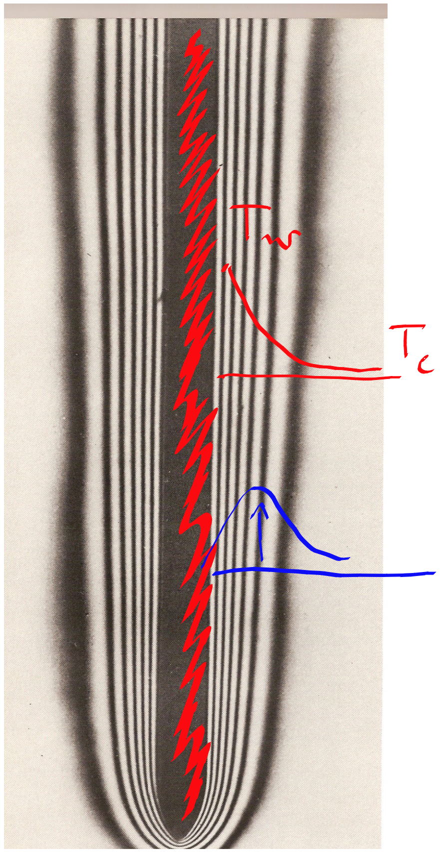
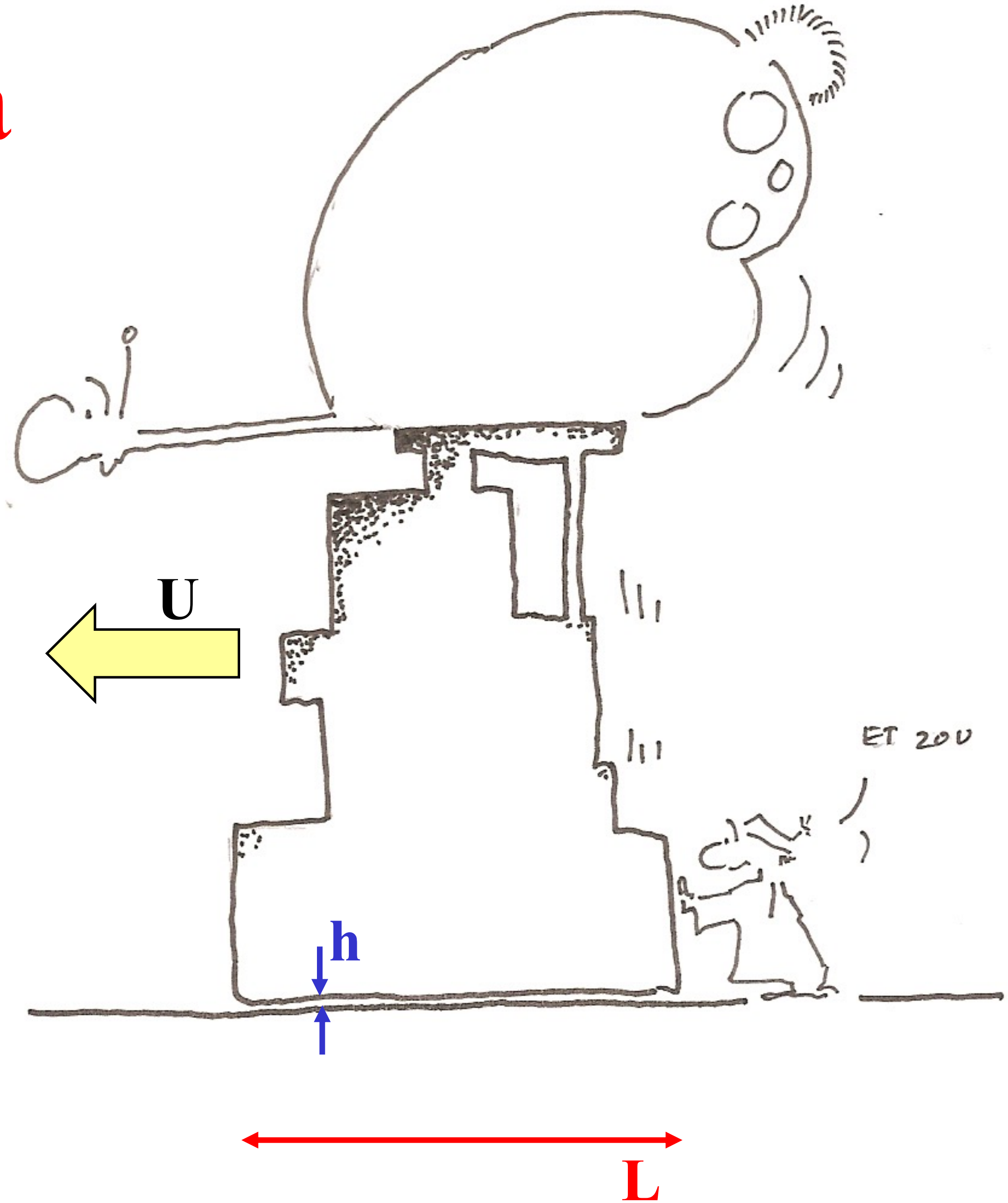
Mais que faire pour des écoulements avec deux échelles spatiales ?



Convection naturelle le long d'une plaque verticale : écoulement laminaire permanent

Lubrification et convoyage hydraulique : butée Michell

Théorie de la lubrification

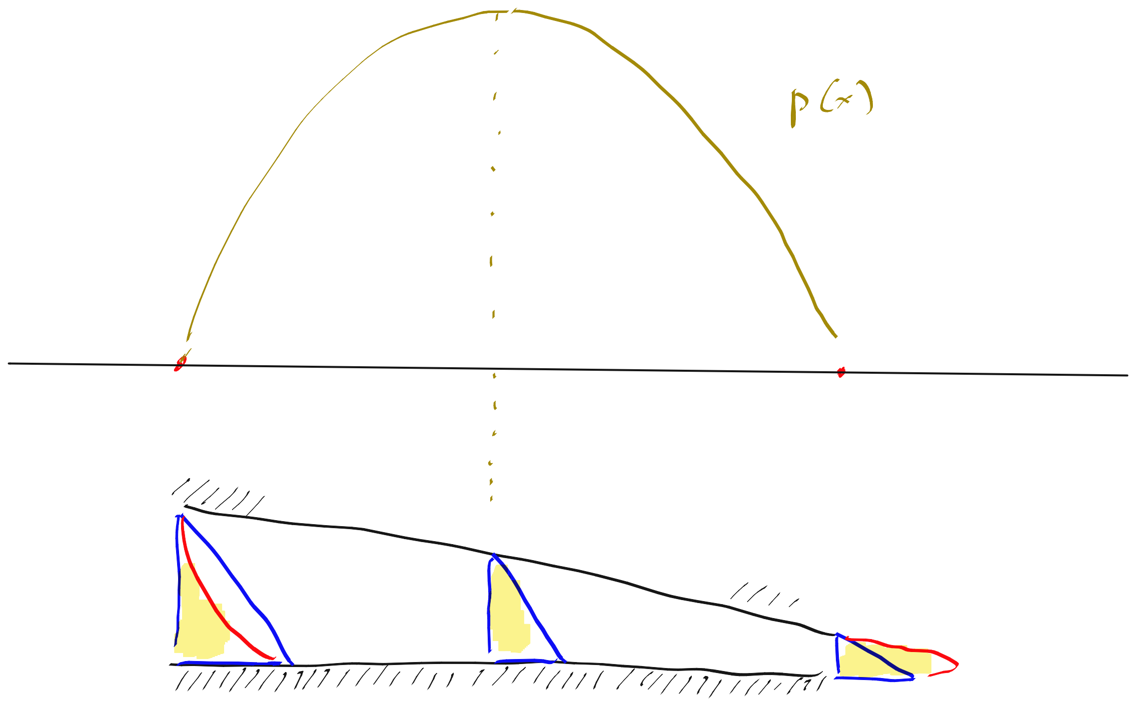


Convoyage hydraulique de charges très importantes :

- ***turbines hydroélectriques***
- ***applications marines***
- ***butées hydrauliques***

Intuitivement…

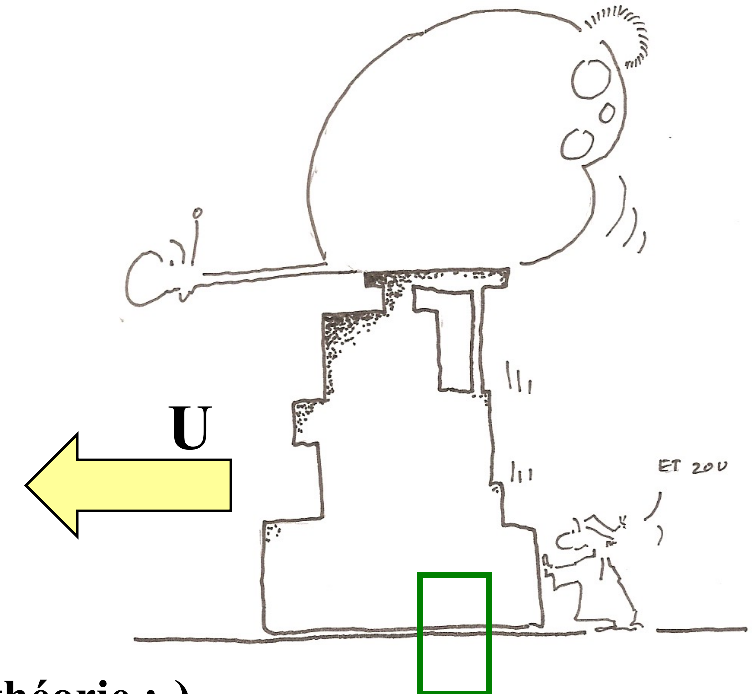
– ECOULEMENT
INCOMPRESSIBLE
STATIONNAIRE
2D

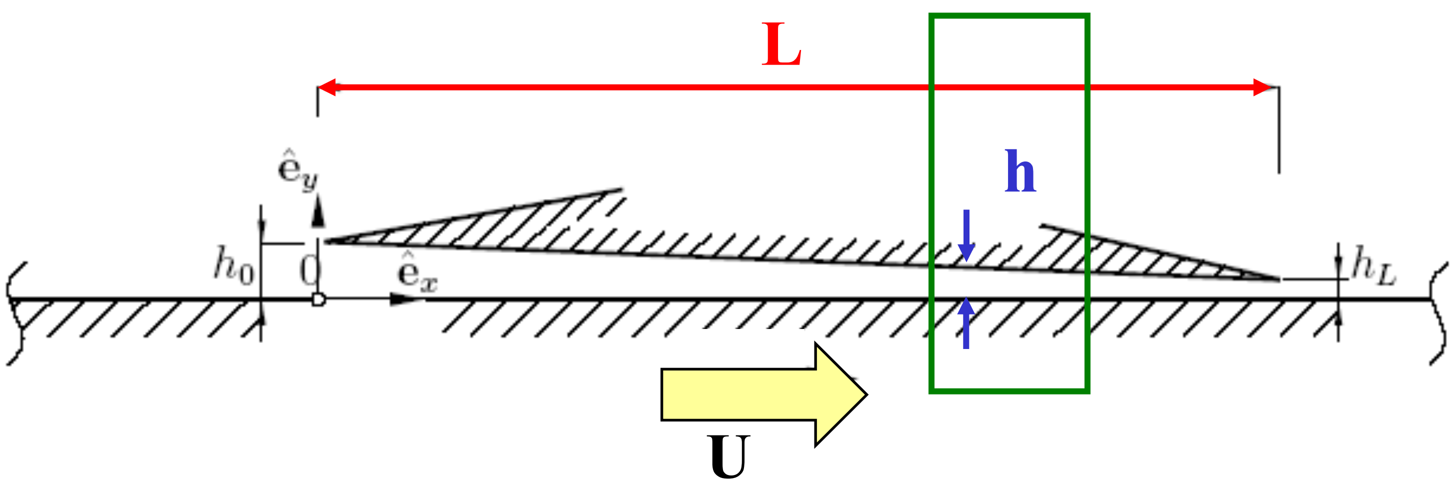


Théorie de la lubrification

$$h \ll L$$

Hypothèse géométrique de base
Valable dans la zone centrale uniquement en théorie :-)





Ecoulements incompressibles plans stationnaires

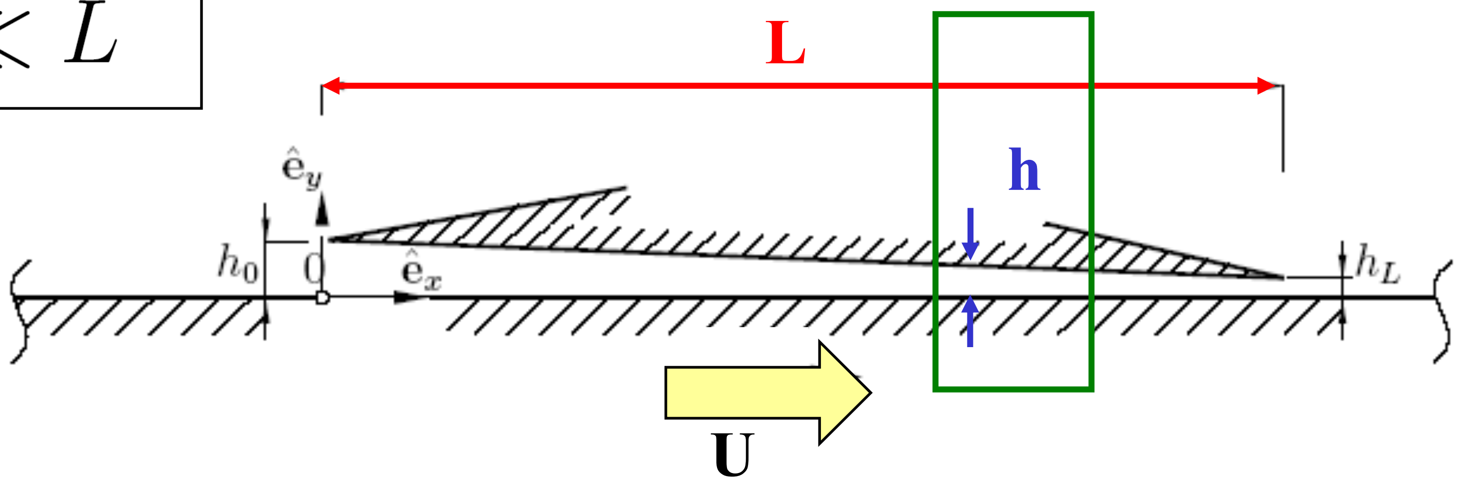
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\, u\, \frac{\partial u}{\partial x} + \rho\, v\, \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu\, \frac{\partial^2 u}{\partial x^2} + \mu\, \frac{\partial^2 u}{\partial y^2}$$

$$\rho\, u\, \frac{\partial v}{\partial x} + \rho\, v\, \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu\, \frac{\partial^2 v}{\partial x^2} + \mu\, \frac{\partial^2 v}{\partial y^2}$$

Que deviennent ces équations ?

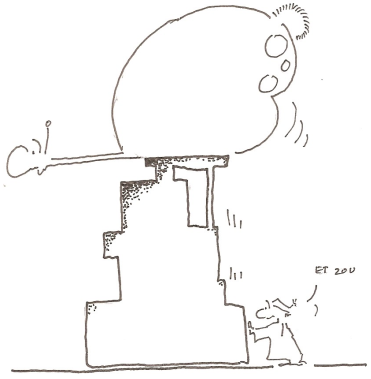
$$h \ll L$$



$$h \ll L$$

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho\, u\, \frac{\partial u}{\partial x} + \rho\, v\, \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \mu\, \frac{\partial^2 u}{\partial x^2} + \mu\, \frac{\partial^2 u}{\partial y^2} \\
\rho\, u\, \frac{\partial v}{\partial x} + \rho\, v\, \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu\, \frac{\partial^2 v}{\partial x^2} + \mu\, \frac{\partial^2 v}{\partial y^2}
\end{aligned}
$$

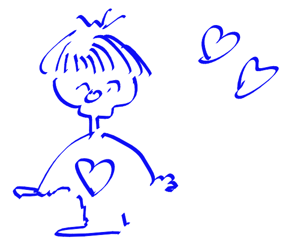
Longueur horizontale caractéristique : L
Longueur verticale caractéristique : h
Vitesse horizontale caractéristique : U



Comment choisir une vitesse verticale caractéristique ?

Définir une vitesse verticale caractéristique…

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



$$\frac{U}{L} = \frac{V}{h}$$

$$V = \frac{h}{L} U \ll U$$

$$\underbrace{\frac{\partial u}{\partial x}}_{\mathcal{O}(U/L)} + \underbrace{\frac{\partial v}{\partial y}}_{\mathcal{O}(V/h)} = 0$$

Il ne faut pas définir de vitesse caractéristique verticale !

$$V = \frac{Uh}{L} \ll U$$

Simplifions…

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\rho \frac{U^2}{L} \qquad \rho \frac{VU}{h} = \rho \frac{U^2}{L} \qquad \mu \frac{U}{L^2} \ll \mu \frac{U}{h^2}$$

$$\frac{\text{INERTIE}}{\text{VISQUEUX}} = \frac{\rho U^2/L}{\mu U/h^2} = \underbrace{\rho \frac{UL}{\mu}}_{Re} \underbrace{\frac{h^2}{L^2}}_{\ll 1} \ll 1$$

EN SUPPOSANT QUE Re RESTE RAISONNABLE

Quand peut-on négliger les termes d'inertie ?

$$\underbrace{\rho\, u \frac{\partial u}{\partial x}}_{\mathcal{O}(\rho U^2/L)} + \underbrace{\rho\, v \frac{\partial u}{\partial y}}_{\mathcal{O}(\rho U^2/L),\ \mathcal{O}(\rho V U/h)} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\mathcal{O}(\mu U/L^2) \ll \mathcal{O}(\mu U/h^2)$$

Hypothèse de lubrification :
Écoulements rampants

$$\frac{\text{Forces d'inertie}}{\text{Forces visqueuses}} = \frac{\rho U^2/L}{\mu U/h^2} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Et l'autre équation ?

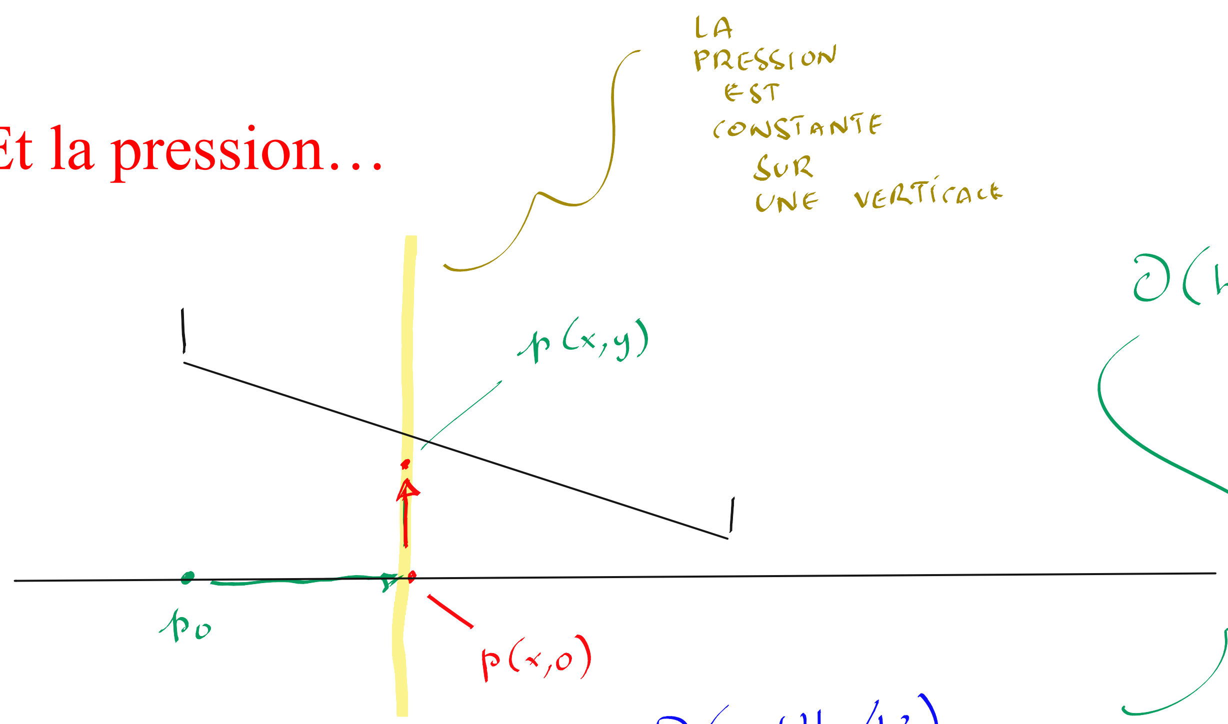
$$\underbrace{\rho u \frac{\partial v}{\partial x}}_{\mathcal{O}(\rho U^2 h/L^2)} + \underbrace{\rho v \frac{\partial v}{\partial y}}_{\mathcal{O}(\rho U^2 h/L^2)} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}$$

$$\mathcal{O}(\mu U h/L^3) \ll \mathcal{O}(\mu U/Lh)$$

On obtient la
même condition…

$$\frac{\text{Forces d'inertie}}{\text{Forces visqueuses}} = \frac{\rho U^2 h/L^2}{\mu U/Lh} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Et la pression…



$$\mathcal{O}\left(h\, \frac{\mu V}{h^2}\right) \sim \mu \frac{Uh}{L} \frac{1}{h^2}$$

$$\mathcal{O}(\mu U / L)$$

$$\mathcal{O}(\mu U L / h^2)$$

$$p(x,y) - p_0 = \boxed{p(x,0) - p_0} + \boxed{y \left.\frac{\partial p}{\partial y}\right|_{y=0}} + y^2 \dots$$

$$\times \frac{\partial p}{\partial x}$$

$$\mathcal{O}\left(L \mu \frac{U}{h^2}\right)$$

C'EST PETIT CAR $y^2 \ll y$

$\mathcal{O}(\mu U L / h^2)$ $\mathcal{O}(\mu U / L)$

$$p(x,y) - p_0 = p(x,0) - p_0 + y \left.\frac{\partial p}{\partial y}\right|_{y=0} + y^2 \dots$$

$$\frac{\mu U L}{h^2} L \gg \frac{\mu U}{L} L$$

$$\mu U \frac{L^2}{h^2} \gg \mu U$$

$$\mu U \gg \mu U \frac{h^2}{L^2} \ll 1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$$

Théorie de la lubrification

$$\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}$$

$\mathcal{O}(\mu U / L h)$

Et la pression ?

$$p(x,y) - p_0 = p(x,0) - p_0 + y \left.\frac{\partial p}{\partial y}\right|_{y=0}$$

$\mathcal{O}(\mu U L / h^2) \gg \mathcal{O}(\mu U L / L^2)$

$\mathcal{O}(\mu U / h^2)$

$$\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}$$

Equations de Reynolds (1889)

Théorie de la lubrification

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$$

Film fluide mince

$$h \ll L$$

Hypothèse de lubrification :
Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Est-ce que l'hypothèse de lubrification est réaliste ?

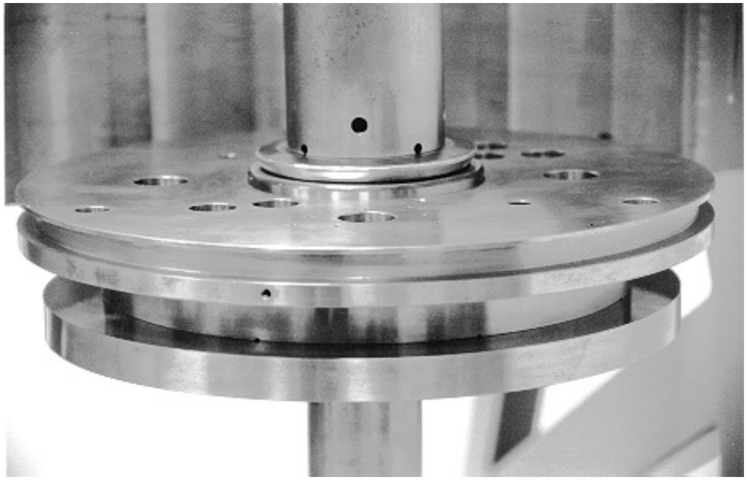
0.0375

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\begin{aligned} L &= 10\ cm \\ h &= 0.5\ mm \\ U &= 1\ m/s \end{aligned}$$

$$\begin{aligned} \rho &= 900\ kg/m^3 \\ \mu &= 60\ 10^{-3}\ Ns/m^2 \end{aligned}$$

Huile SAE50 à 60 degrés



Huile SAE 50
C’est quoi ?

Transport maritime

Marine LCX

Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

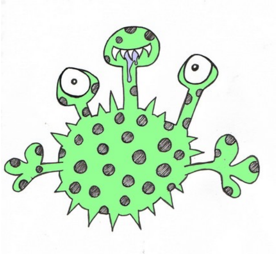
Grades offerts :
SAE 50

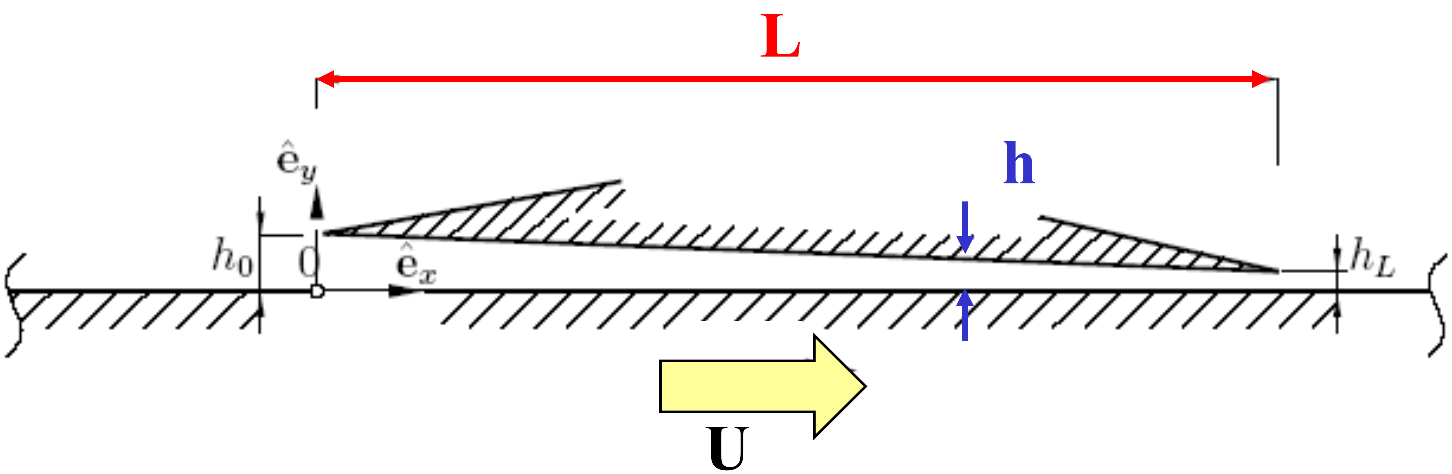
Fiche technique
Fiche signalétique

-i- calcul de u(x,y)

$$\left\{\begin{array}{rcl} \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y} & = & 0 \\ -\dfrac{dp}{dx}+\mu\,\dfrac{\partial^2 u}{\partial y^2} & = & 0 \end{array}\right.$$

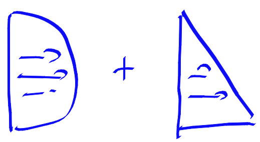
$$u(x,y) = -\frac{dp}{dx}\,\frac{h^2}{2\mu}\,\frac{y}{h}\left(1-\frac{y}{h}\right)+U\left(1-\frac{y}{h}\right)$$





Calcul du profil de vitesse…

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$



$$u(x,y) = -\frac{dp}{dx}\,\frac{h^2}{2\mu}\left(1-\frac{y}{h}\right)\frac{y}{h} + U\left(1-\frac{y}{h}\right)$$

$$\left[\frac{N}{m^3}\right]\left[\frac{m^2}{N\,s}\;m^2\right]$$

$$[m/s]$$

$$\underline{\underline{\tau}} = 2\mu\,\underline{\underline{d}}$$

$$\left[\frac{N}{m^2}\right]\left[\frac{N\,s}{m^2}\right]\left[\frac{1}{s}\right]$$

Calcul du profil de pression…

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = \int_0^h \frac{\partial u}{\partial x}\, dy + \int_0^h \frac{\partial v}{\partial y}\, dy$$

$$= \underbrace{\frac{d}{dx} \int_0^{h(x)} u \, dy}_{Q(x)} + \underbrace{\left[v \right]_0^h}_{=0}$$

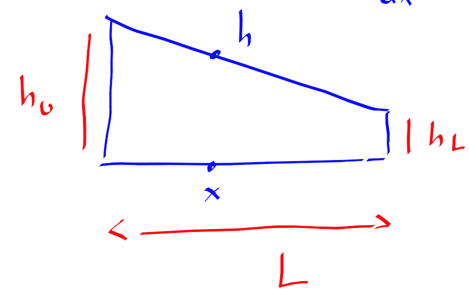
$$u(x,y) = -\frac{dp}{dx} \frac{h^2}{2\mu} \left(1 - \frac{y}{h}\right) \frac{y}{h} + U \left(1 - \frac{y}{h}\right)$$

$$0 = \frac{d}{dx} \left[-\frac{dp}{dx}(x) \frac{h^2(x)}{2\mu} \underbrace{\int_0^{h(x)} \left(1 - \frac{y}{h(x)}\right) \frac{y}{h(x)} \, dy}_{h(x)/6} + U \frac{h(x)}{2} \right]$$

$$0 = \frac{d}{dx}\left[-\frac{dp}{dx}(x)\,\frac{h^2(x)}{2\mu}\int_0^{h(x)} \left(1-\frac{y}{h(x)}\right)\frac{y}{h(x)}\,dy \;+\; U\,\frac{h(x)}{2}\right]$$

$$\int_0^{h(x)} \left(1-\frac{y}{h(x)}\right)\frac{y}{h(x)}\,dy = \frac{h(x)}{6}$$

$$\frac{dh}{dx} = \frac{h_L - h_0}{L}$$



$$0 = \frac{d}{dx}\left[-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2}\right]$$

$$\frac{d}{dx}\left[\frac{dp}{dx}(x)\;h^3(x)\right] = 6\mu U \frac{dh}{dx}$$

$$\left(\frac{dh}{dx}\right)^2 \frac{d}{dh}\left[\frac{dp}{dh}\,h^3\right] = 6\mu U\frac{dh}{dx}$$

$$-\frac{d}{dh}\left[\frac{dp}{dh}\,h^3\right] = \underbrace{\frac{6\mu U L}{h_0 - h_L}}_{C}$$

$$-\frac{dp}{dh}\,h^3 = C\left[h + A\right]$$

$$-\frac{dp}{dh} = C\left[\frac{1}{h^2} + \frac{A}{h^3}\right]$$

$$p(h) = C\left[\frac{1}{h} + \frac{A}{2h^2} + B\right]$$

2 CONDITIONS AUX LIMITES

$$p(0) = p_0$$

$$p(L) = p_0 \quad :-)$$

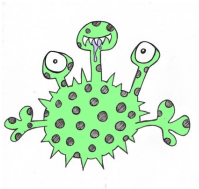
-ii- calcul de p(x)

$$
\begin{cases}
-\dfrac{dp}{dx} + \mu \dfrac{\partial^2 u}{\partial y^2} = 0 \\
\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0
\end{cases}
$$

$$0 = \int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} dy$$

$$0 = \frac{d}{dx} \overbrace{\int_0^h u(x,y)dy}^{Q(x)} + \Big[v(x,y) \Big]_0^h$$

En utilisant l'expression de u(x,y)



Equation classique de Reynolds (1889)

$$0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2} \right)$$

$$0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2}\right)$$

$$\frac{d}{dx}\left(h^3(x)\,\frac{dp}{dx}(x)\right) = 6\mu U \boxed{\frac{dh}{dx}}$$

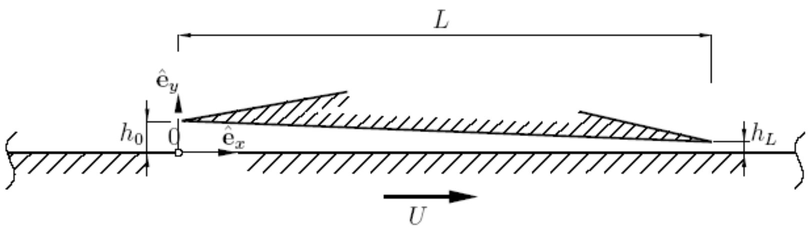
$$-\frac{d}{dh}\left(h^3\frac{dp}{dh}(h)\right) = \frac{6\mu UL}{h_0 - h_L}$$

$$-h^3\frac{dp}{dh}(h) = \frac{6\mu UL}{h_0 - h_L}(h + A)$$

$$-\frac{dp}{dh}(h) = \frac{6\mu UL}{h_0 - h_L}\left(\frac{1}{h^2} + \frac{A}{h^3}\right)$$

$$p(h) = \frac{6\mu UL}{h_0 - h_L}\left(B + \frac{1}{h} + \frac{A}{2h^2}\right)$$

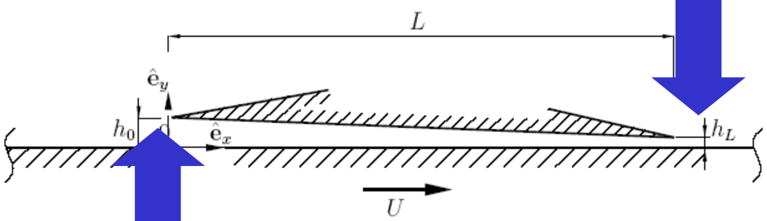
Palier plat



$$\frac{x}{L} = \frac{h_0 - h(x)}{h_0 - h_L}$$

$$\frac{dh}{dx} = -\frac{h_0 - h_L}{L}$$

$$p(h_L) = p_0$$



$$p(h_0) = p_0$$

Deux conditions aux limites

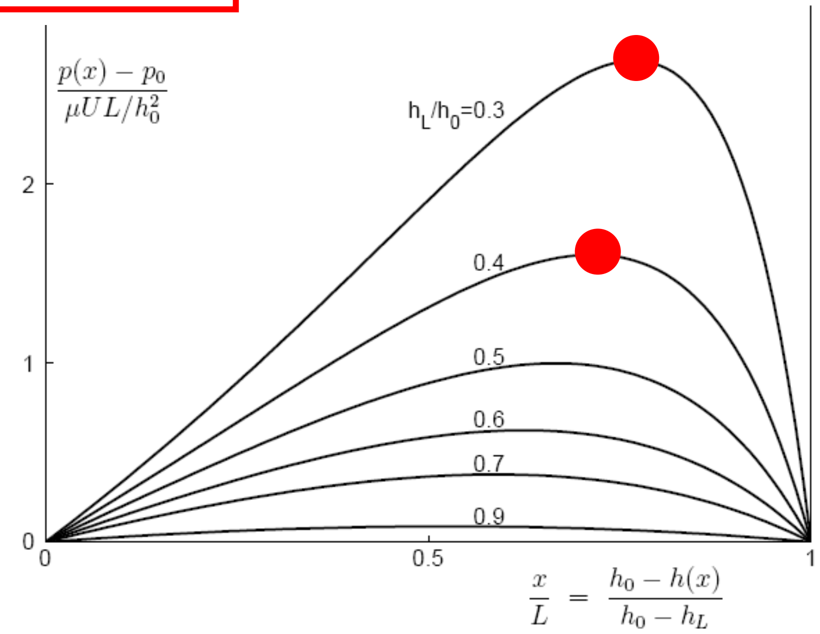
$$p(h) = \frac{6\mu U L}{h_0 - h_L}\left(B + \frac{1}{h} + \frac{A}{2h^2}\right)$$

Deux constantes

$$p(h) - p_0 = \frac{6\mu U L(h_0 - h)(h - h_L)}{(h_0^2 - h_L^2)h^2}$$

Où la pression est-elle maximale ?

$$h = \frac{2\,h_0\,h_L}{(h_0 + h_L)}$$

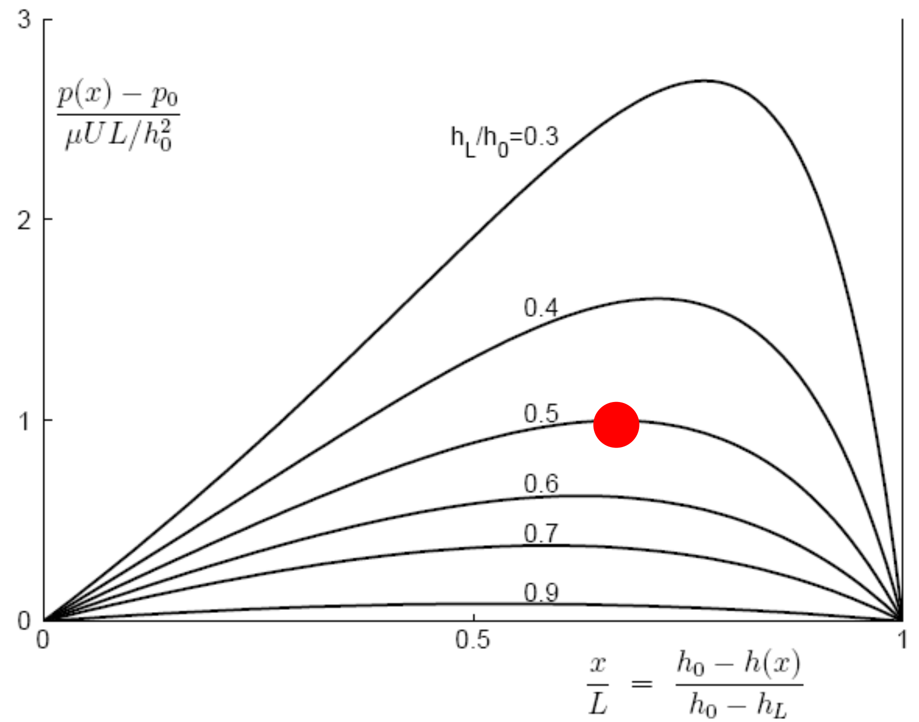


Cette pression peut être énorme !

$$\begin{aligned} L &= 10\ cm \\ h_0 &= 0.1\ mm \\ h_L &= 0.05\ mm \\ U &= 10\ m/s \end{aligned}$$

$$\begin{aligned} \rho &= 900\ kg/m^3 \\ \mu &= 0.1\ Ns/m^2 \end{aligned}$$

Huile SAE50 à 50 degrés

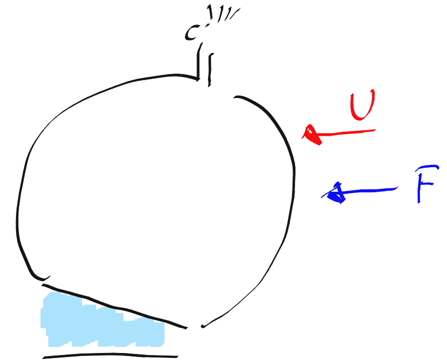


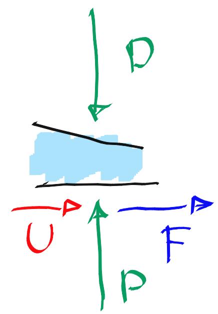
$$p_{\max} - p_0 = \frac{3}{2}\frac{\mu\, U\, L}{h_0\, h_L}\frac{(h_0 - h_L)}{(h_0 + h_L)}$$

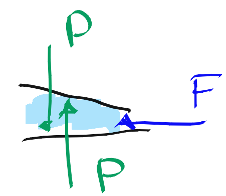
10^7 Pascal

Charge utile

$$P = \int_0^L p(x) - p_0 \, dx$$







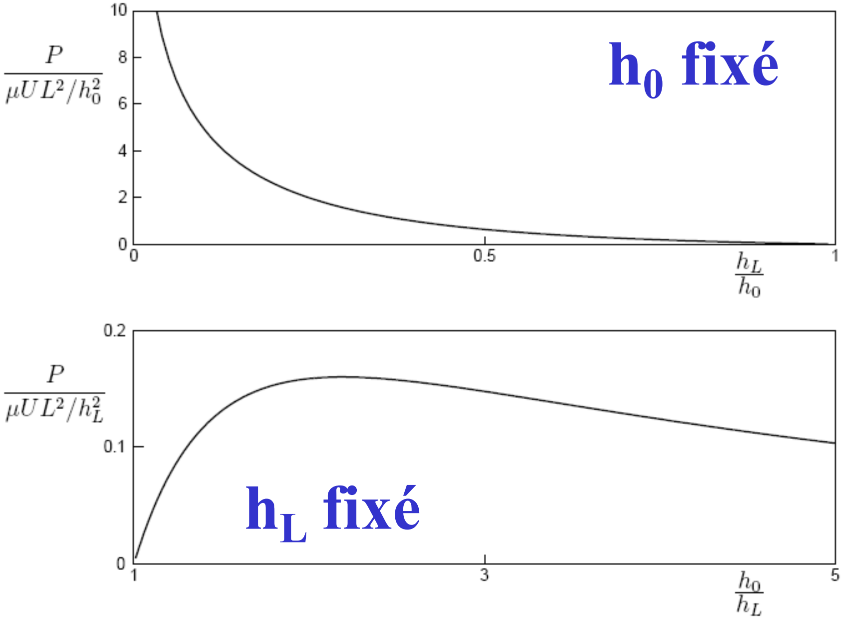
$$F = -\mu \int_0^L \frac{\partial u}{\partial y} \, dx$$

Force de poussée

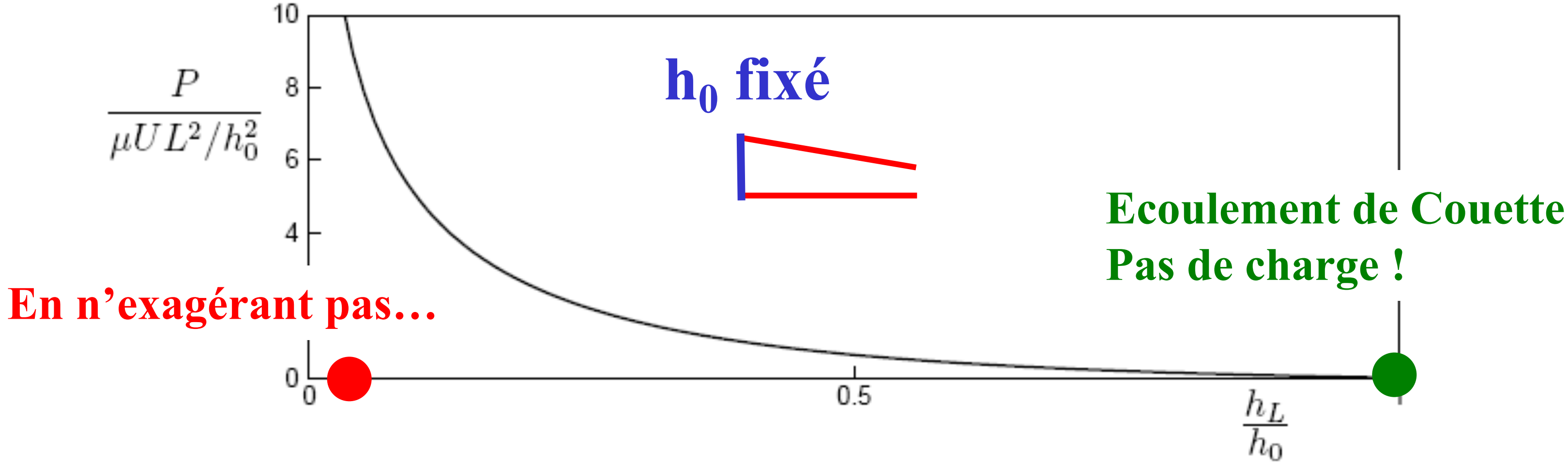
Puissance dissipée

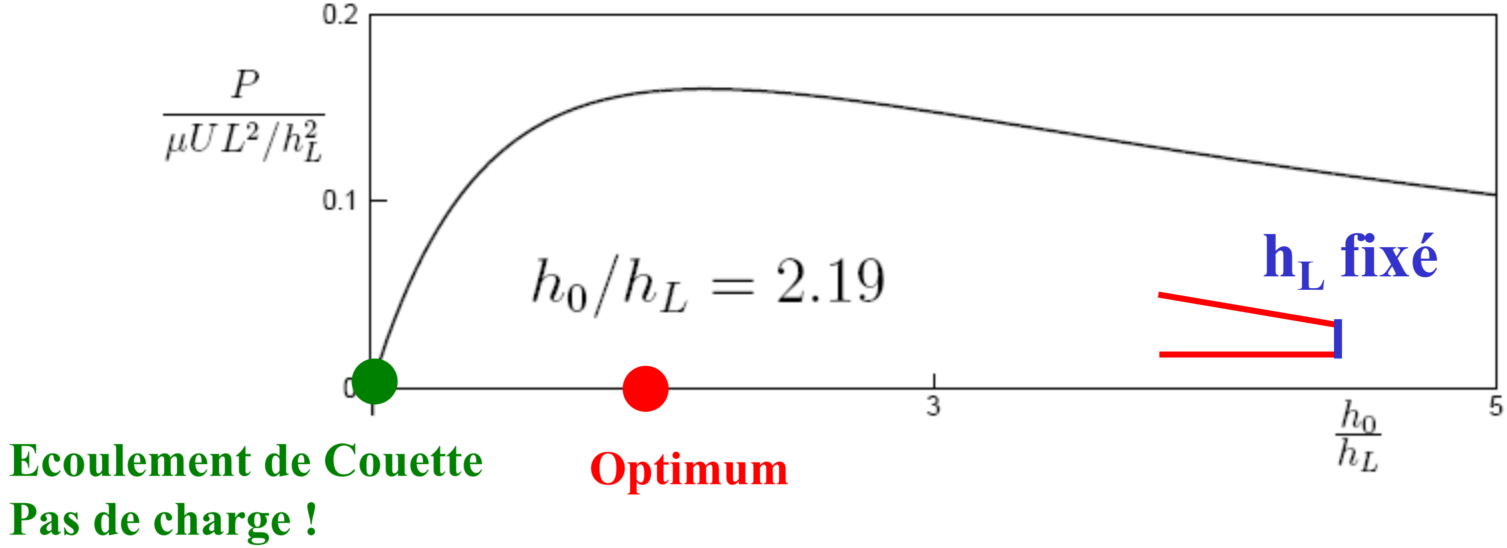
$$F \, U$$

Charge utile



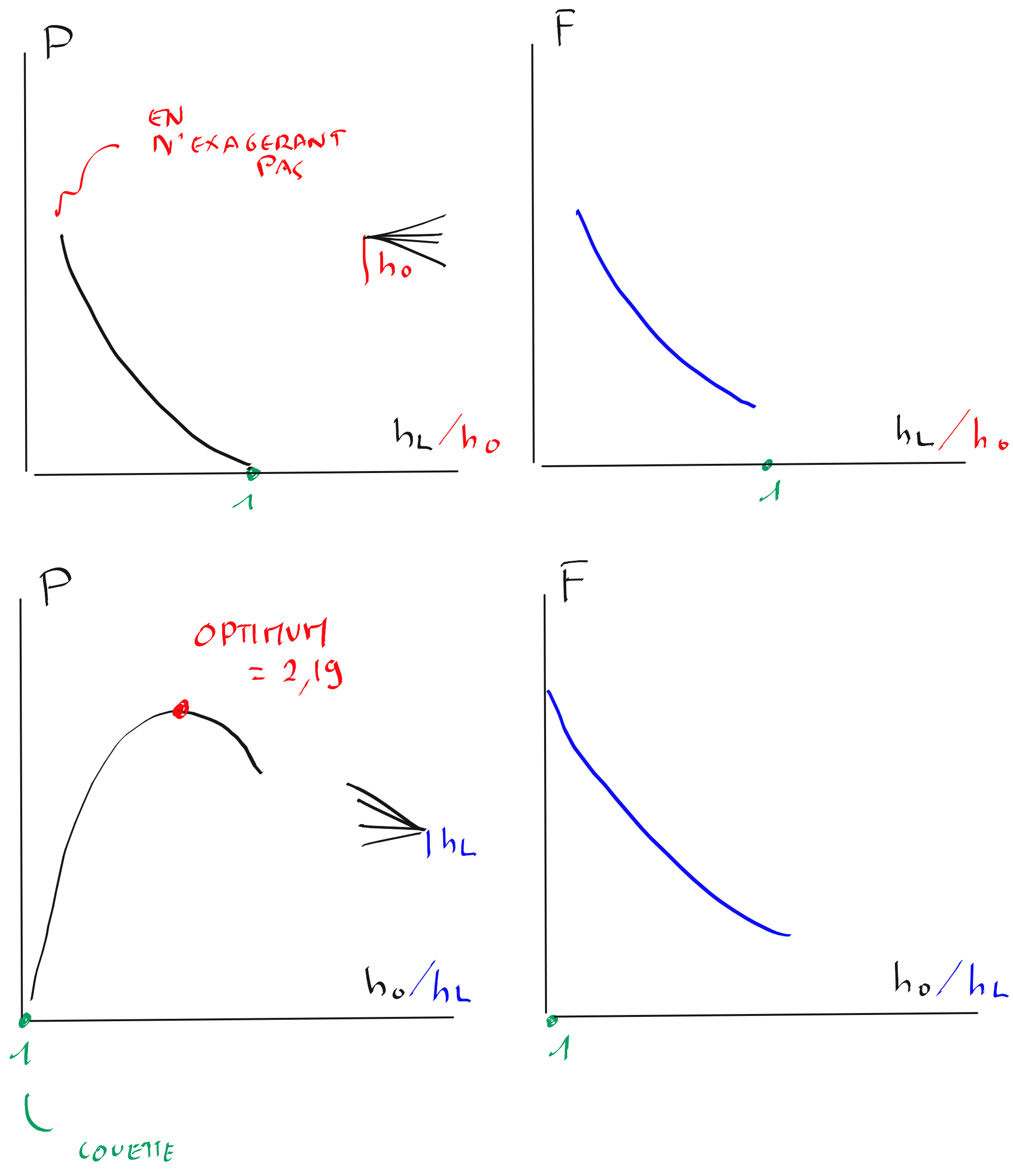
$$
\begin{aligned}
P &= \int_0^L (p(x) - p_0)\, dx \\
&= -\frac{L}{(h_0 - h_L)} \int_{h_0}^{h_L} (p(h) - p_0)\, dh \\
&= -\frac{L}{(h_0 - h_L)} \frac{6\,\mu\, U\, L}{(h_0^2 - h_L^2)} \int_{h_0}^{h_L} \left[(h_0 + h_L)\frac{1}{h} - \frac{h_0\, h_L}{h^2} - 1 \right] dh \\
&= -6\,\mu\, U\, L^2 \left[\frac{1}{(h_0 - h_L)^2} \log\left(\frac{h_L}{h_0}\right) + \frac{2}{(h_0^2 - h_L^2)} \right]
\end{aligned}
$$



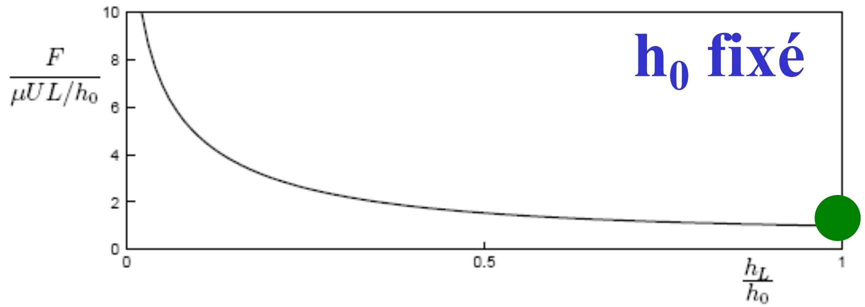


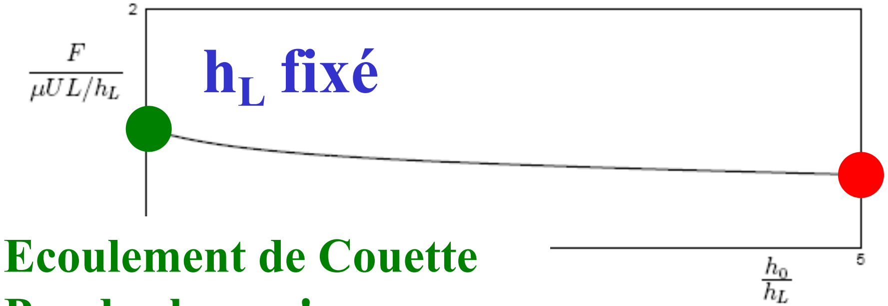
Rapport optimal…

Quatre petits graphes !



Force exercée par le fluide sur la partie mobile





$$
\begin{aligned}
F &= -\int_0^L \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} dx \\
&= \frac{\mu U L}{(h_0 - h_L)} \int_{h_0}^{h_L} \left[\frac{6}{h^2}\frac{h_0\, h_L}{(h_0 + h_L)} - \frac{4}{h}\right] dh \\
&= -\mu U L \left[\frac{6}{(h_0 + h_L)} + \frac{4}{(h_0 - h_L)} \log\left(\frac{h_L}{h_0}\right)\right]
\end{aligned}
$$

La force diminue de façon monotone lorsque le rapport augmente…

La puissance consommée est dissipée…

$$F\,U = -\frac{\mu\,U^2\,L}{h_0}\left[\frac{6}{(1+h_L/h_0)} + \frac{4}{(1-h_L/h_0)}\log\left(\frac{h_L}{h_0}\right)\right]$$

Embêtant…

S'assurer que l'huile est bien refroidie car la viscosité (et donc la charge utile) décroît rapidement avec la température…

…en chaleur !

A propos de la viscosité de notre huile SAE 50

Transport maritime

Marine LCX

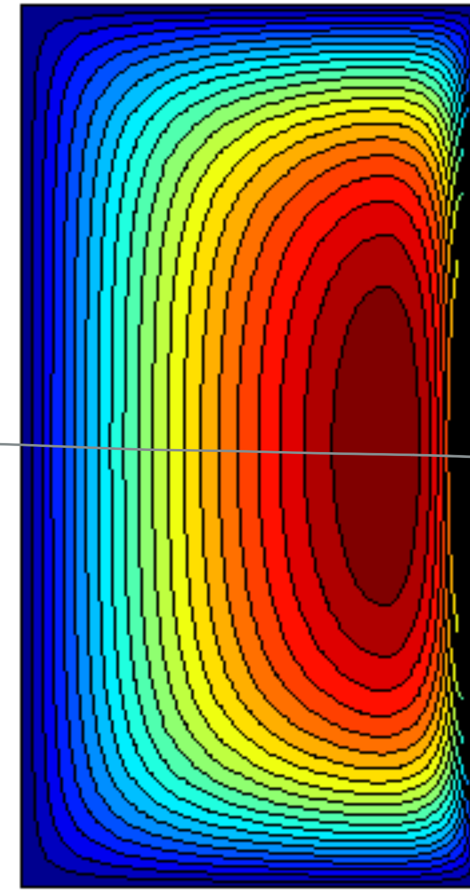
Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

Grades offerts :
SAE 50

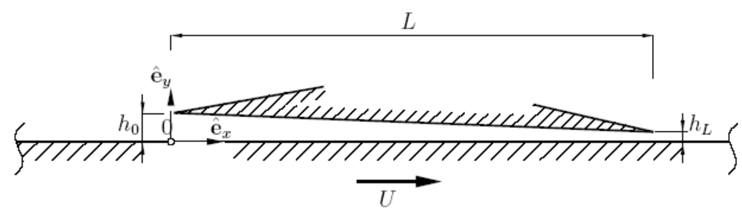
Fiche techniqu
Fiche signalét

$$
\begin{aligned}
T &= 20^0 C & \mu &= 1.100\ Ns/m^2 \\
T &= 40^0 C & \mu &= 0.210\ Ns/m^2 \\
T &= 50^0 C & \mu &= 0.100\ Ns/m^2 \\
T &= 60^0 C & \mu &= 0.060\ Ns/m^2 \\
T &= 80^0 C & \mu &= 0.025\ Ns/m^2 \\
T &= 100^0 C & \mu &= 0.013\ Ns/m^2
\end{aligned}
$$

Analyse « tridimensionnelle » du palier plat



pression sous un palier dont la largeur vaut le double de la longueur



Lubrification 2D ½

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial p}{\partial z} = 0$$

Théorie de la lubrification

Film fluide mince

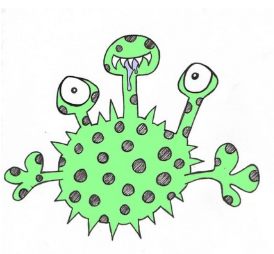
$$h \ll L$$

Hypothèse de lubrification : Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\left\{\begin{array}{rcl} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} & = & 0 \\ -\dfrac{\partial p}{\partial x} + \mu \dfrac{\partial^2 u}{\partial z^2} & = & 0 \\ -\dfrac{\partial p}{\partial y} + \mu \dfrac{\partial^2 v}{\partial z^2} & = & 0 \end{array}\right.$$

-i- calcul
de u(x,y,z)
et de v(x,y,z)



$$u(x,y,z) = -\frac{\partial p}{\partial x}\frac{h^2}{2\mu}\frac{z}{h}\left(1-\frac{z}{h}\right) + U\left(1-\frac{z}{h}\right)$$

$$v(x,y,z) = -\frac{\partial p}{\partial y}\frac{h^2}{2\mu}\frac{z}{h}\left(1-\frac{z}{h}\right)$$

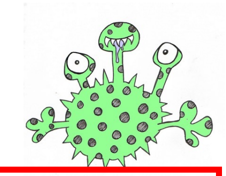
-ii- calcul de p(x,y)

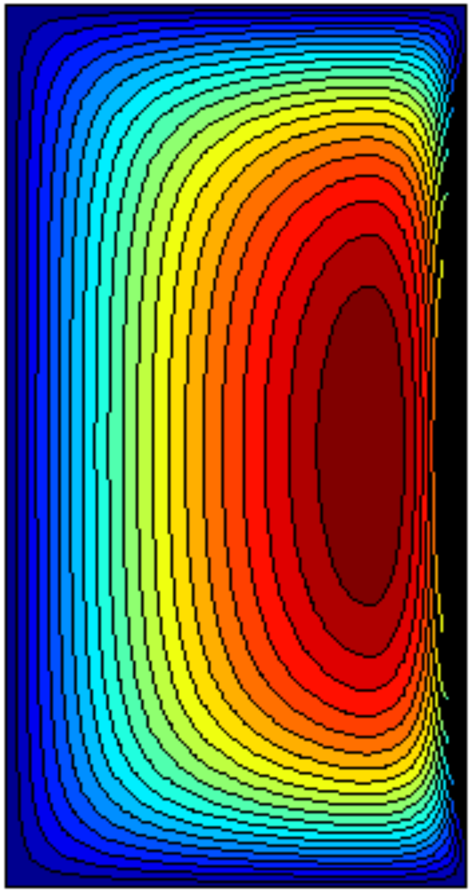
$$\left\{\begin{array}{rcl} -\dfrac{\partial p}{\partial x} + \mu\,\dfrac{\partial^2 u}{\partial z^2} &=& 0 \\ -\dfrac{\partial p}{\partial y} + \mu\,\dfrac{\partial^2 v}{\partial z^2} &=& 0 \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} &=& 0 \end{array}\right.$$

$$\int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\, dz = 0$$

$$\frac{\partial}{\partial x}\int_0^h u(x,y,z)dz + \frac{\partial}{\partial y}\int_0^h v(x,y,z)dz + \Big[\, w(x,y,z)\,\Big]_0^h = 0$$

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) = 6\mu U\frac{dh}{dx}$$

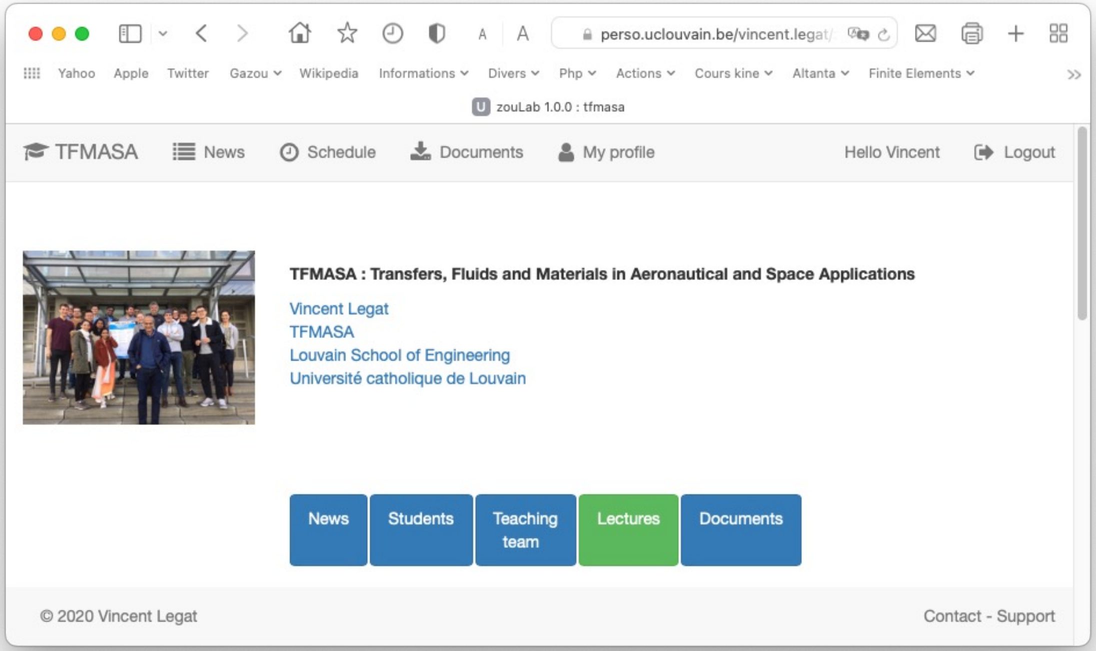




-iiii- calcul numérique par différences finies de p(x,y)

$$h^3\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+3h^2\left(\frac{h_L-h_0}{L}\right)\frac{\partial p}{\partial x} = 6\mu U\left(\frac{h_L-h_0}{L}\right)$$

Un triple diplôme belgo-germano-français ?



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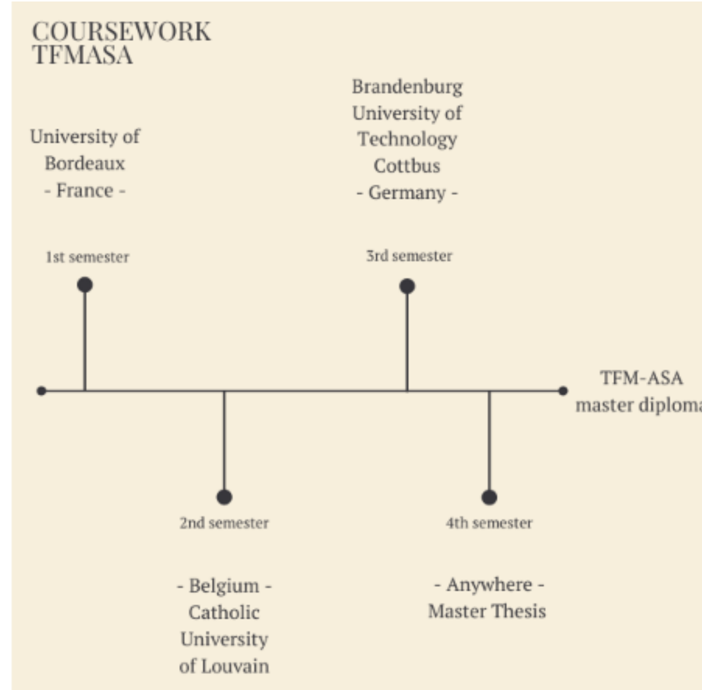
The students will benefit from top quality training in Mechanical and Aerospace Engineering. They will spend an entire semester in each university. Many industrial partners are directly involved through internships for students, conferences and even courses.



Et tout cela
en deux années !

Master in Transferts, Fluids and Materials for Aeronautical and Space Applications

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