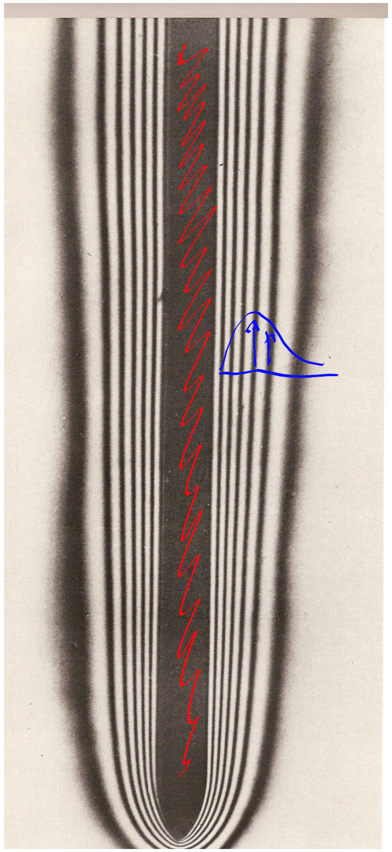
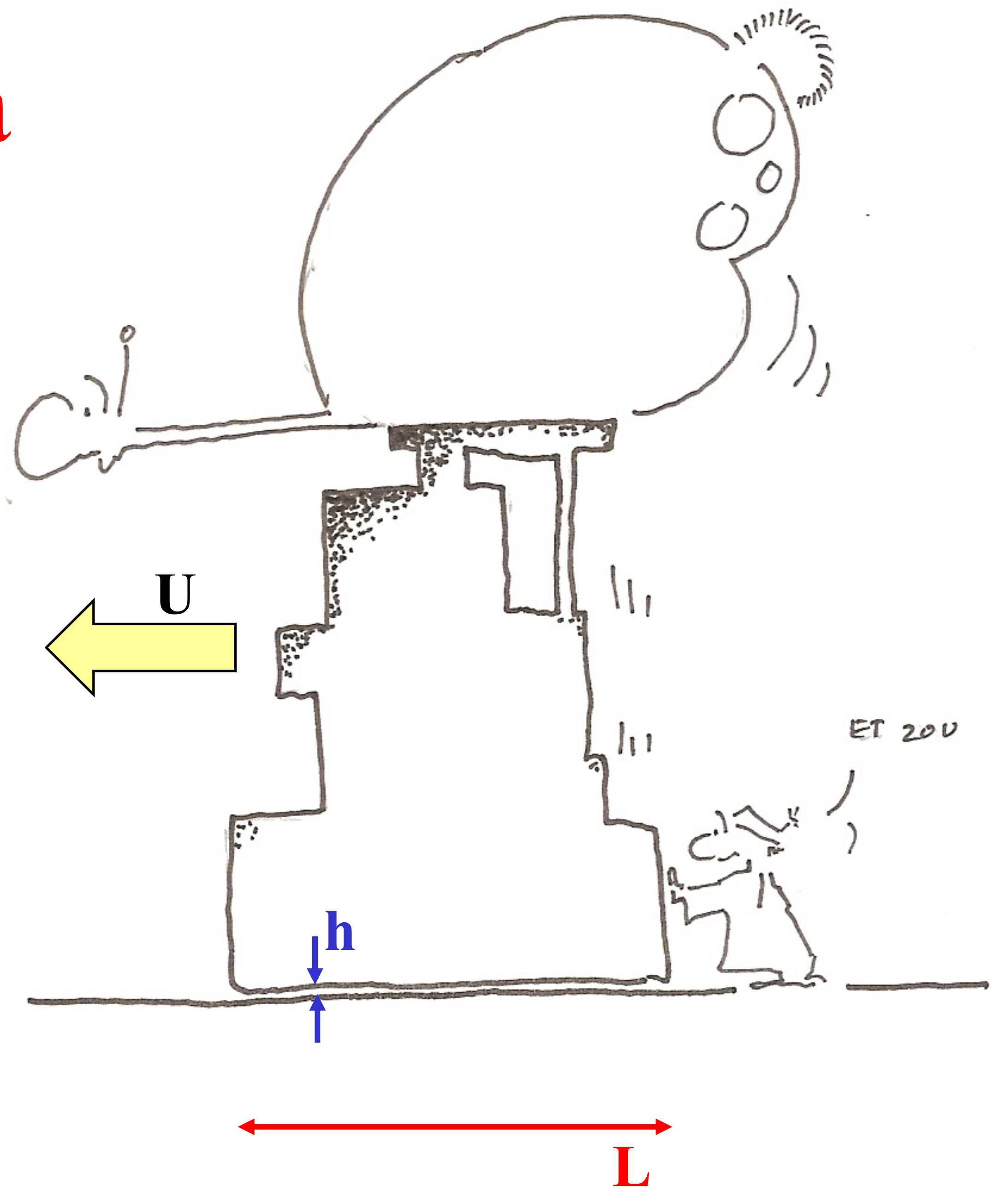
Mais que faire pour des écoulements avec deux échelles spatiales ?



Convection naturelle le long d'une plaque verticale : écoulement laminaire permanent

Lubrification et convoyage hydraulique : butée Michell

Théorie de la lubrification

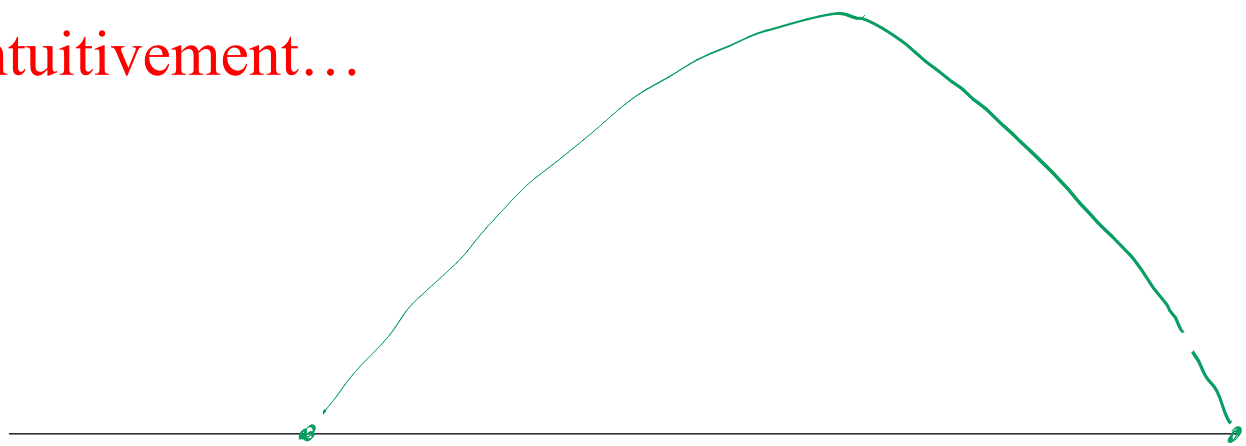


Convoyage hydraulique de charges très importantes :

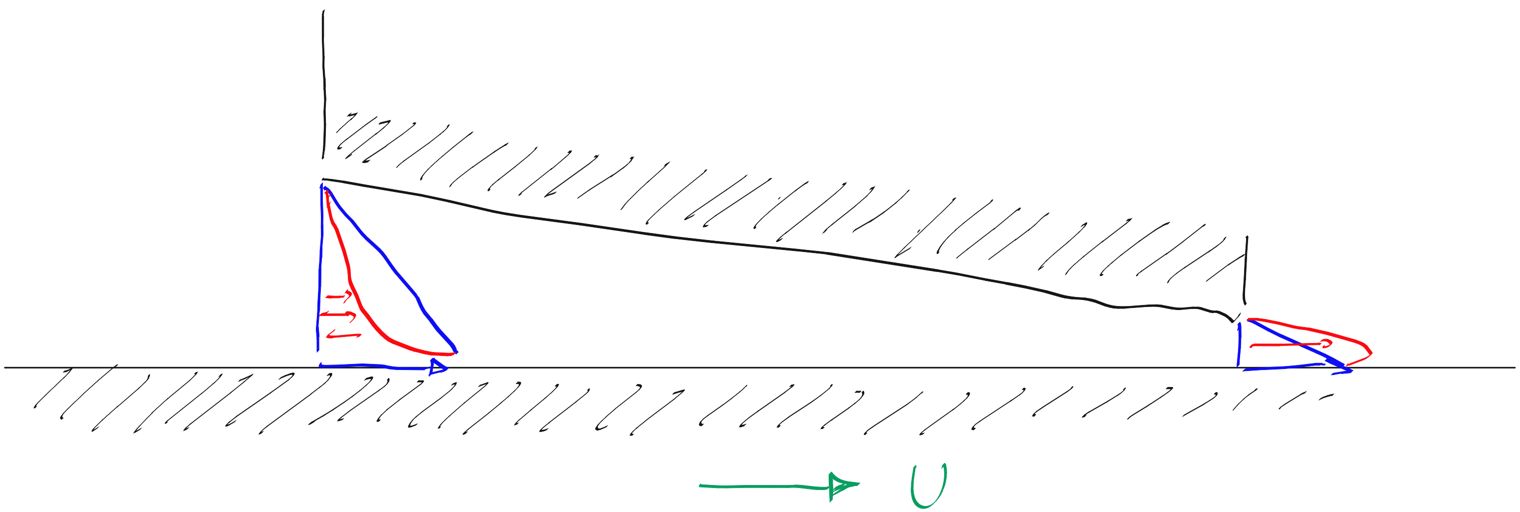
- ***turbines hydroélectriques***
- ***applications marines***
- ***butées hydrauliques***

Intuitivement…

- ECOULEMENT STATIONNAIRE INCOMPRESSIBLE 2D



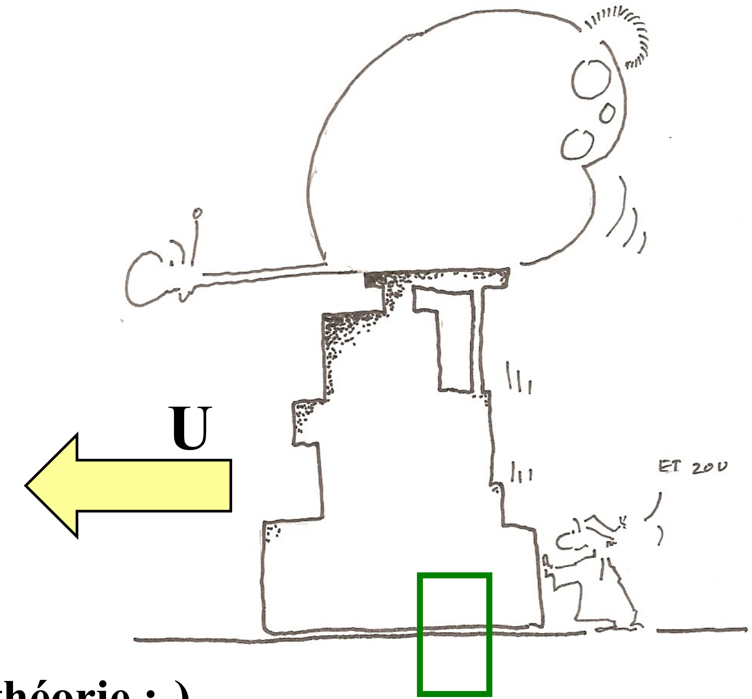
MERCI ANTOINE D'INCOURT :-)

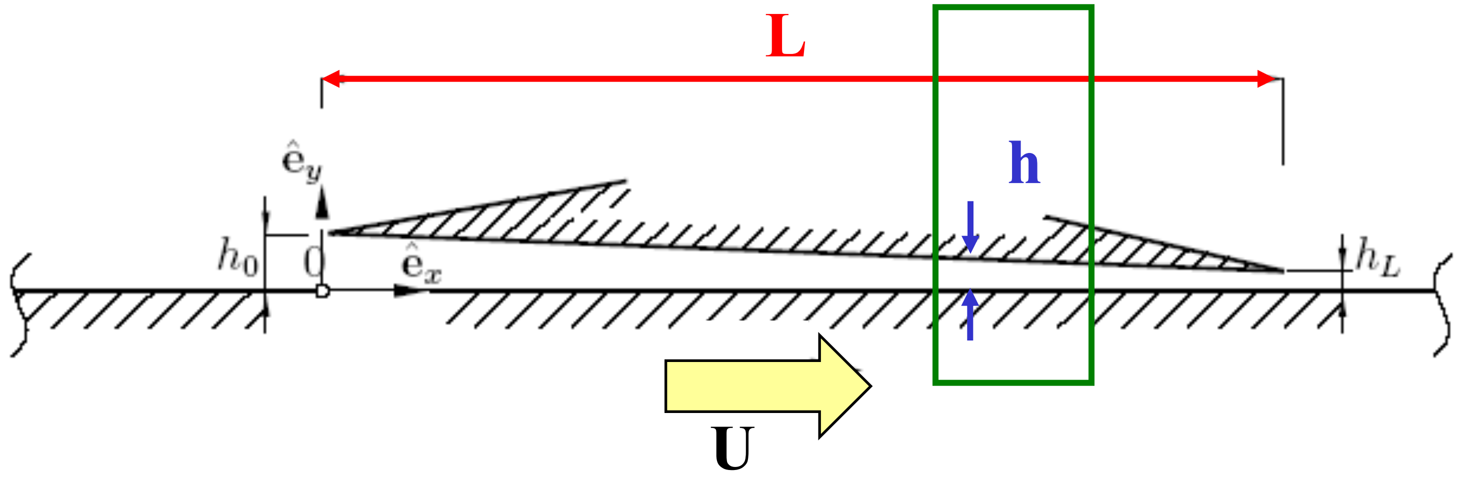


Théorie de la lubrification

$$h \ll L$$

Hypothèse géométrique de base
Valable dans la zone centrale uniquement en théorie :-)





Ecoulements incompressibles plans stationnaires

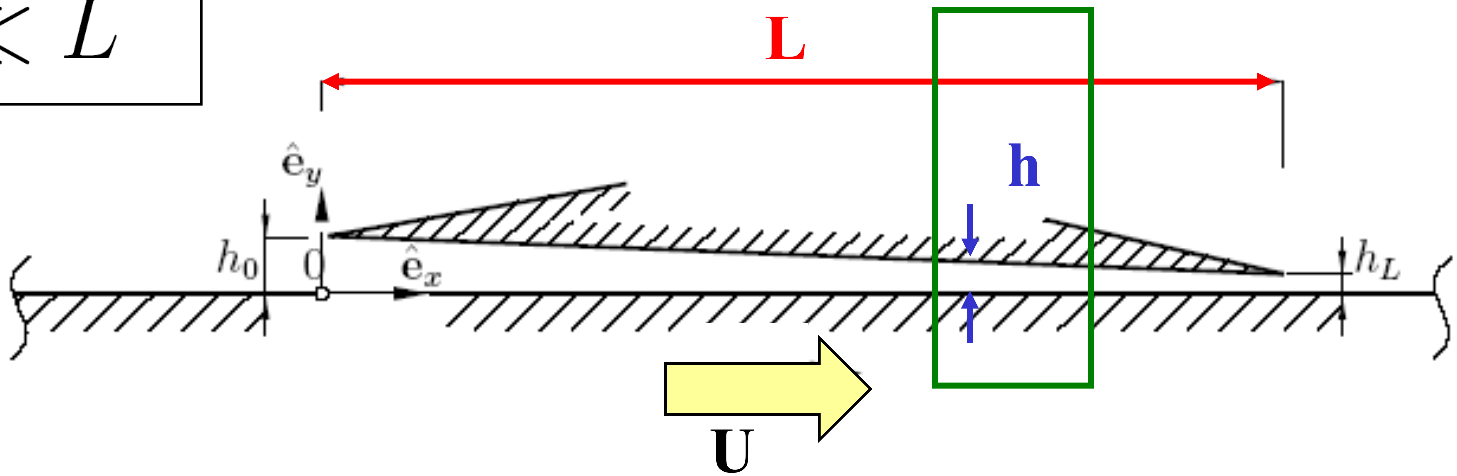
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}$$

Que deviennent ces équations ?

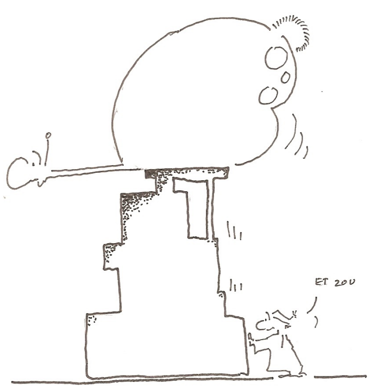
$$h \ll L$$



$$h \ll L$$

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} \\
\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}
\end{aligned}$$

Longueur horizontale caractéristique : L
Longueur verticale caractéristique : h
Vitesse horizontale caractéristique : U



Comment choisir une vitesse verticale caractéristique ?

Définir une vitesse verticale caractéristique…

$$h \ll L$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{U}{L} = \frac{V}{h}$$

$$V = \frac{h}{L} U \ll U$$

$$\underbrace{\frac{\partial u}{\partial x}}_{\mathcal{O}(U/L)} + \underbrace{\frac{\partial v}{\partial y}}_{\mathcal{O}(V/h)} = 0$$

Il ne faut pas définir de vitesse caractéristique verticale !

$$V = \frac{Uh}{L} \ll U$$

Simplifions…

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\partial(\rho U^2/L) \qquad \partial(\rho V U/h) \qquad \partial\left(\mu \frac{U}{L^2}\right) \ll \partial\left(\mu \frac{U}{h^2}\right)$$

$$V = U\frac{h}{L}$$

$$\partial\left(\rho \frac{Uh}{L}\frac{U}{h}\right) = \partial\left(\rho \frac{U^2}{L}\right)$$

$$\frac{\text{INERTIE}}{\text{VISQUEUX}} = \frac{\rho U^2/L}{\mu U/h^2} = \underbrace{\frac{\rho U L}{\mu}}_{Re}\frac{h^2}{L^2} \ll 1$$

$$\frac{h^2}{L^2} \ll 1$$

EN SUPPOSANT QUE Re RESTE ACCEPTABLE

Quand peut-on négliger les termes d'inertie ?

$$\underbrace{\rho\, u \frac{\partial u}{\partial x}}_{\mathcal{O}(\rho U^2/L)} + \underbrace{\rho\, v \frac{\partial u}{\partial y}}_{\mathcal{O}(\rho U^2/L),\ \mathcal{O}(\rho V U/h)} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\mathcal{O}(\mu U/L^2) \ll \mathcal{O}(\mu U/h^2)$$

Hypothèse de lubrification :
Écoulements rampants

$$\frac{\text{Forces d'inertie}}{\text{Forces visqueuses}} = \frac{\rho U^2/L}{\mu U/h^2} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Et l'autre équation ?

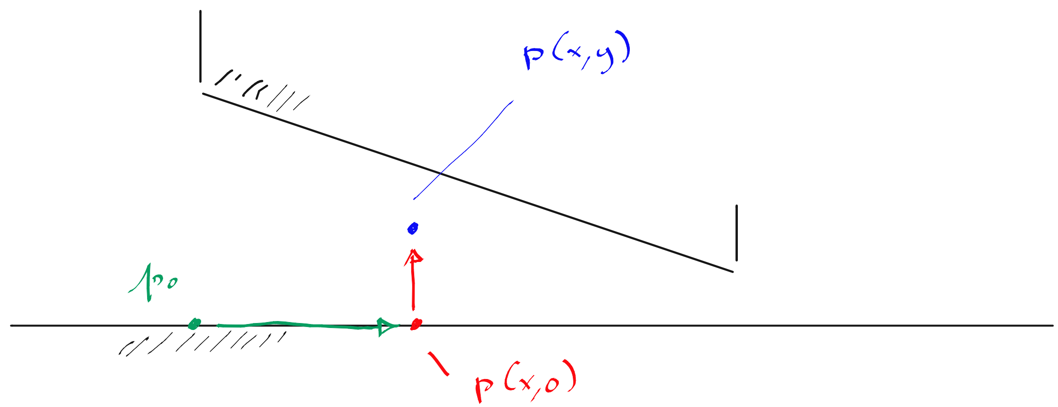
$$\underbrace{\rho\, u \frac{\partial v}{\partial x}}_{\mathcal{O}(\rho U^2 h/L^2)} + \underbrace{\rho\, v \frac{\partial v}{\partial y}}_{\mathcal{O}(\rho U^2 h/L^2)} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}$$

$$\mathcal{O}(\mu U h/L^3) \ll \mathcal{O}(\mu U/Lh)$$

On obtient la
<u>même condition</u>…

$$\frac{\text{Forces d'inertie}}{\text{Forces visqueuses}} = \frac{\rho U^2 h/L^2}{\mu U/Lh} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Et la pression…



$$\mathcal{O}\left(h\,\mu\,\frac{V}{h^2}\right)$$

$$\mathcal{O}\left(h\,\mu\,\frac{Uh}{L}\,\frac{1}{h^2}\right)$$

$$\mathcal{O}\left(\mu\,\frac{U}{L}\right)$$

$$p(x,y) - p_0 = \boxed{p(x,0) - p_0} + \boxed{y\,\frac{\partial p}{\partial y}\Big|_{(x,0)}} + y^2 \dots$$

C'EST PETIT

CAR $y^2 \ll y$:-)

$$\mathcal{O}\left(\mu\,\frac{UL}{h^2}\right)$$

$$\mathcal{O}(L) \times \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} \sim \mathcal{O}\left(\mu\,\frac{U}{h^2}\right)$$

$\partial\left(\mu \frac{U}{L}\right)$

$$p(x,y) - p_0 = p(x,0) - p_0 + y \left.\frac{\partial p}{\partial y}\right|_{(x,0)} + y^2 \ldots$$

$\partial\left(\mu \frac{UL}{h^2}\right)$

C'EST PETIT CAR $y^2 \ll y$:-)

$$\mu \frac{U}{L} \frac{h^2}{L} \ll \mu \frac{UL}{h^2} \frac{h^2}{L}$$

$$\frac{h^2}{L^2} \mu U \ll \mu U$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$$

Théorie de la lubrification

$$\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}$$

$\mathcal{O}(\mu U / L h)$

Et la pression ?

$$p(x,y) - p_0 = p(x,0) - p_0 + y \left.\frac{\partial p}{\partial y}\right|_{y=0}$$

$\mathcal{O}(\mu U L / h^2) \gg \mathcal{O}(\mu U L / L^2)$

$\mathcal{O}(\mu U / h^2)$

$$\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

Equations de Reynolds (1889)

Théorie de la lubrification

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$$

Film fluide mince

$$h \ll L$$

Hypothèse de lubrification : Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Est-ce que l'hypothèse de lubrification est réaliste ?

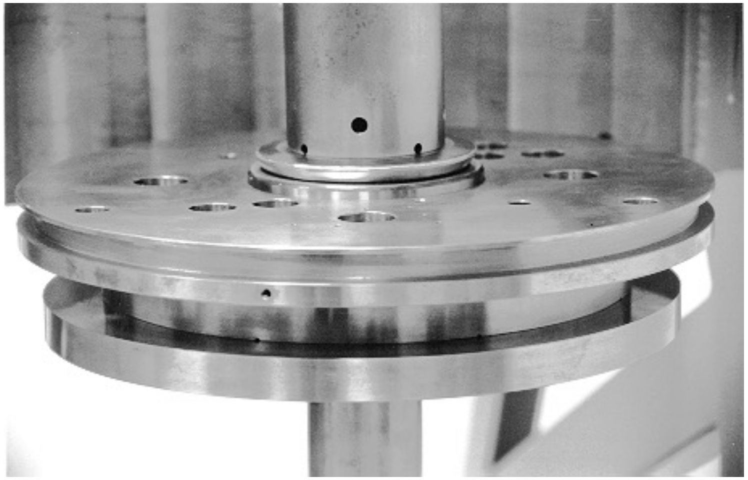
0.0375

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\begin{aligned} L &= 10\ cm \\ h &= 0.5\ mm \\ U &= 1\ m/s \end{aligned}$$

$$\begin{aligned} \rho &= 900\ kg/m^3 \\ \mu &= 60\ 10^{-3}\ Ns/m^2 \end{aligned}$$

Huile SAE50 à 60 degrés



Huile SAE 50
C’est quoi ?

Transport maritime

Marine LCX

Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

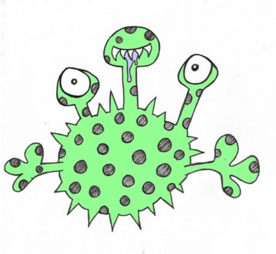
Grades offerts :
SAE 50

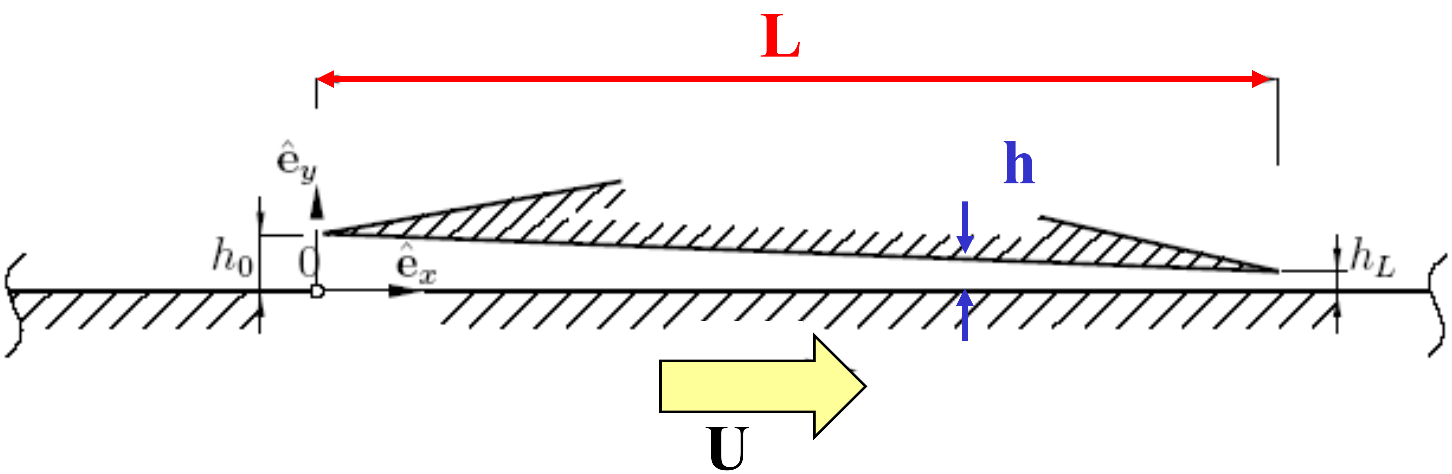
Fiche technique
Fiche signalétique

$$\left\{ \begin{array}{rcl} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} & = & 0 \\ -\dfrac{dp}{dx} + \mu \dfrac{\partial^2 u}{\partial y^2} & = & 0 \end{array} \right.$$

-i- calcul de u(x,y)

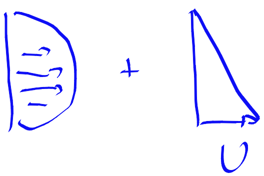
$$u(x,y) = -\frac{dp}{dx}\frac{h^2}{2\mu}\frac{y}{h}\left(1 - \frac{y}{h}\right) + U\left(1 - \frac{y}{h}\right)$$





Calcul du profil de vitesse…

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$



$$u(x,y) = -\frac{dp}{dx}\frac{h^2}{2\mu}\left(1-\frac{y}{h}\right)\frac{y}{h} + U\left(1-\frac{y}{h}\right)$$

$$\left[\frac{N}{m^3}\right]\left[\frac{m^2\, m^2}{N\, s}\right]$$

$$\left[\frac{m}{s}\right]$$

$$\underline{\underline{\tau}} = 2\mu\, \underline{\underline{d}}$$

$$\left[\frac{N}{m^2}\right] \qquad \left[\frac{1}{s}\right]$$

Calcul du profil de pression…

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = \int_0^h \frac{\partial u}{\partial x}\, dy + \int_0^h \frac{\partial v}{\partial y}\, dy$$

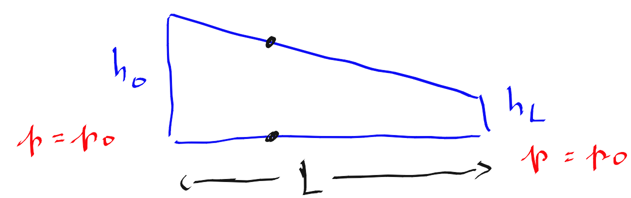
$$= \frac{d}{dx} \underbrace{\int_0^h u \, dy}_{Q(x)} + \underbrace{\big[v \big]_0^h}_{=0}$$

$$\boxed{u(x,y) = -\frac{dp}{dx} \frac{h^2}{2\mu} \left(1 - \frac{y}{h}\right) \frac{y}{h} + U \left(1 - \frac{y}{h}\right)}$$

$$0 = \frac{d}{dx} \left[-\frac{dp}{dx}(x) \frac{h^2(x)}{2\mu} \underbrace{\int_0^{h(x)} \left(1 - \frac{y}{h}\right) \frac{y}{h} \, dy}_{\frac{h(x)}{6}} + U \frac{h}{2} \right]$$

$$0 = \frac{d}{dx}\left[-\frac{dp}{dx}(x)\,\frac{h^2(x)}{2\mu}\underbrace{\int_0^{h(x)} \left(1-\frac{y}{h}\right)\frac{y}{h}\,dy}_{\frac{h(x)}{6}} + U\frac{h}{2}\right]$$

$$\frac{dh}{dx} = \frac{h_L - h_o}{L}$$



$$0 = \frac{d}{dx}\left[-\frac{dp}{dx}\,\frac{h^3}{12\mu} + \frac{Uh}{2}\right]$$

$$\frac{d}{dx}\left[\frac{dp}{dx}(x)\,h^3(x)\right] = 6\mu U \frac{dh}{dx}$$

$$\left(\frac{dh}{dx}\right)^2 \frac{d}{dh}\left[\frac{dp}{dh}\,h^3\right] = 6\mu U \frac{dh}{dx}$$

$$-\frac{d}{dh}\left[\frac{dp}{dh}\,h^3\right] = \underbrace{\frac{6\mu U L}{h_o - h_L}}_{C}$$

$$-\frac{dp}{dh}\,h^3 = C\left[h + A\right]$$

$$-\frac{dp}{dh} = C\left[\frac{1}{h^2} + \frac{A}{h^3}\right]$$

$$p(h) = C\left[\frac{1}{h} + \frac{A}{2h^2} + B\right]$$

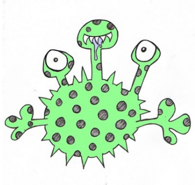
-ii- calcul de p(x)

$$\begin{cases} -\dfrac{dp}{dx} + \mu \dfrac{\partial^2 u}{\partial y^2} = 0 \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \end{cases}$$

$$0 = \int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} dy$$

$$0 = \frac{d}{dx} \overbrace{\int_0^h u(x,y)dy}^{Q(x)} + \Big[v(x,y) \Big]_0^h$$

En utilisant l'expression de u(x,y)



Equation classique de Reynolds (1889)

$$0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2} \right)$$

$$0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2}\right)$$

$$\frac{d}{dx}\left(h^3(x)\,\frac{dp}{dx}(x)\right) = 6\mu U \boxed{\frac{dh}{dx}}$$

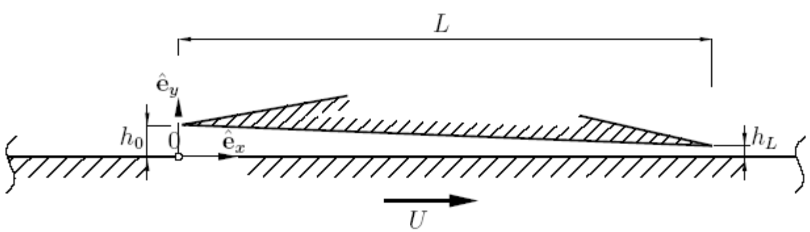
$$-\frac{d}{dh}\left(h^3\frac{dp}{dh}(h)\right) = \frac{6\mu U L}{h_0 - h_L}$$

$$-h^3\frac{dp}{dh}(h) = \frac{6\mu U L}{h_0 - h_L}(h + A)$$

$$-\frac{dp}{dh}(h) = \frac{6\mu U L}{h_0 - h_L}\left(\frac{1}{h^2} + \frac{A}{h^3}\right)$$

$$p(h) = \frac{6\mu U L}{h_0 - h_L}\left(B + \frac{1}{h} + \frac{A}{2h^2}\right)$$

Palier plat

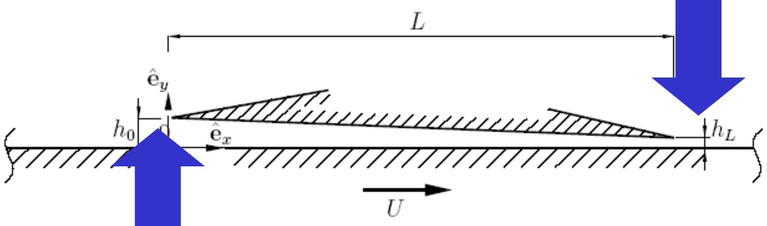


$$\frac{x}{L} = \frac{h_0 - h(x)}{h_0 - h_L}$$

$$\frac{dh}{dx} = -\frac{h_0 - h_L}{L}$$

$$p(h_L) = p_0$$

Deux conditions aux limites



$$p(h_0) = p_0$$

$$p(h) = \frac{6\mu U L}{h_0 - h_L}\left(\boxed{B} + \frac{1}{h} + \frac{\boxed{A}}{2h^2}\right)$$

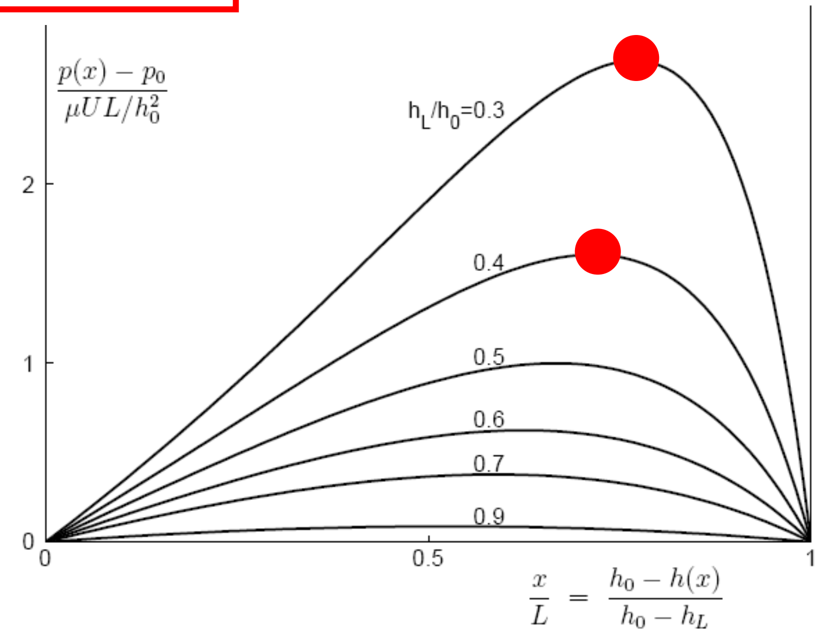
Deux constantes

$$\downarrow$$

$$p(h) - p_0 = \frac{6\mu U L (h_0 - h)(h - h_L)}{(h_0^2 - h_L^2)h^2}$$

Où la pression est-elle maximale ?

$$h = \frac{2\,h_0\,h_L}{(h_0 + h_L)}$$

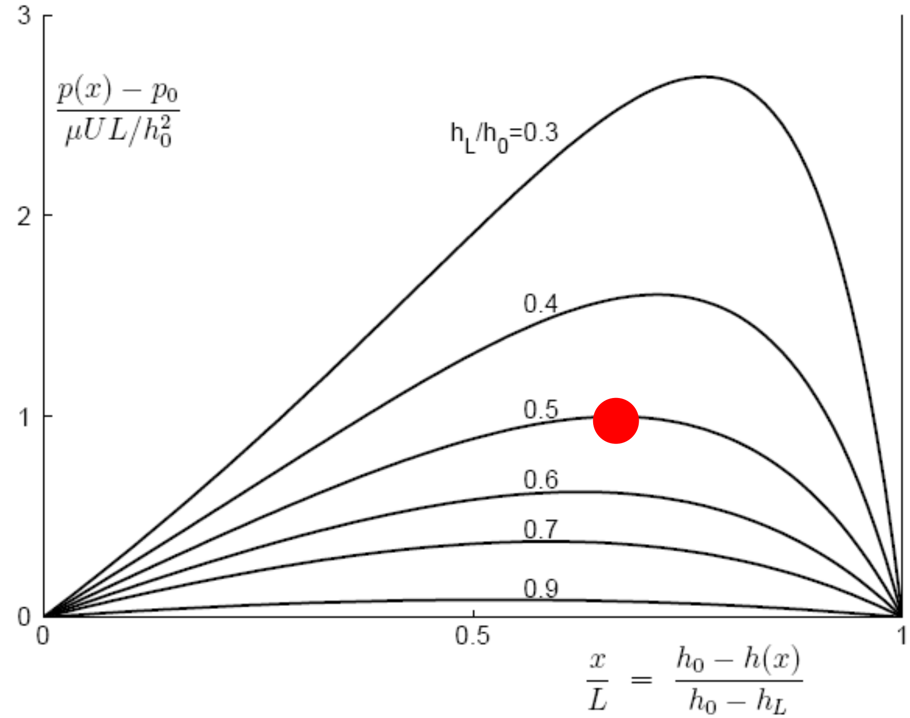


Cette pression peut être énorme !

$$
\begin{aligned}
L &= 10\ cm \\
h_0 &= 0.1\ mm \\
h_L &= 0.05\ mm \\
U &= 10\ m/s
\end{aligned}
$$

$$
\begin{aligned}
\rho &= 900\ kg/m^3 \\
\mu &= 0.1\ Ns/m^2
\end{aligned}
$$

Huile SAE50 à 50 degrés

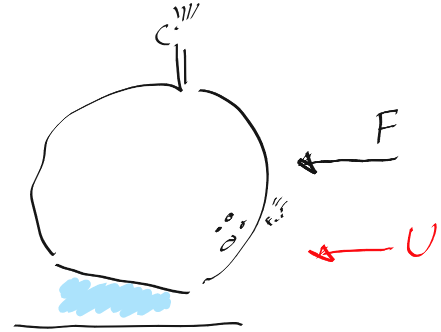


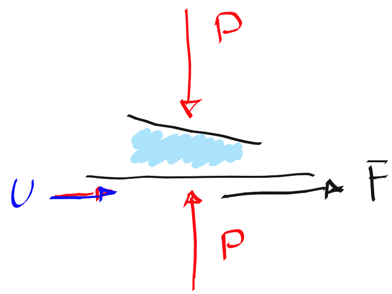
$$p_{\max} - p_0 = \frac{3}{2} \frac{\mu U L}{h_0 h_L} \frac{(h_0 - h_L)}{(h_0 + h_L)}$$

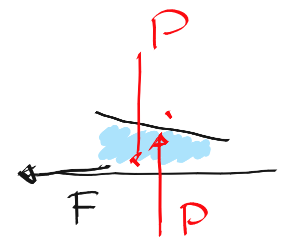
10^7 Pascal

Charge utile

$$P = \int_0^L p(x) - p_0 \, dx$$







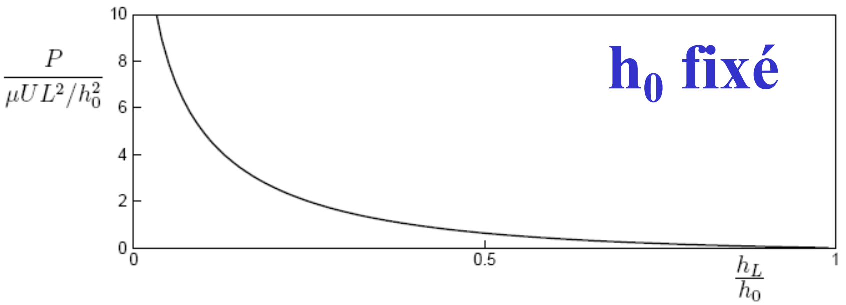
$$F = -\mu \int_0^L \frac{\partial u}{\partial y} \, dx$$

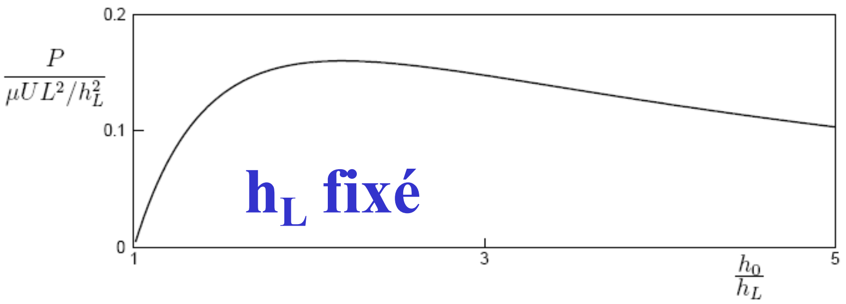
Force de poussée

Puissance dissipée

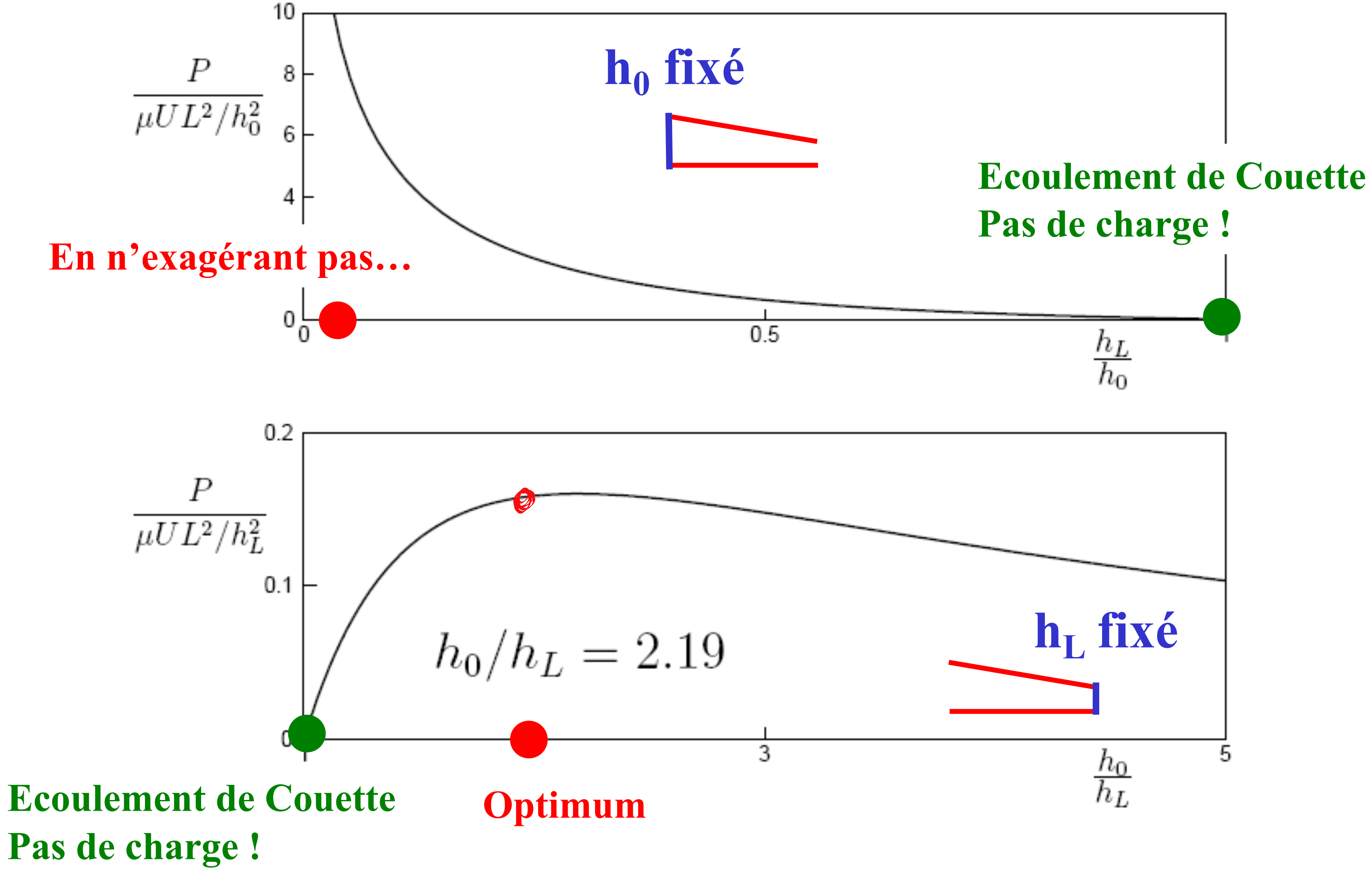
$$FU$$

Charge utile



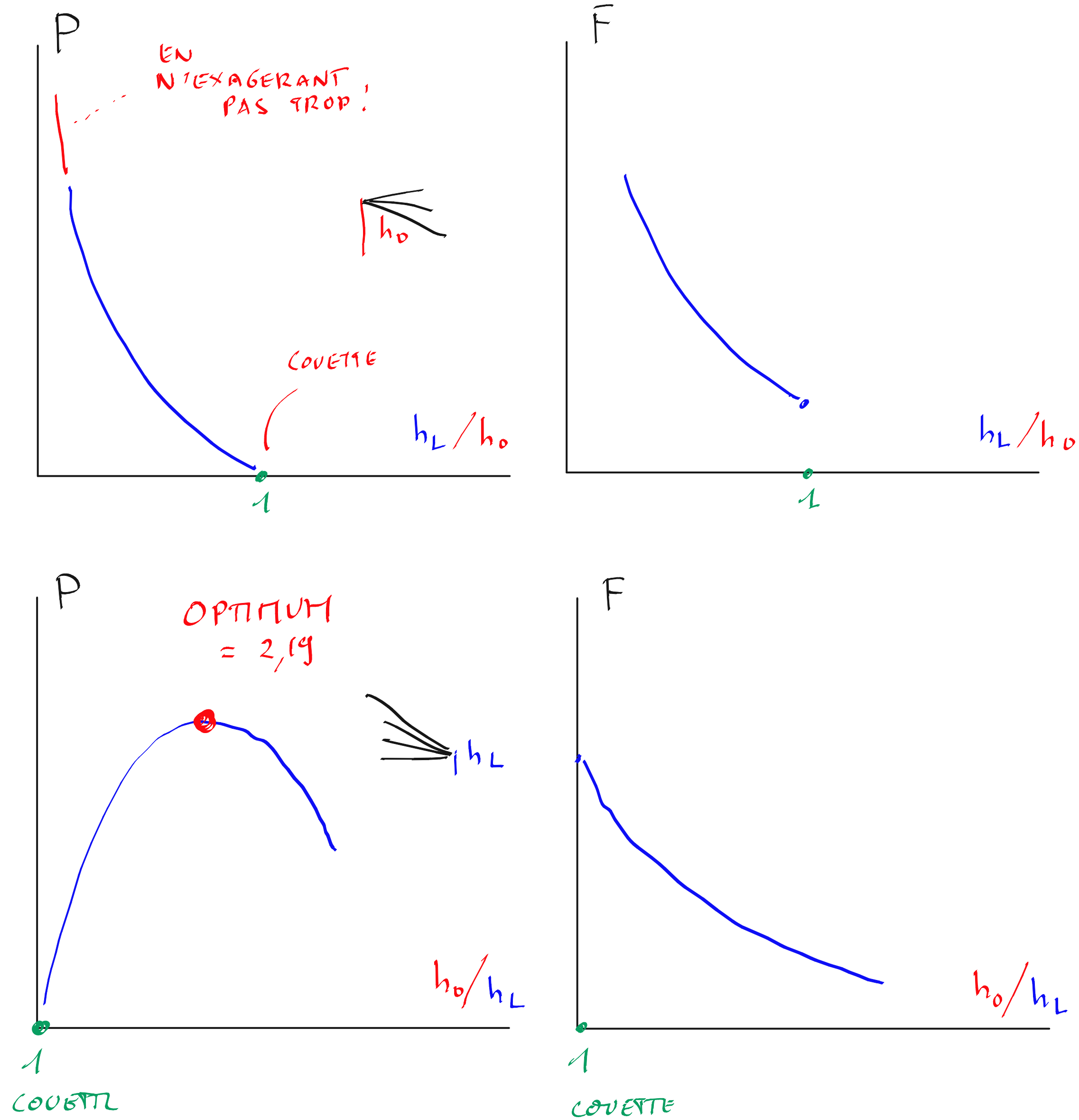


$$
\begin{aligned}
P &= \int_0^L (p(x) - p_0)\, dx \\
&= -\frac{L}{(h_0 - h_L)} \int_{h_0}^{h_L} (p(h) - p_0)\, dh \\
&= -\frac{L}{(h_0 - h_L)} \frac{6\,\mu\, U\, L}{(h_0^2 - h_L^2)} \int_{h_0}^{h_L} \left[(h_0 + h_L)\frac{1}{h} - \frac{h_0\, h_L}{h^2} - 1 \right] dh \\
&= -6\,\mu\, U\, L^2 \left[\frac{1}{(h_0 - h_L)^2} \log\left(\frac{h_L}{h_0}\right) + \frac{2}{(h_0^2 - h_L^2)} \right]
\end{aligned}
$$

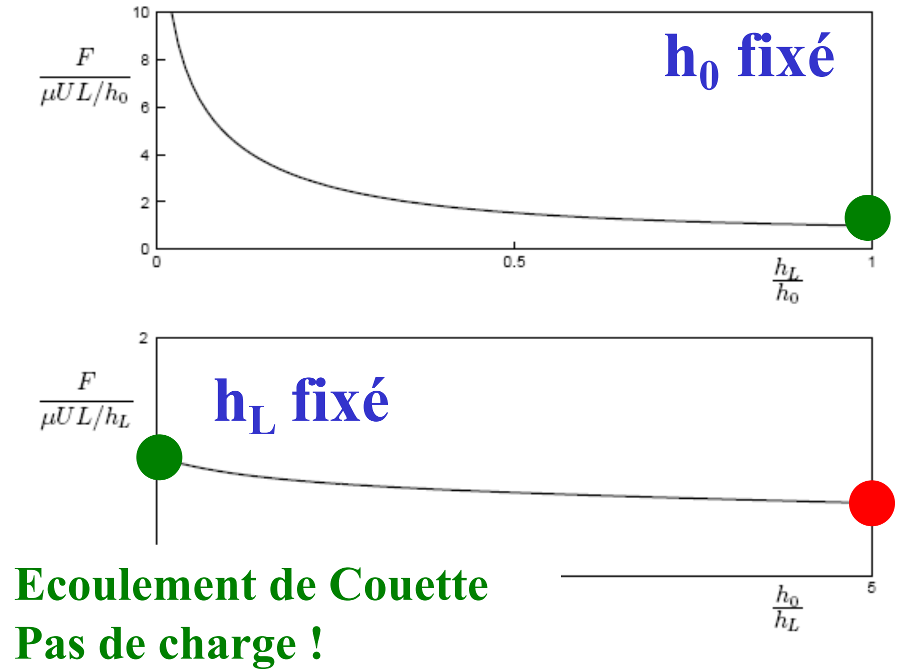


Rapport optimal…

Quatre petits graphes !



Force exercée par le fluide sur la partie mobile



La force diminue de façon monotone lorsque le rapport augmente…

$$
\begin{aligned}
F &= -\int_0^L \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} dx \\
&= \frac{\mu U L}{(h_0 - h_L)} \int_{h_0}^{h_L} \left[\frac{6}{h^2}\,\frac{h_0\, h_L}{(h_0 + h_L)} - \frac{4}{h}\right] dh \\
&= -\mu U L \left[\frac{6}{(h_0 + h_L)} + \frac{4}{(h_0 - h_L)} \log\left(\frac{h_L}{h_0}\right)\right]
\end{aligned}
$$

La puissance consommée est dissipée…

$$F\,U = -\frac{\mu\,U^2\,L}{h_0}\left[\frac{6}{(1+h_L/h_0)} + \frac{4}{(1-h_L/h_0)}\log\left(\frac{h_L}{h_0}\right)\right]$$

Embêtant…

S'assurer que l'huile est bien refroidie car la viscosité (et donc la charge utile) décroît rapidement avec la température…

…en chaleur !

A propos de la viscosité de notre huile SAE 50

Transport maritime

Marine LCX

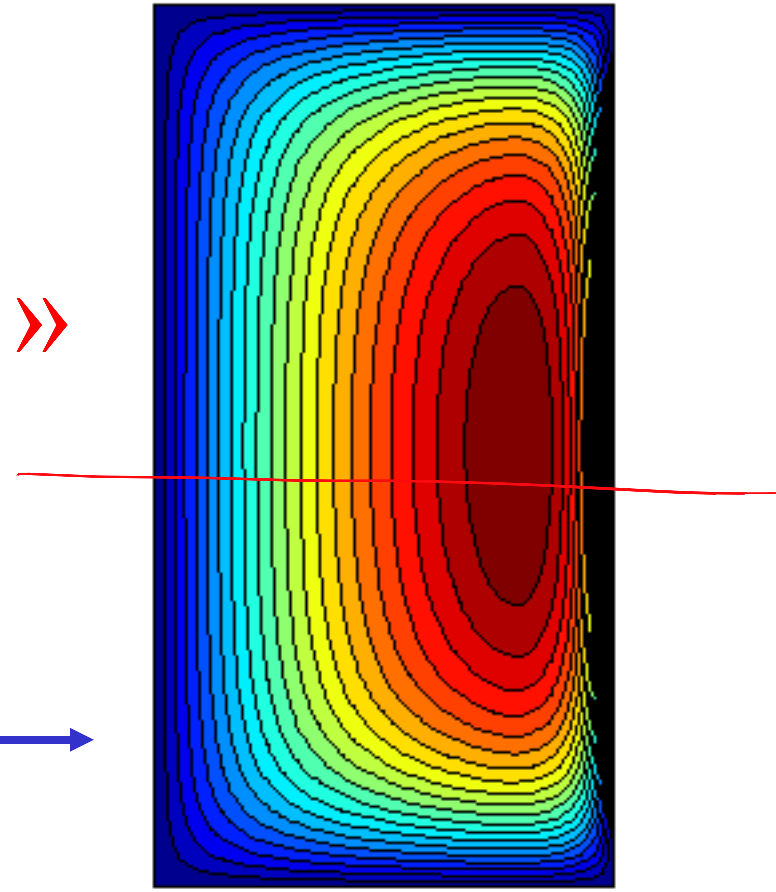
Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

Grades offerts :
SAE 50

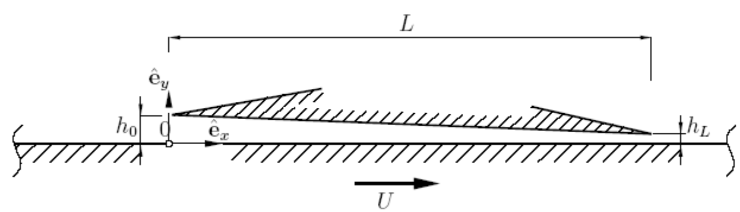
Fiche techniq
Fiche signalét

$$
\begin{aligned}
T &= 20^0 C & \mu &= 1.100 \; Ns/m^2 \\
T &= 40^0 C & \mu &= 0.210 \; Ns/m^2 \\
T &= 50^0 C & \mu &= 0.100 \; Ns/m^2 \\
T &= 60^0 C & \mu &= 0.060 \; Ns/m^2 \\
T &= 80^0 C & \mu &= 0.025 \; Ns/m^2 \\
T &= 100^0 C & \mu &= 0.013 \; Ns/m^2
\end{aligned}
$$

Analyse « tridimensionnelle » du palier plat



pression sous un palier dont la largeur vaut le double de la longueur



Lubrification 2D ½

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial p}{\partial z} = 0$$

Théorie de la lubrification

Film fluide mince

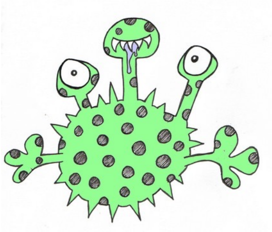
$$h \ll L$$

Hypothèse de lubrification :
Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0 \\ -\dfrac{\partial p}{\partial x} + \mu \dfrac{\partial^2 u}{\partial z^2} = 0 \\ -\dfrac{\partial p}{\partial y} + \mu \dfrac{\partial^2 v}{\partial z^2} = 0 \end{cases}$$

-i- calcul de u(x,y,z) et de v(x,y,z)



$$u(x,y,z) = -\frac{\partial p}{\partial x}\frac{h^2}{2\mu}\frac{z}{h}\left(1-\frac{z}{h}\right) + U\left(1-\frac{z}{h}\right)$$

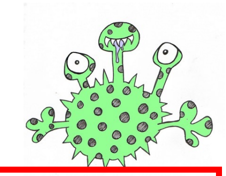
$$v(x,y,z) = -\frac{\partial p}{\partial y}\frac{h^2}{2\mu}\frac{z}{h}\left(1-\frac{z}{h}\right)$$

-ii- calcul de p(x,y)

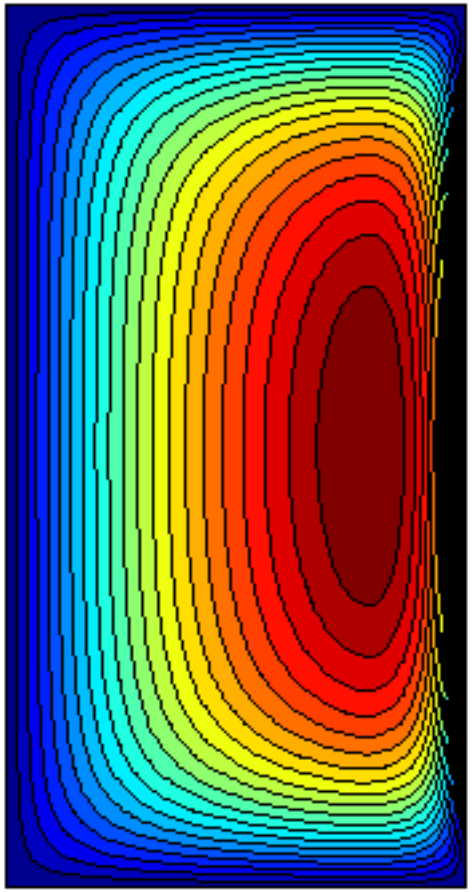
$$\left\{\begin{array}{rcl} -\dfrac{\partial p}{\partial x} + \mu\,\dfrac{\partial^2 u}{\partial z^2} & = & 0 \\ -\dfrac{\partial p}{\partial y} + \mu\,\dfrac{\partial^2 v}{\partial z^2} & = & 0 \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} & = & 0 \end{array}\right.$$

$$\int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\,dz \quad = \quad 0$$

$$\frac{\partial}{\partial x}\int_0^h u(x,y,z)dz + \frac{\partial}{\partial y}\int_0^h v(x,y,z)dz + \Big[\, w(x,y,z) \,\Big]_0^h \quad = \quad 0$$



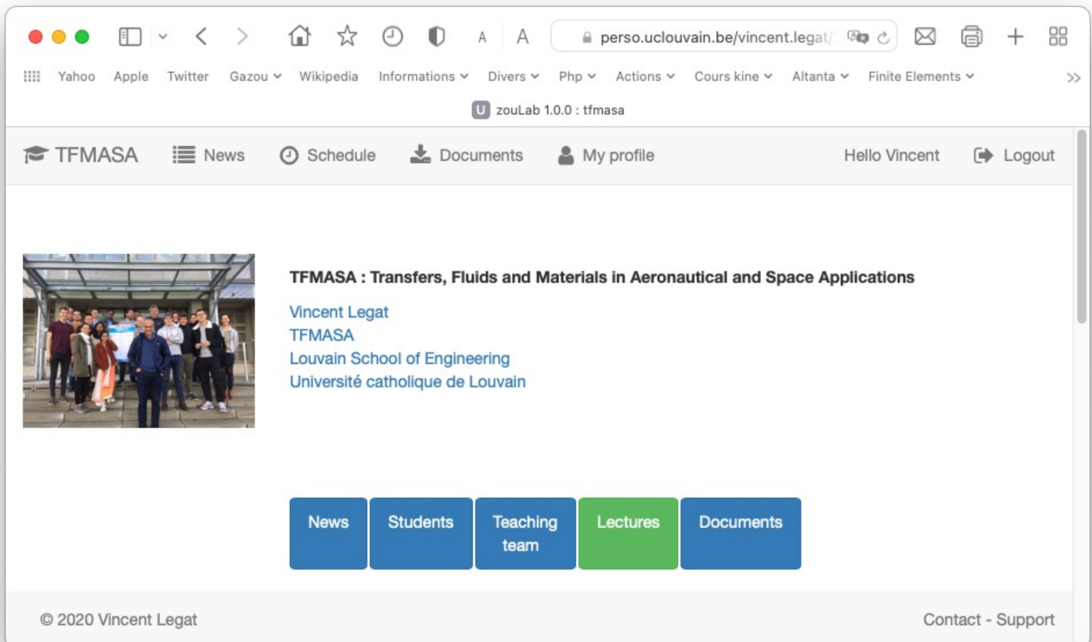
$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) \quad = \quad 6\mu U\frac{dh}{dx}$$



-iiii- calcul numérique par différences finies de p(x,y)

$$h^3\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+3h^2\left(\frac{h_L-h_0}{L}\right)\frac{\partial p}{\partial x} = 6\mu U\left(\frac{h_L-h_0}{L}\right)$$

Un triple diplôme belgo-germano-français ?



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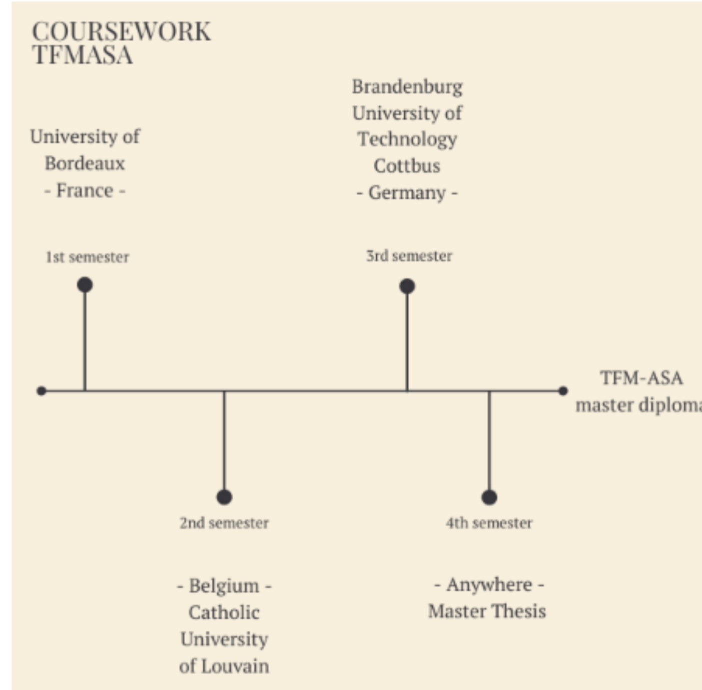

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