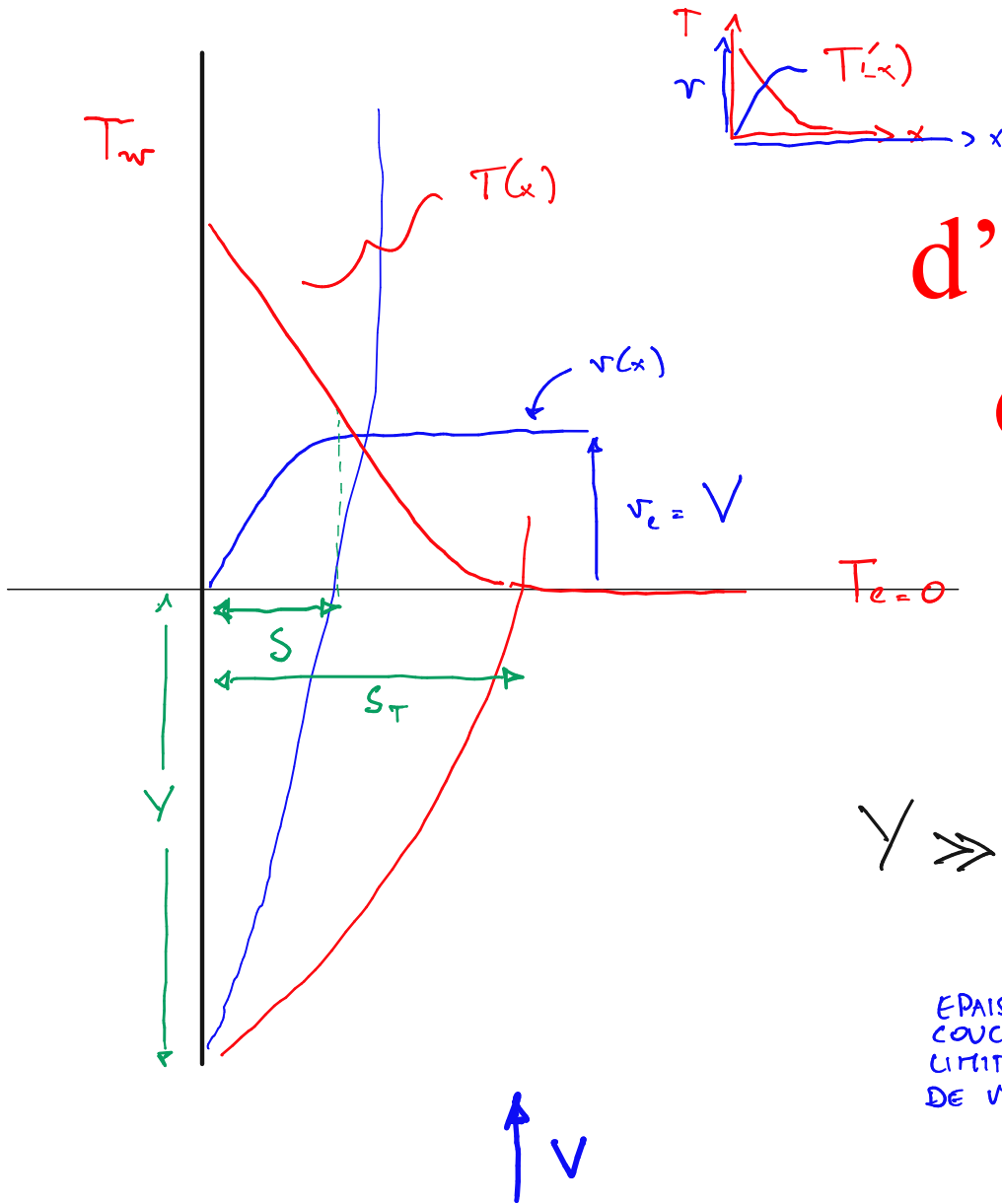


Mais, tout d'abord, un peu de convection forcée

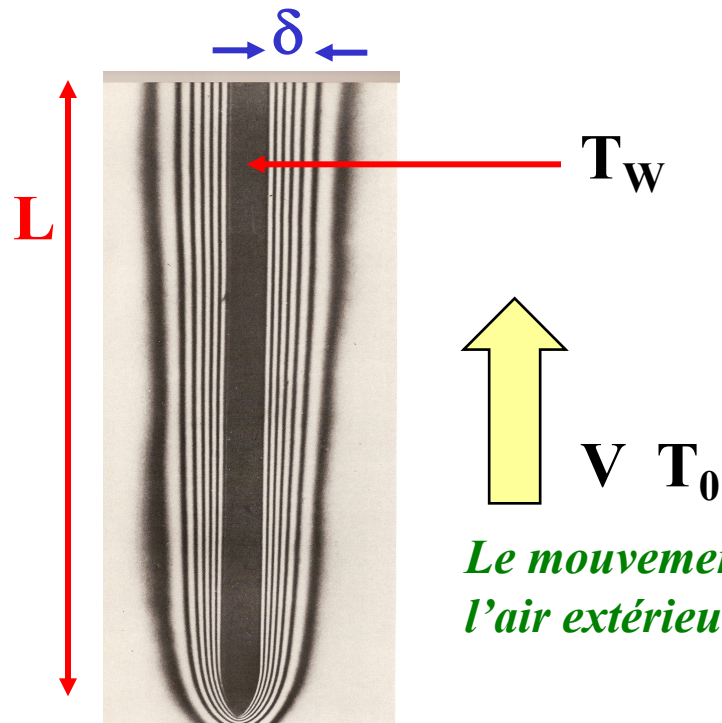


$Y \Rightarrow \delta, \delta_T$

EPAISSEUR  
COUCHE  
LIMITE  
DE VITESSE

EPAISSEUR  
COUCHE  
LIMITE  
THERMIQUE





*Le mouvement vertical de l'air extérieur est forcé*

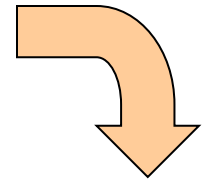
Mais, tout d'abord, un peu de convection forcée

*Plus facile car, il est ici possible de découpler le problème de l'écoulement et le problème thermique !*

Problème de l'écoulement

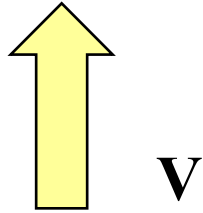
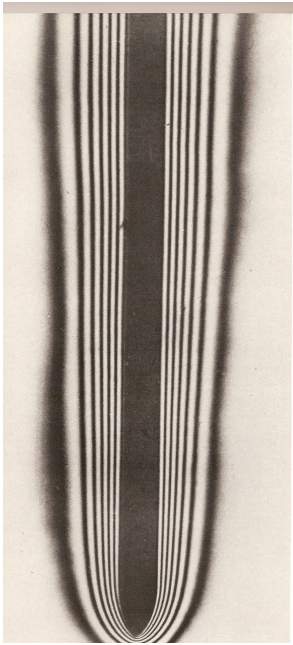
$$\nabla \cdot \mathbf{v} = 0,$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g},$$



$$\rho c \frac{DT}{Dt} = 2\mu \mathbf{d} : \mathbf{d} + r + \nabla \cdot (k \nabla T).$$

Problème thermique



# -i- problème de l'écoulement

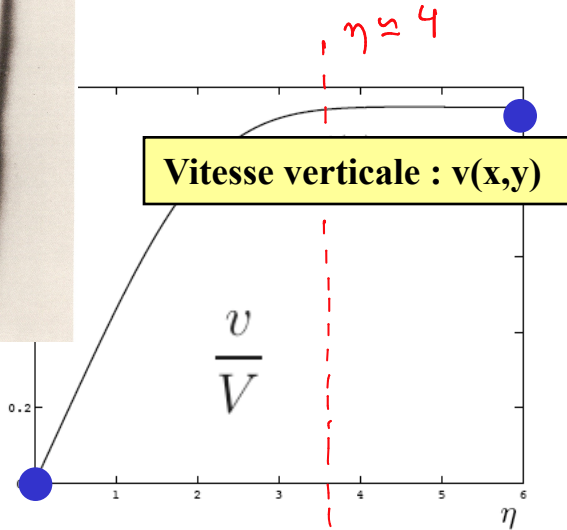
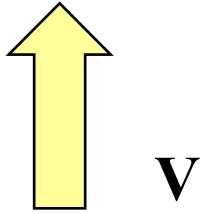
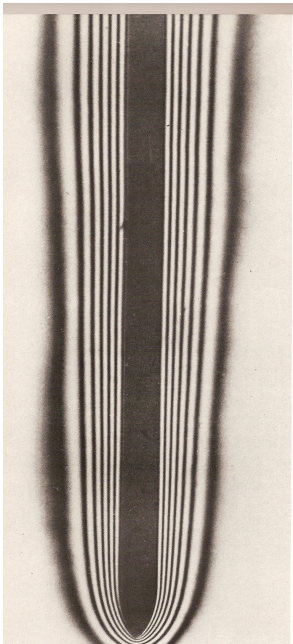
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

**On introduit**  
**une variable de similitude basée**  
**une estimation de la couche limite**

$$\eta(x, y) = \frac{x}{\delta(y)} = \frac{x}{\sqrt{\frac{2\nu y}{V}}}$$

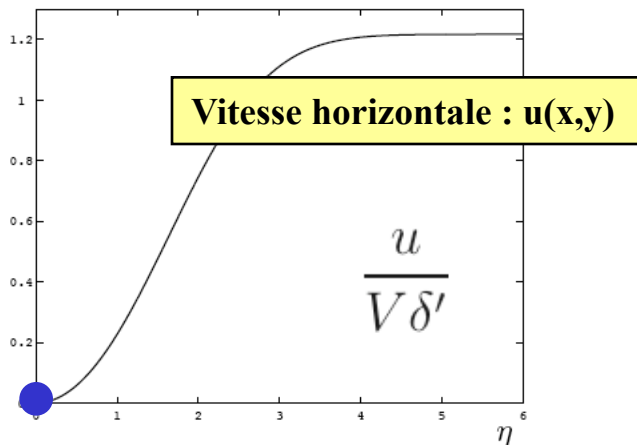


# Solution de Blasius



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

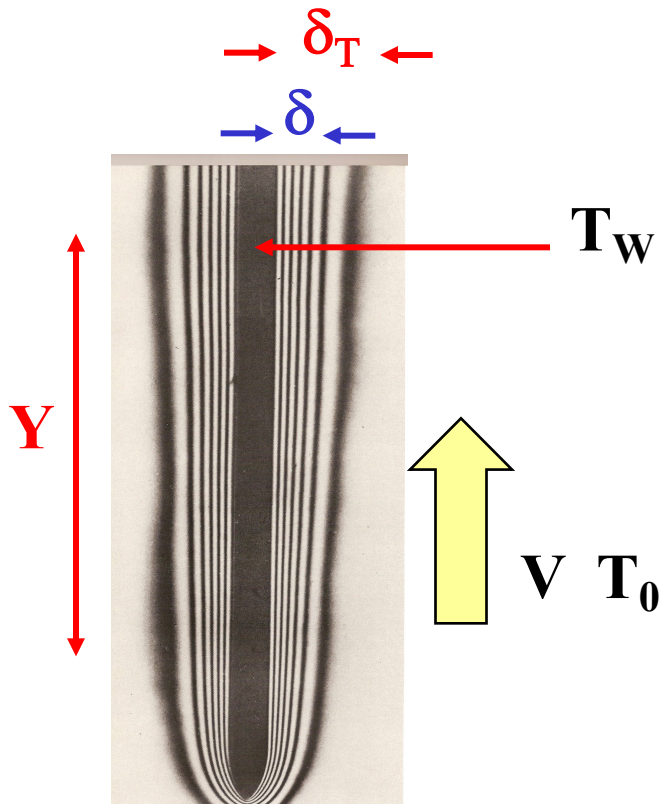


L'estimation de l'ordre de grandeur de la couche limite était bien adéquat !



$$\eta(x, y) = \frac{x}{\delta(y)} = \frac{x}{\sqrt{\frac{2\nu y}{V}}}$$

## -ii- problème thermique



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$\delta \ll Y$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\delta_T \ll Y$$

Près de la plaque, les effets conductifs sont dominants...

C'est la zone dite de couche limite thermique

Loin de la plaque, la conduction est négligeable

On définit l'épaisseur thermique comme le lieu géométrique où la conduction et la convection sont du même ordre de grandeur.

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} + \alpha \frac{\partial^2 T}{\partial y^2}$$

$$\frac{S}{Y} = \sqrt{\frac{1}{Re}}$$

$$\frac{U}{S} = \frac{V}{Y}$$

$$\frac{U \Delta T}{S} = \frac{V \Delta T}{Y}$$

$$\propto \frac{\Delta T}{S^2_T}$$

$$\frac{S_T}{Y} = \sqrt{\frac{1}{Pe}} = \sqrt{\frac{1}{Re Pr}}$$

$$\frac{V \Delta T}{Y}$$

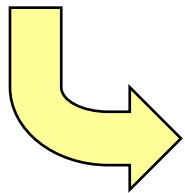
$$\frac{\cancel{V \Delta T}}{Y} \frac{S_T^2}{\cancel{\alpha \Delta T}} = 1$$

$$\frac{S_T^2}{Y^2} = \frac{\alpha}{V Y}$$

Et  $\delta_T$  ?

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} + \alpha \frac{\partial^2 T}{\partial y^2}$$

*Lieu où l'ordre de la convection et de la conduction sont identiques*



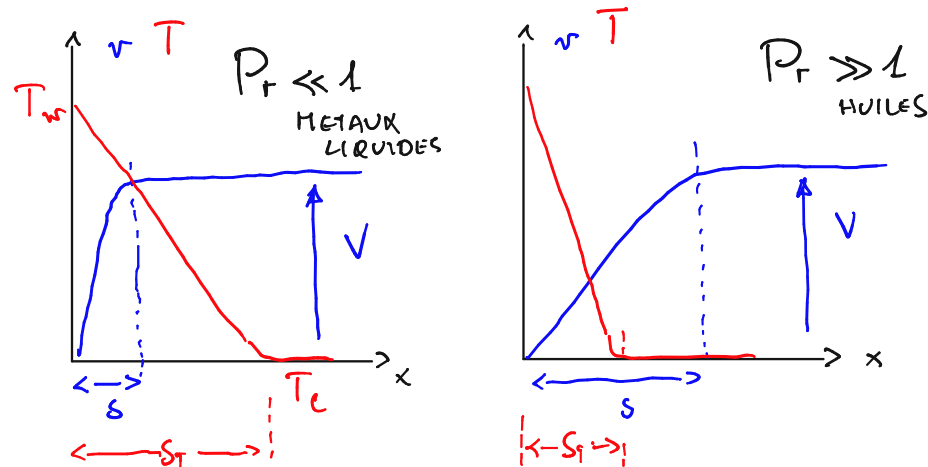
$$\frac{\text{Convection}}{\text{Conduction}} = \frac{V \Delta T / Y}{\alpha \Delta T / \delta_T^2} = \frac{V Y}{\underbrace{\alpha}_{Pe_Y}} \frac{\delta_T^2}{Y^2} = 1$$

Et  $\delta_T$  ?

$$\frac{\delta_T}{Y} = \sqrt{\frac{1}{Pe_Y}} = \sqrt{\frac{1}{Pr Re_Y}}$$

# Epaisseurs de couches limites et le nombre de Prandtl

$$\frac{\delta_T}{\delta} = \sqrt{\frac{1}{Pr}}$$



En fait, c'est uniquement l'ordre de grandeur...

Lorsqu'on calcule la vraie épaisseur pour Blasius, c'est plutôt un exposant 1/3 !

Car l'ordre de grandeur de vitesse est un peu surestimé en prenant U !

Métaux liquides	$Pr \ll 1$
Gaz	$Pr = 0.7$
Eau	$Pr = 2 \dots 7$
Huiles	$Pr \gg 1$

*Attention : Prantl est une fonction de la température (davantage pour les liquides que pour les gaz) !*

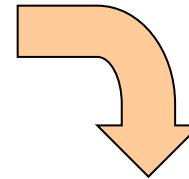


# Soyons un peu plus général : et la dissipation visqueuse ?

Problème de l'écoulement

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

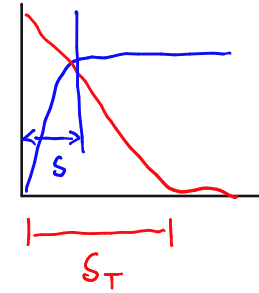


$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{c} \left( \frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$

Problème thermique

# Eckert : dissipation visqueuse ?

$$\underbrace{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}_{\text{TRANSPORT ADVECTION CONVECTION}} = \underbrace{\frac{\nu}{c} \left( \frac{\partial v}{\partial x} \right)^2}_{\text{DISSIPATION VISQUEUSE}} + \underbrace{\alpha \frac{\partial^2 T}{\partial x^2}}_{\text{CONDUCTION}}$$



TRANSPORT  
ADVECTION  
CONVECTION

$$\frac{1}{c} \frac{V^2}{S^2}$$

DISSIPATION  
VISQUEUSE

$$\propto \frac{\Delta T}{S_T^2}$$

CONDUCTION

$$\frac{\text{DISSIPATION}}{\text{CONDUCTION}} = \frac{1}{c} \frac{V^2}{S^2} \frac{S_T^2}{\alpha \Delta T} = \underbrace{\frac{1}{c} \frac{V^2}{\Delta T}}_{E_c} \underbrace{\frac{S_T^2}{\alpha S^2}}_{\frac{1}{Pr}}$$

$\frac{1}{\alpha}$  is labeled  $Pr$  and  $\frac{S_T^2}{S^2}$  is labeled  $1/Pr$ .

# Eckert : dissipation visqueuse ?

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{c} \left( \frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$

*Dissipation visqueuse*
*Conduction*

$$\frac{\text{■}}{\text{■}} = \frac{\nu V^2}{\alpha c \Delta T} \underbrace{\left( \frac{\delta_T}{\delta} \right)^2}_{Pr^{-1}} = \frac{V^2}{c \Delta T} = Ec$$

# Nombre d'Eckert

$$Ec = \frac{u_e^2}{c(T_w - T_e)}$$

caractérise un écoulement  
d'un fluide !

**Energie cinétique**

---

**Energie interne**



Picture was taken on August 22, 2000

$$Pr = 1$$

$$Ec \ll 1$$

$$Pr = 1$$

$$Ec \not\ll 1$$

*Couches  
limites  
identiques*

$$Pr \neq 1$$

$$Ec \ll 1$$

$$Pr \neq 1$$

$$Ec \not\ll 1$$

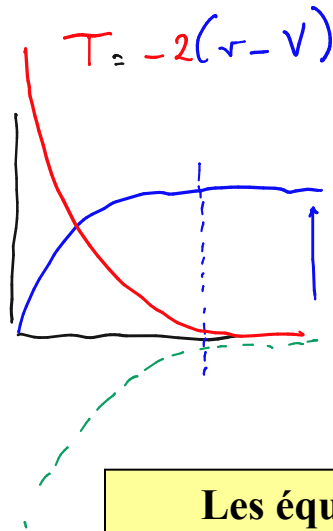
*Dissipation  
visqueuse  
négligeable*



**Quatre cas  
possibles !**

$Pr = 1$	$Pr = 1$
$Ec \ll 1$	$Ec \ll 1$

$Pr \neq 1$	$Pr \neq 1$
$Ec \ll 1$	$Ec \ll 1$



$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nu \frac{\partial^2 T}{\partial x^2}$$

↖  $ca Pr = 1 :-)$

**Les équations d'énergie et de quantité de mouvement expriment les mêmes opérateurs différentiels pour la température et la vitesse verticale !**

$Pr = 1$

$Ec \ll 1$

$$T = Av + B$$

Si, si, c'est aussi simple !

# Relation de Crocco

$Pr = 1$ $Ec \ll 1$	$Pr = 1$ $Ec \ll 1$
$Pr \neq 1$ $Ec \ll 1$	$Pr \neq 1$ $Ec \ll 1$

$$cT + \frac{v^2}{2} = Av + B$$

On a les mêmes opérateurs différentiels pour l'énergie interne totale et la vitesse verticale !

$$Pr = 1$$

$$Ec \ll 1$$

$$v \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = v \left[ \nu \frac{\partial^2 v}{\partial x^2} \right]$$

$$c \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = c \left[ \frac{\nu}{c} \left( \frac{\partial v}{\partial x} \right)^2 + \nu \frac{\partial^2 T}{\partial x^2} \right]$$

$$\frac{\partial}{\partial x} \left( \frac{v^2}{2} \right) = v \frac{\partial v}{\partial x}$$

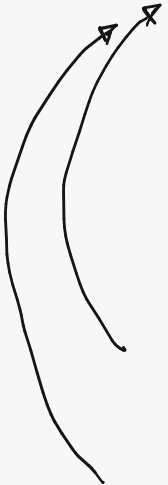
$$cT + \frac{v^2}{2} = Av + B$$

$$c \frac{\partial}{\partial x} \left( cT + \frac{v^2}{2} \right) + v \frac{\partial}{\partial y} \left( cT + \frac{v^2}{2} \right) = \frac{\partial}{\partial x^2} \left( cT + \frac{v^2}{2} \right)$$

$$= \frac{\partial}{\partial x} \left[ v \frac{\partial v}{\partial x} \right]$$

$$= \frac{\partial v^2}{\partial x^2} + v \frac{\partial^2 v}{\partial x^2}$$

$$+ \left[ c \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \left[ \frac{\partial v^2}{\partial x^2} + v \frac{\partial^2 v}{\partial x^2} \right] + \left[ \frac{\partial T}{\partial x^2} \right]$$





# Relation de Crocco

$Pr = 1$ $Ec \ll 1$	$Pr = 1$ $Ec \ll 1$
$Pr \neq 1$ $Ec \ll 1$	$Pr \neq 1$ $Ec \ll 1$

$$cT + \frac{v^2}{2} = Av + B$$

On a les mêmes opérateurs différentiels pour l'énergie interne totale et la vitesse verticale !

$$Pr = 1$$

$$Ec \ll 1$$

$$v \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = v \left[ \nu \frac{\partial^2 v}{\partial x^2} \right]$$

$$c \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = c \left[ \frac{\nu}{c} \left( \frac{\partial v}{\partial x} \right)^2 + \nu \frac{\partial^2 T}{\partial x^2} \right]$$

$Pr = 1$	$Pr = 1$
$Ec \ll 1$	$Ec \ll 1$
$Pr \neq 1$	$Pr \neq 1$
$Ec \ll 1$	$Ec \ll 1$

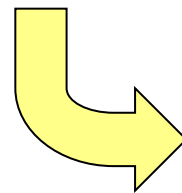
$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2}$$

↑  
!!!

$$\theta(\eta) = \frac{T - T_w}{T_e - T_w}$$

On procède comme pour Blasius !



$$Pr f \theta' + \theta'' = 0$$

Solution de similitude

$Pr \neq 1$
$Ec \ll 1$

$$\exp(g) \underbrace{Pr f}_{g'(n)} \Theta' + \exp(g) \Theta'' = 0$$

$$\underbrace{(\exp g)'} \Theta' + \exp(g) \Theta'' = 0$$

$$(\exp(g) \Theta')' = 0$$

$$\exp(g) \Theta' = A$$

$$\Theta' = A \exp(-g)$$

$$\Theta = A \int_0^n \exp(-g) + B$$

$$Pr f \theta' + \theta'' = 0$$

Calcul de la solution de similitude

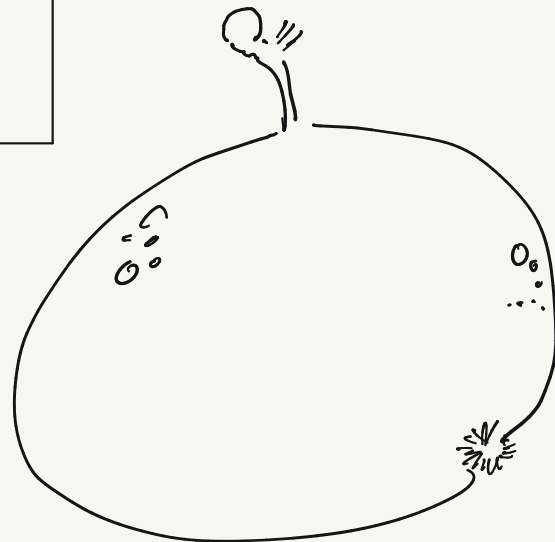
$$\Theta(\eta) = A \int_0^\eta \exp \left[ -Pr \int_0^\zeta f(\xi) d\xi \right] d\zeta + B$$

$$\Theta(\eta) = \frac{T - T_w}{T_c - T_w}$$

$$\Theta(0) = 0$$

$$\Theta(\eta \rightarrow \infty) = 1$$

$$\Theta(\eta) = \frac{\int_0^\eta \exp \left[ -Pr \int_0^\zeta f(\xi) d\xi \right] d\zeta}{\int_0^\infty \exp \left[ -Pr \int_0^\zeta f(\xi) d\xi \right] d\zeta}$$



$$\begin{array}{l} Pr = 1 \\ Ec \ll 1 \end{array}$$

$$\begin{array}{l} Pr = 1 \\ Ec \lll 1 \end{array}$$

$$\begin{array}{l} Pr \neq 1 \\ Ec \ll 1 \end{array}$$

$$\begin{array}{l} Pr \neq 1 \\ Ec \lll 1 \end{array}$$

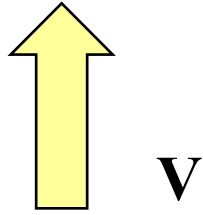
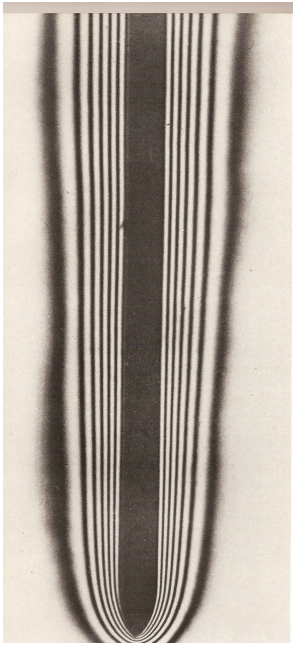
$$\begin{array}{l} Pr \neq 1 \\ Ec \lll 1 \end{array}$$

**Le cas le plus général  
(et donc aussi le plus probable !)**

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{c} \left( \frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$

**Pas de solution analytique...**

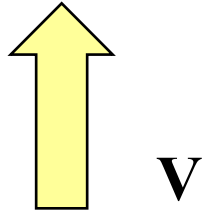
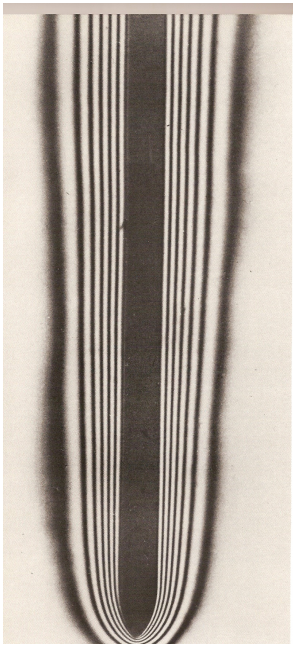


$$\begin{aligned}
 \tau_w &= \mu \left. \frac{\partial v}{\partial x} \right|_{x=0} \\
 &= \mu V \underbrace{f'' \Big|_{\eta=0}}_{0,4696} \underbrace{\frac{1}{5}}_{\sqrt{\frac{V}{2 \nu y}}} \\
 &= 0,332 \sqrt{\frac{\mu^2 V^3}{\nu y}} \underbrace{\sqrt{\frac{\nu^2 \rho^2 V^3}{\nu y}}}_{\sqrt{\frac{\nu}{V y}}} \\
 &= \frac{\rho V^2}{2} \underbrace{0,664}_{\underbrace{\sqrt{\frac{\nu}{V y}}}_{Re^{-1/2}}}
 \end{aligned}$$

$$\begin{aligned}
 C_f &= \frac{\tau_w}{\rho V^2 / 2} \\
 &= 0,664 Re^{-1/2}
 \end{aligned}$$



# Calcul de la force de trainée



$$\begin{aligned}
 q_w &= -k \left. \frac{\partial T}{\partial x} \right|_{x=0} \\
 &= -k (T_e - T_w) \underbrace{\Theta' \Big|_{\eta=0}}_{\substack{\approx 0,332 \sqrt{2} Pr^{1/3} \\ \text{VARIABLE si } Pr > 0,01}} \frac{1}{s} \sqrt{\frac{V}{2 \nu y}} \\
 &= -k (T_e - T_w) 0,332 \sqrt{\frac{V}{\nu y}} Pr^{1/3} \\
 &= -\rho c V (T_e - T_w) 0,332 \underbrace{\frac{k}{\rho c}}_{\alpha} \underbrace{\sqrt{\frac{V}{V^2 \nu y}}}_{\frac{1}{U} \sqrt{\frac{U}{\nu y}}} Pr^{1/3} \\
 &= -\rho c V (T_e - T_w) 0,332 Pr^{-1} Re^{-1/2} \\
 &= -\rho c V (T_e - T_w) 0,332 Pr^{-2/3} Re^{-1/2}
 \end{aligned}$$



$$\begin{aligned}
 St &= \frac{-q_w}{\rho c V (T_e - T_w)} \\
 &= 0,332 Pr^{-2/3} Re^{-1/2}
 \end{aligned}$$

# Calcul du flux de chaleur

$$C_f = 2 Pr^{2/3} St$$

FORCE  
DE FROTTEMENT  
ADIMENSIONNELLE

FLUX DE  
CHALEUR  
ADIMENSIONNEL

$$C_f = \frac{\tau_w}{\rho V^2 / 2}$$
$$= 0,664 Re^{-1/2}$$



$$St = \frac{-q_w}{\rho c V (T_e - T_w)}$$
$$= 0,332 Pr^{-2/3} Re^{-1/2}$$



# Analogie de Reynolds