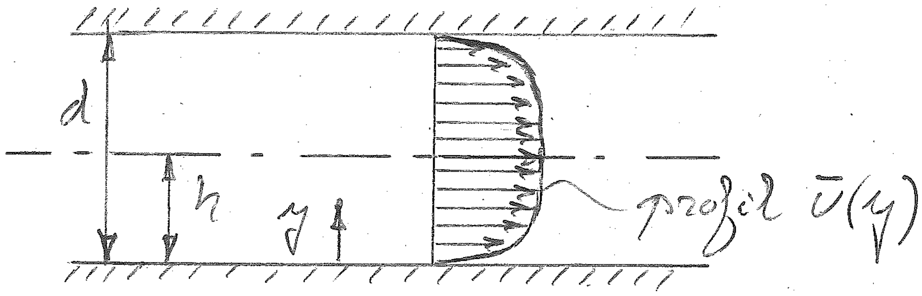
LMECA 1321 Cours 10/11/21

Ecoulement turbulent établi en canal



$$\begin{cases} U(x,y,z,t) \\ V(x,y,z,t) \\ W(x,y,z,t) \\ P(x,y,z,t) \end{cases} \xRightarrow[\text{temporelle}]{\text{moyenne}} \begin{cases} \bar{U}(y) \\ \bar{V}(y)=0 \\ \bar{W}(y)=0 \\ \bar{P}(x,y) \end{cases}$$

$$\bar{\varphi} \triangleq \frac{1}{T}\int_{t_0-T/2}^{t_0+T/2} \varphi(t)\,dt \quad \equiv \text{moyenne de Reynolds}$$

avec $T \gg T_F$ ≡ temps caractéristique des fluctuations turbulentes "les plus lentes"
(i.e. celles associées aux grandes structures)

t_0 = n'importe quel temps vu que l'écoulement établi est statistiquement stationnaire.

$$\varphi = \bar{\varphi} + \varphi'$$

moyenne

fluctuation relativement à la moyenne

$\hookrightarrow \overline{\varphi'} = 0$

$\hookrightarrow \bar{\varphi} = \bar{\bar{\varphi}}$

<u>variance de la fluctuation</u>

$$\overline{\varphi'\varphi'} = \overline{(\varphi')^2} \triangleq \frac{1}{T} \int_{t_0 - T/2}^{t_0 + T/2} \varphi'(t)\,\varphi'(t)\,dt$$

$\hookrightarrow$ <u>écart type moyen</u> (= "root mean square" = rms)

$$\varphi'_{rms} \triangleq \left(\overline{\varphi'\varphi'}\right)^{1/2}$$

<u>Commutation</u> :

$$\overline{\left(\frac{\partial \varphi}{\partial x_i}\right)} = \frac{\partial}{\partial x_i} \bar{\varphi}$$

$u' \rightarrow \overline{u'u'}$

$v' \rightarrow \overline{v'v'}$

$w' \rightarrow \overline{w'w'}$

$$\bar{k} \triangleq \frac{\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right)}{2}$$

$\equiv$ énergie cinétique de la turbulence

(= "turbulent kinetic energy" $\equiv$ tke)

$$\varphi = \bar{\varphi} + \varphi'$$

$$\psi = \bar{\psi} + \psi'$$

$$\hookrightarrow \varphi\psi = (\bar{\varphi} + \varphi')(\bar{\psi} + \psi') = \bar{\varphi}\,\bar{\psi} + \varphi'\bar{\psi} + \bar{\varphi}\,\psi' + \varphi'\psi'$$

$$\hookrightarrow \overline{\varphi\psi} = \bar{\varphi}\,\bar{\psi} + 0 + 0 + \overline{\varphi'\psi'}$$

$$= \bar{\varphi}\,\bar{\psi} + \overline{\varphi'\psi'}$$

- <u>covariance des fluctuations φ' et ψ'</u>

$$\overline{\varphi'\psi'} \triangleq \frac{1}{T} \int_{t_0 - T/2}^{t_0 + T/2} \varphi'(t)\, \psi'(t)\, dt$$

$$\left.\begin{matrix} u' \\ v' \end{matrix}\right. \Rightarrow \overline{u'v'}$$

<u>Canal</u> $\begin{cases} \bar{u}, \overline{u'u'}, \overline{v'v'}, \overline{w'w'}, \overline{u'v'} \equiv \text{fonctions de } y \\ \bar{v}, \overline{u'w'}, \overline{v'w'} \equiv 0 \end{cases}$

$$\hookrightarrow \begin{cases} \dfrac{d}{dy}\overline{u'v'} = -\dfrac{\partial \bar{P}}{\partial x} + \nu \dfrac{d^2\bar{u}}{dy^2} & (1) \\[2ex] \dfrac{d}{dy}\overline{v'v'} = -\dfrac{\partial \bar{P}}{\partial y} & (2) \end{cases}$$

(2)

$$\hookrightarrow \frac{\partial}{\partial y}\left(\bar{P} + \overline{v'v'}\right) = 0$$

$$\hookrightarrow \bar{P}(x,y) + \overline{v'v'}(y) = \bar{P}(x,0) \triangleq \bar{P}_w(x)$$

$$\boxed{\bar{P}(x,y) = \bar{P}_w(x) - \overline{v'v'}(y)}$$

$$\hookrightarrow \frac{\partial \bar{P}}{\partial x} = \frac{d\bar{P}_w}{dx}(x)$$

(1)

$$\hookrightarrow \frac{d}{dy}\left(\nu \frac{d\bar{u}}{dy} + \left(-\overline{u'v'}\right)\right) = \frac{d\bar{P}_w}{dx}(x)$$

$\hookrightarrow \dfrac{\partial \bar{P}}{\partial x} = \dfrac{d\bar{P}_w}{dx}$ est constant !

$$\hookrightarrow \nu \frac{d\bar{u}}{dy} + \left(-\overline{u'v'}\right) = -\frac{d\bar{P}_w}{dx}(h-y) = -\frac{d\bar{P}_w}{dx} h \left(1 - \frac{y}{h}\right)$$

$$\underbrace{\mu \frac{d\bar{u}}{dy}}_{\bar{\tau}} + \underbrace{\left(-\rho \overline{u'v'}\right)}_{\bar{\tau}^t} = -\frac{d\bar{p}_w}{dx} h \left(1 - \frac{y}{h}\right)$$

$$\bar{\tau} + \bar{\tau}^t = \tau_w \left(1 - \frac{y}{h}\right)$$

contrainte effective due à la turbulence

contrainte due à la viscosité moléculaire

En effet :

- $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

  $$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uu) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw) = \ldots$$

- $$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

  $$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(vu) + \frac{\partial}{\partial y}(vv) + \frac{\partial}{\partial z}(vw) = \ldots$$

- $$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

  $$\frac{\partial w}{\partial t} + \frac{\partial}{\partial x}(wu) + \frac{\partial}{\partial y}(wv) + \frac{\partial}{\partial z}(ww) = \ldots$$

$\Downarrow$ Moyenne de Reynolds en canal

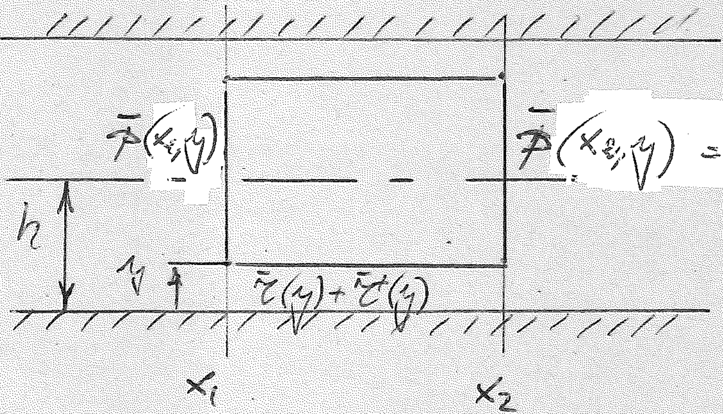
- $$\frac{\partial \bar{u}}{\partial x} + 0 + 0 = 0$$

- $$0 + 0 + \frac{d}{dy}\left(\overline{u'v'}\right) + 0 = -\frac{\partial \bar{P}}{\partial x} + \nu\frac{d^2\bar{u}}{dy^2}$$

- $$0 + 0 + \frac{d}{dy}\left(\overline{v'v'}\right) + 0 = -\frac{\partial \bar{P}}{\partial y} + 0$$

- $$0 + 0 + 0 + 0 = 0 + 0$$

Approche par volume de contrôle



$$\bar p(x_2,y) = \bar p(x_1,y) + \frac{\partial \bar p}{\partial x}(x_2-x_1)$$

volume avec $0<y<h$: $$2\left(\bar\tau(y)+\bar\tau^t(y)\right)(x_2-x_1) = 2\int_y^h \bar p(x_1,y)\,dy - 2\int_y^h \bar p(x_2,y)\,dy$$
$$= -2\,\frac{\partial \bar p}{\partial x}(x_2-x_1)\,(h-y)$$

↳ $$\bar\tau(y)+\bar\tau^t(y) = -\frac{\partial\bar p}{\partial x}(h-y)$$

volume avec $y=0$: $$\bar\tau_w = -\frac{d\bar p_w}{dx}\,h$$ et $$\frac{\partial \bar p}{\partial x} = \frac{d\bar p_w}{dx} \equiv \text{constante}$$

↳ $$\bar\tau(y)+\bar\tau^t(y) = \bar\tau_w\left(1-\frac{y}{h}\right)$$ ok idem!

Rappel

$$-\frac{\partial \bar p}{\partial x} = -\frac{d\bar p_w}{dx} \triangleq \left(\frac12\rho\bar U_m^2\right)\lambda\,\frac1d$$

coefficient de pertes de charge

$$\frac{\bar\tau_w}{h}$$

coefficient de frottement pariétal

↳ $$\frac{\bar\tau_w}{h} = \left(\frac12\rho\bar U_m^2\right)\lambda\,\frac{1}{2h} \rightarrow C_f \triangleq \frac{\bar\tau_w}{\frac12\rho\bar U_m^2} = \frac{\lambda}{2}$$

$$\nu \frac{d\bar{u}}{dy} + (-\overline{u'v'}) = \frac{\bar{\tau}_w}{\rho}\left(1 - \frac{y}{h}\right)$$

$$\bar{u}_\tau \triangleq \sqrt{\frac{\bar{\tau}_w}{\rho}}$$ ≡ vitesse de frottement pariétal

(≡ "friction velocity")

↳ $$\nu \frac{d\bar{u}}{dy} + (-\overline{u'v'}) = \bar{u}_\tau^2\left(1 - \frac{y}{h}\right)$$

↳ voir figures syllabus

Aussi

$$y^+ \triangleq \frac{y\,\bar{u}_\tau}{\nu}$$ ≡ distance adimensionelle à la paroi

$$h^+ \triangleq Re_\tau = \frac{h\,\bar{u}_\tau}{\nu}$$ ≡ nombre de Reynolds basé sur la vitesse de frottement

(Rappel: $Re_d = \frac{d\,\bar{u}_m}{\nu}$)

$$u^+ = \frac{\bar{u}}{\bar{u}_\tau}$$ ≡ profil de vitesse adimensionel

Dans la région proche de la paroi ($\eta \triangleq \frac{y}{h} \leq 0.15$)

$$u^+ = f(y^+)$$ ≡ "loi de la paroi"

(≡ "law of the wall")

↳ voir prochain cours.