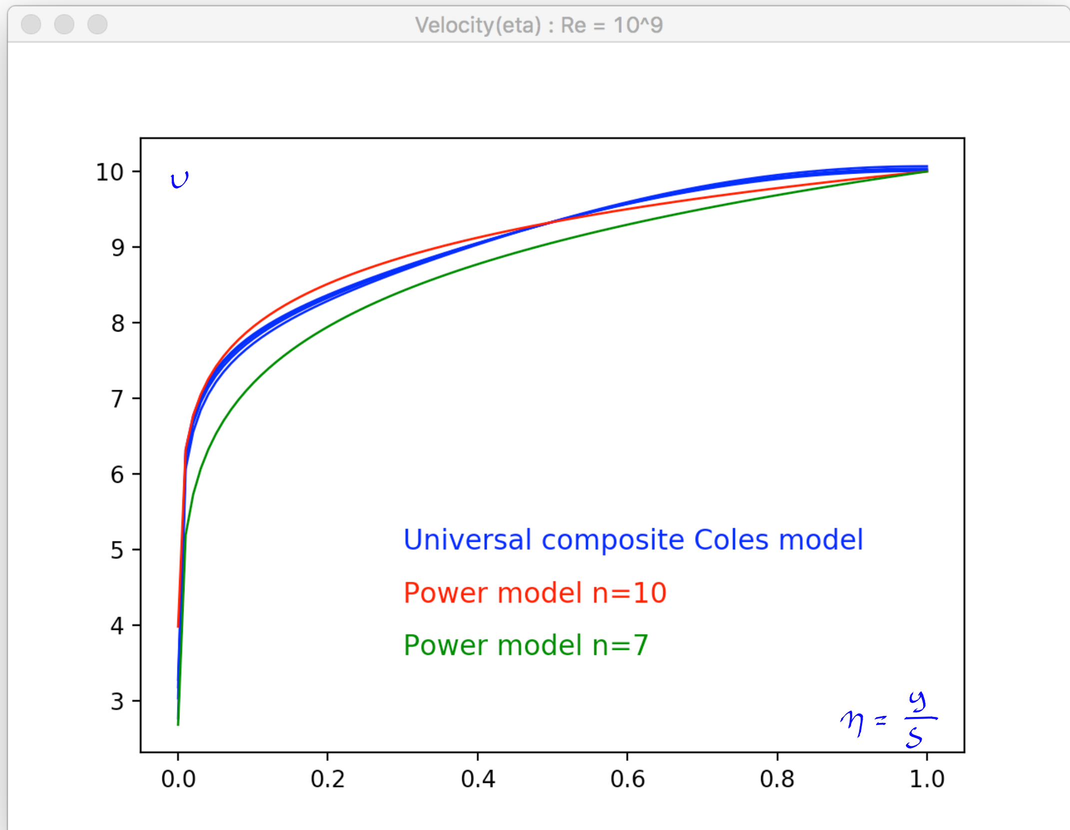
$$\boxed{\frac{\Theta(x)}{\delta(x)} = \frac{7}{72}}$$

$\alpha \triangleq 1/n$

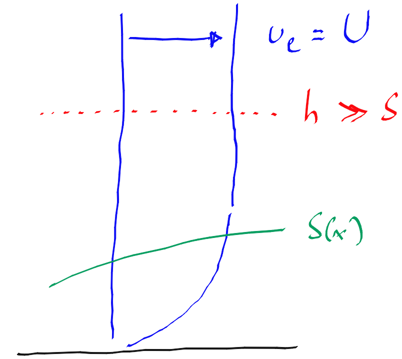
$$\Theta = \delta \int_0^1 \eta^{\alpha}(1-\eta^{\alpha})\, d\eta$$

$$= \delta \left[ \frac{\eta^{\alpha+1}}{\alpha+1} - \frac{\eta^{2\alpha+1}}{2\alpha+1} \right]_0^1$$

$$= \delta \left[ \frac{1}{\alpha+1} - \frac{1}{2\alpha+1} \right] = \delta \left[ \frac{\cancel{2}\alpha \cancel{+1} - \cancel{\alpha} - \cancel{1}}{(\alpha+1)(2\alpha+1)} \right] = \delta \left[ \frac{1/n}{(1/n+1)(2/n+1)} \right]$$



# von Karman le retour !




$$\int_0^h u \frac{\partial u}{\partial x} + \int_0^h v \frac{\partial u}{\partial y} = \underbrace{\int_0^h \frac{1}{\rho} \frac{\partial \tau}{\partial y}\, dy}_{\left[\frac{\tau}{\rho}\right]_0^h = -\frac{\tau_w}{\rho}}$$

$$\boxed{\int_0^h \int_0^y \underbrace{\frac{\partial v}{\partial y}}_{-\frac{\partial u}{\partial x}} dy \, \frac{\partial u}{\partial y}\, dy}$$

$$\int_0^h \int_0^y -\frac{\partial u}{\partial x}\, dy \, \frac{\partial u}{\partial y}\, dy$$

$$= \underbrace{\left[ \int_0^y -\frac{\partial u}{\partial x}\, dy \; u \right]_0^h}_{-\int_0^h u_e \frac{\partial u}{\partial x}} - \int_0^h -\frac{\partial u}{\partial x}\, u \, dy$$

$$\underbrace{\underbrace{\int_0^h 2u \frac{\partial u}{\partial x} - u_e \frac{\partial u}{\partial x}\, dy}_{\frac{\partial}{\partial x}\left(u(u-u_e)\right)}}_{u_e^2 \frac{d}{dx}(-\theta)}$$

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2}$$

$$\underbrace{\frac{d\theta}{dx}(x)}_{\sim\ \frac{7}{72} S'(x)} = \underbrace{\frac{\tau_w(x)}{\rho \bar{U}_e^2}}_{\frac{C_f(x)}{2}\ \simeq\ \frac{1}{100} \underbrace{Re_S^{-1/6}(x)}_{[S(x)]^{-1/6} \left[\frac{\bar{U}_e}{\nu}\right]^{-1/6}}}$$

$$\bar{u}(\eta) = U\eta^{1/n}$$

$$S'(x)\,[S(x)]^{1/6} = \frac{72}{700}\left[\frac{\bar{U}_e}{\nu}\right]^{-1/6}$$

# Un rapport constant qui devrait pas être constant…

# Obtenir une équation différentielle ordinaire

$$S'(x)\,[S(x)]^{1/6} = \frac{72}{700}\left[\frac{U_e}{\nu}\right]^{-1/6}$$

$$\frac{6}{7}[S(x)]^{7/6} = \frac{72}{700}\, x \left[\frac{U_e}{\nu}\right]^{-1/6}$$

$$\left[\frac{S(x)}{x}\right]^{7/6} = \frac{72}{600}\left[\frac{U_e\, x}{\nu}\right]^{-1/6}$$

$$S(x) = x\left[\left(\frac{72}{600}\right)^{6}\frac{1}{Re(x)}\right]^{1/7}$$

$$= x\;0{,}162\,[Re(x)]^{-1/7}$$

# Et zou !

$$\delta(x) = 0{,}162 \times \left[ Re(x) \right]^{-1/7}$$

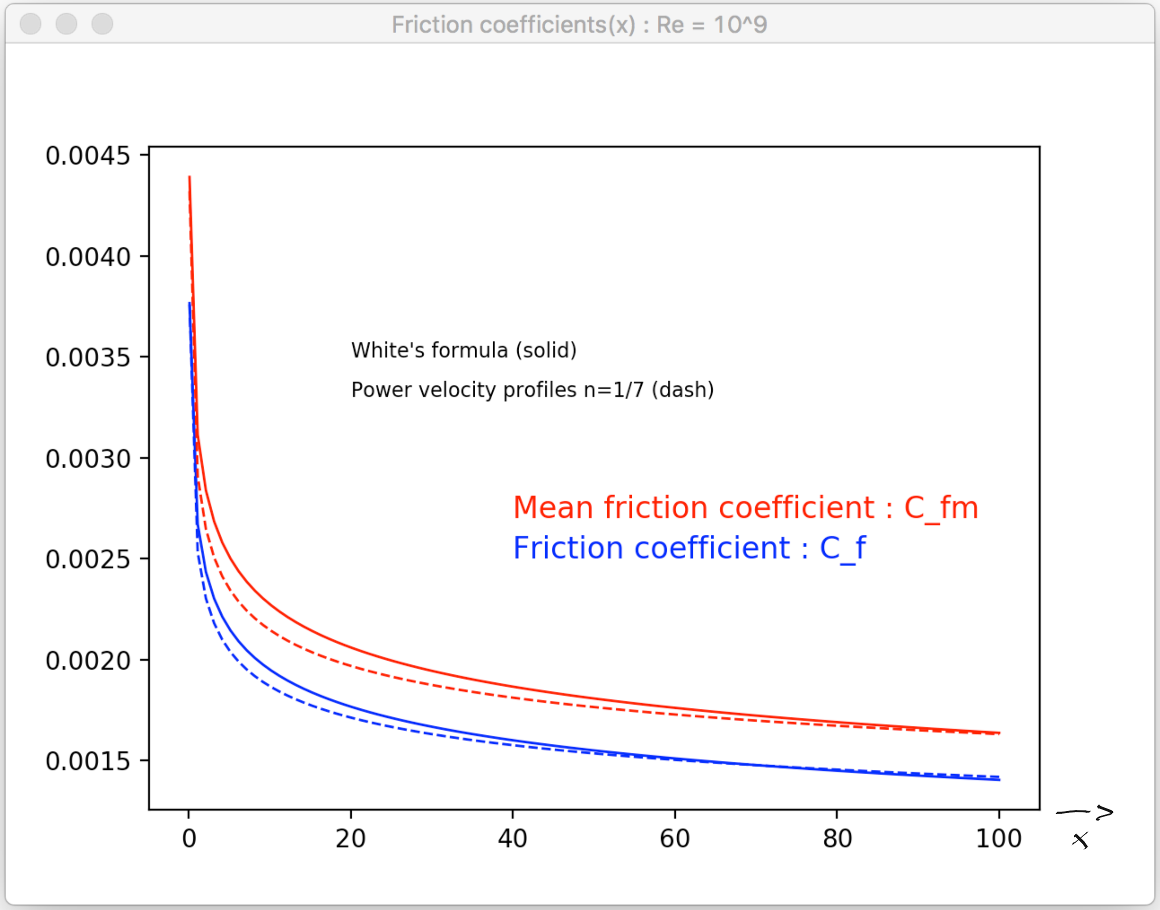
$$\theta(x) = \frac{7}{72}\, 0{,}162 \times \left[ Re(x) \right]^{-1/7}$$

$$C_f(x) = 2\,\frac{d\theta}{dx} = \frac{14}{72}\, 0{,}162\, \underbrace{\underbrace{\frac{d}{dx}\left[ x^{6/7} \right]}_{\frac{6}{7}\, x^{-1/7}} \left[ \frac{U_e}{\nu} \right]^{-1/7}}_{\frac{6}{7} \left[ Re(x) \right]^{-1/7}}$$

$$= 0{,}271 \left[ Re(x) \right]^{-1/7}$$

$$C_{fm}(x) = \frac{2\,\theta}{x} = \frac{14}{72}\, 0{,}162 \left[ Re(x) \right]^{-1/7}$$

# Et voilà
# le travail



# Mais oui pratiquement ?

$$F_{DRAG} = \underbrace{bL}_{2\,10^3}\ \frac{1}{2}\ \underset{10^3}{\rho}\ \underset{10^2}{\bar{v}_c^{\,2}}\ \underbrace{C_{f,m}(L)}_{1{,}63\ 10^{-3}}$$

$$F_{DRAG} = 1{,}63\ 10^{5}\ [N]$$

$$P = 1{,}63\ 10^{6}\ [Watt]$$

```
==== Rough turbulent boundary layer ======================
L/eps     =  5.0000000e+04
C_(fm)    =  3.5927299e-03 (Schlichting's formula)
Drag      =  3.5927299e+05 [N]
Power     =  3.5927299e+06 [W]
```

# Bateaux lisses et rugueux

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==== Smooth turbulent boundary layer =====================
C_(fm)    =  1.6367140e-03 (Power approximation n=1/7)
C_(fm)    =  1.6304878e-03 (White's approximated formula)
Drag      =  1.6304878e+05 [N]
Power     =  1.6304878e+06 [W]
```
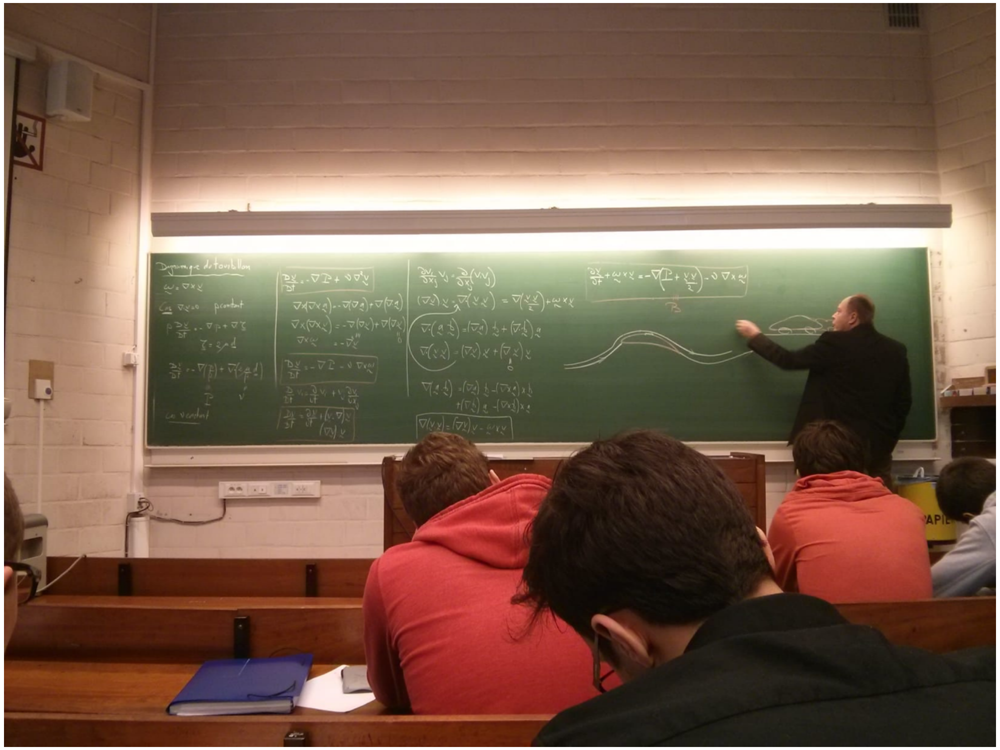

# Et la fameuse relation implicite exacte !



# Et la fameuse relation implicite exacte !