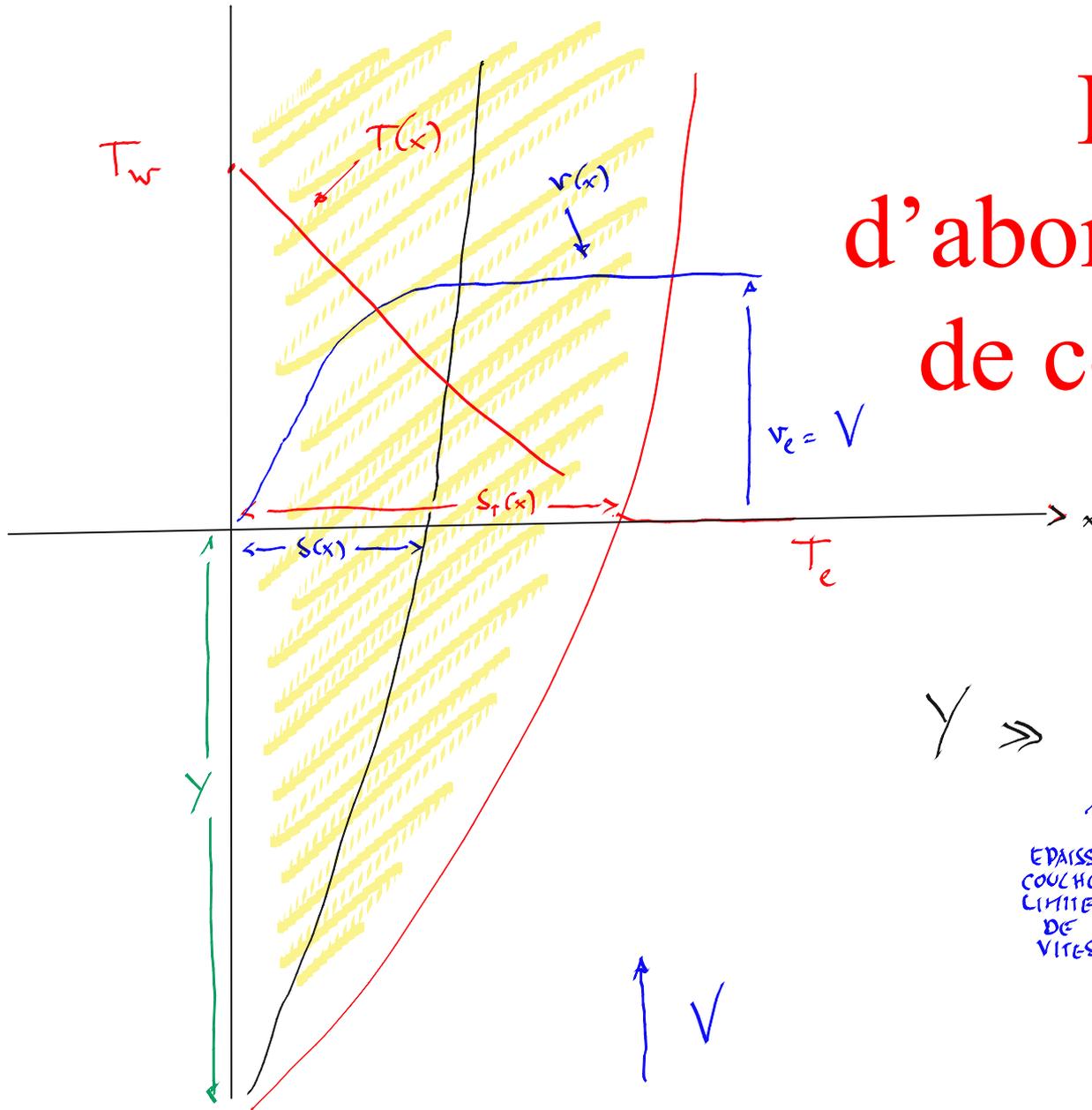


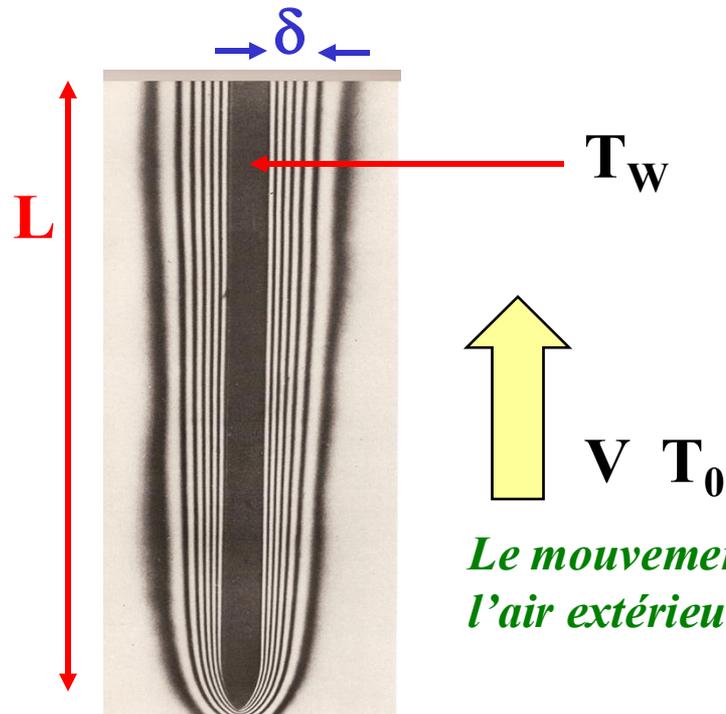
Mais, tout d'abord, un peu de convection forcée



$Y \gg S, S_T$

ÉPAISSEUR
COUCHE
LIMITE
DE
VITESSE

ÉPAISSEUR
DE LA
COUCHE
LIMITE
THERMIQUE



Le mouvement vertical de l'air extérieur est forcé

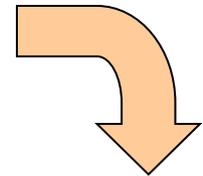
Mais, tout d'abord, un peu de convection forcée

Plus facile car, il est ici possible de découpler le problème de l'écoulement et le problème thermique !

Problème de l'écoulement

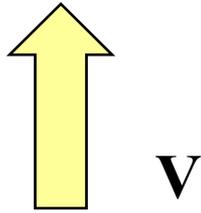
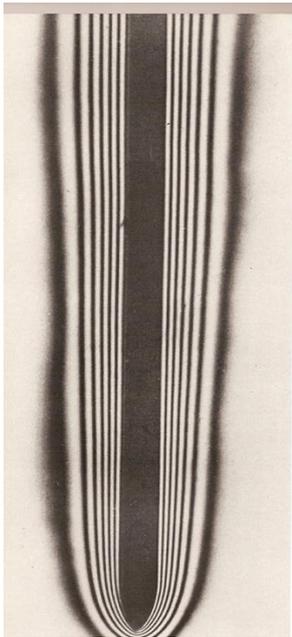
$$\nabla \cdot \mathbf{v} = 0,$$

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g},$$



$$\rho c \frac{DT}{Dt} = 2\mu \mathbf{d} : \mathbf{d} + r + \nabla \cdot (k \nabla T).$$

Problème thermique



-i- problème de l'écoulement

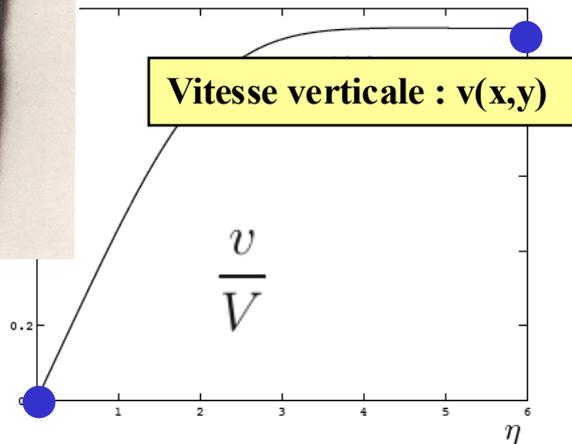
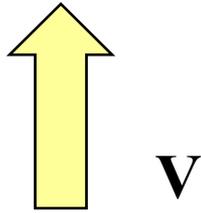
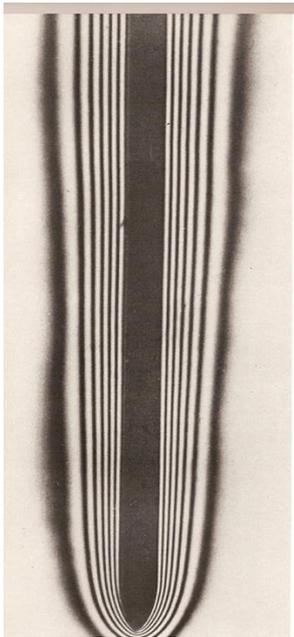
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

On introduit
une variable de similitude basée
une estimation de la couche limite

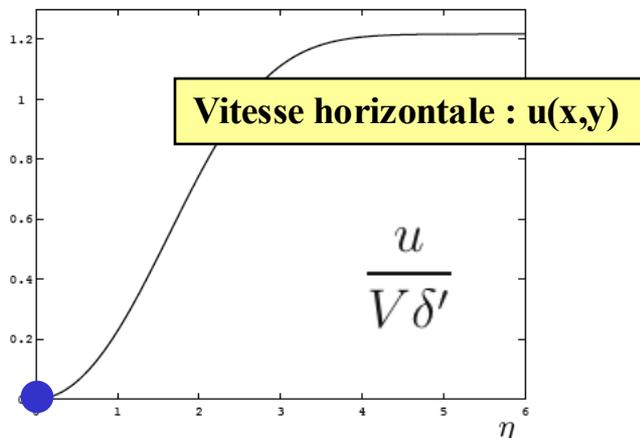
$$\eta(x, y) = \frac{x}{\delta(y)} = \frac{x}{\sqrt{\frac{2\nu y}{V}}}$$



Solution de Blasius



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

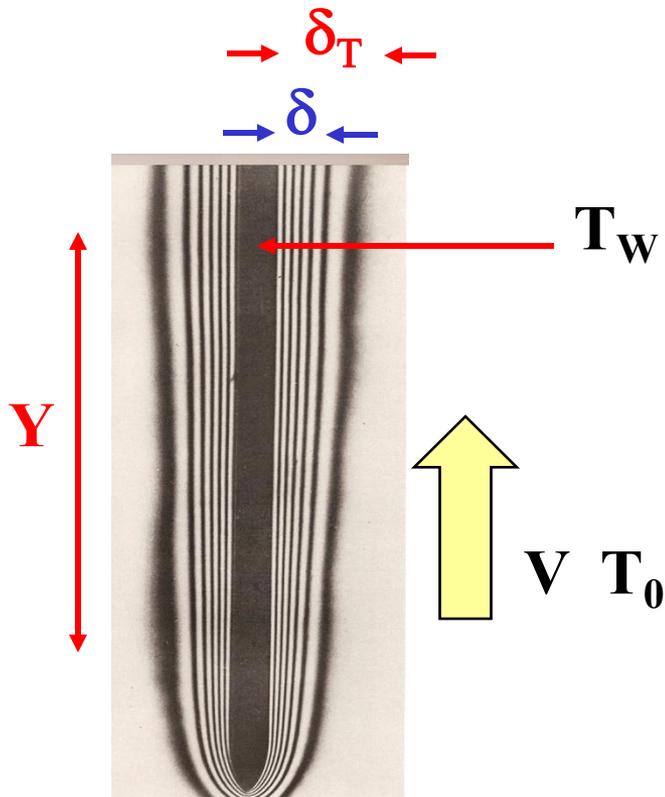


L'estimation de l'ordre de grandeur de la couche limite était bien adéquat !



$$\eta(x, y) = \frac{x}{\delta(y)} = \frac{x}{\sqrt{\frac{2\nu y}{V}}}$$

-ii- problème thermique



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$\delta \ll Y$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\delta_T \ll Y$$

Près de la plaque, les effets conductifs sont dominants...

C'est la zone dite de couche limite thermique

Loin de la plaque, la conduction est négligeable

On définit l'épaisseur thermique comme le lieu géométrique où la conduction et la convection sont du même ordre de grandeur.

$$\underbrace{u \frac{\partial T}{\partial x}}_{\frac{U \Delta T}{\delta}} + \underbrace{v \frac{\partial T}{\partial y}}_{\frac{V \Delta T}{\gamma}} = \underbrace{\alpha \frac{\partial^2 T}{\partial x^2}}_{\alpha \frac{\Delta T}{S_T^2}} + \cancel{\alpha \frac{\partial^2 T}{\partial y^2}}_{\alpha \frac{\Delta T}{\gamma^2}}$$

$$\frac{U}{\delta} = \frac{V}{\gamma}$$

$$\frac{V \Delta T}{\gamma}$$

$$\frac{S}{\gamma} = \sqrt{\frac{1}{Re}}$$

$$\frac{S_T}{\gamma} = \sqrt{\frac{1}{Pe}} = \sqrt{\frac{1}{Re Pr}}$$

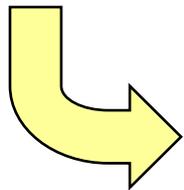
$$\frac{\delta_T}{\delta} = \frac{V \Delta T}{\gamma} \frac{S_T^2}{\alpha \Delta T} \approx 1$$

Et δ_T ?

$$\frac{S_T^2}{\gamma^2} = \frac{\alpha}{V \gamma}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} + \alpha \frac{\partial^2 T}{\partial y^2}$$

Lieu où l'ordre de la convection et de la conduction sont identiques



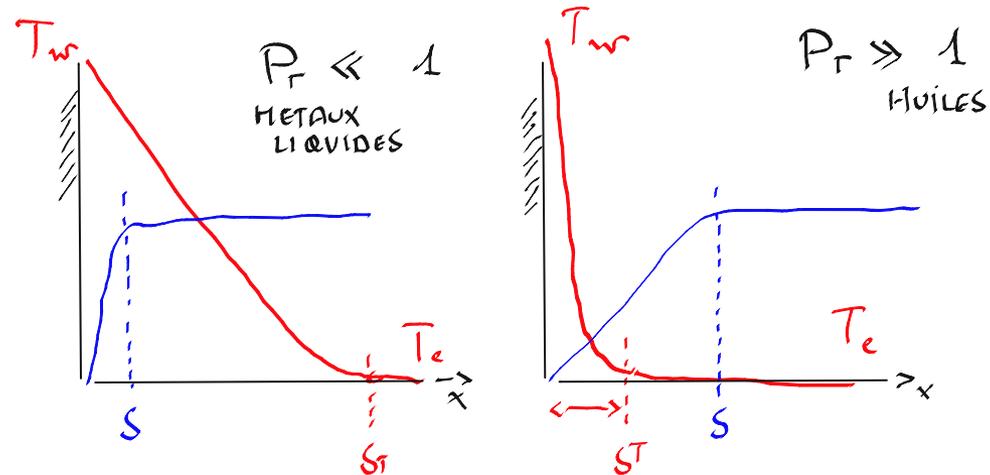
$$\frac{\text{Convection}}{\text{Conduction}} = \frac{V \Delta T / Y}{\alpha \Delta T / \delta_T^2} = \frac{V Y}{\underbrace{\alpha}_{Pe_Y}} \frac{\delta_T^2}{Y^2} = 1$$

Et δ_T ?

$$\frac{\delta_T}{Y} = \sqrt{\frac{1}{Pe_Y}} = \sqrt{\frac{1}{Pr Re_Y}}$$

Epaisseurs de couches limites et le nombre de Prandtl

$$\frac{\delta_T}{\delta} = \sqrt{\frac{1}{Pr}}$$



En fait, c'est uniquement l'ordre de grandeur...

Lorsqu'on calcule la vraie épaisseur pour Blasius, c'est plutôt un exposant 1/3 !

Car l'ordre de grandeur de vitesse est un peu surestimé en prenant U !

Métaux liquides	$Pr \ll 1$
Gaz	$Pr = 0.7$
Eau	$Pr = 2 \dots 7$
Huiles	$Pr \gg 1$

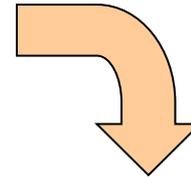
Attention : Prandtl est une fonction de la température (davantage pour les liquides que pour les gaz) !

Soyons un peu plus général : et la dissipation visqueuse ?

Problème de l'écoulement

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

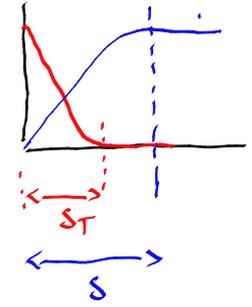


$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{c} \left(\frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$

Problème thermique

Eckert : dissipation visqueuse ?

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{\epsilon} \left(\frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$



$\frac{1}{c} \frac{V^2}{\delta^2}$ DISSIPATION VISQUEUSE

$\alpha \frac{\Delta T}{\delta_T^2}$

DIFFUSION
CONDUCTION
THERMIQUE

$$\frac{\frac{\nu}{\epsilon} \left(\frac{\partial v}{\partial x} \right)^2}{\alpha \frac{\partial^2 T}{\partial x^2}} = \frac{1}{c} \frac{V^2}{\delta^2} \frac{\delta_T^2}{\alpha \Delta T} = \frac{\rho \nu}{\alpha} \frac{\delta_T^2}{\delta^2} \frac{V^2}{c \Delta T} = \frac{1}{Pr} \frac{Ec}{Pr}$$

Eckert : dissipation visqueuse ?

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{c} \left(\frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$

Dissipation visqueuse
Conduction

$$\frac{\text{■}}{\text{■}} = \frac{\nu V^2}{\alpha c \Delta T} \underbrace{\left(\frac{\delta_T}{\delta} \right)^2}_{Pr^{-1}} = \frac{V^2}{c \Delta T} = Ec$$

Nombre d'Eckert

$$Ec = \frac{u_e^2}{c(T_w - T_e)}$$

caractérise un écoulement
d'un fluide !

Energie cinétique

Energie interne



Picture was taken on August 22, 2000

$$Pr = 1$$

$$Ec \ll 1$$

$$Pr = 1$$

$$Ec \not\ll 1$$

*Couches
limites
identiques*

$$Pr \neq 1$$

$$Ec \ll 1$$

$$Pr \neq 1$$

$$Ec \not\ll 1$$

*Dissipation
visqueuse
négligeable*



**Quatre cas
possibles !**

$Pr = 1$ $Ec \ll 1$	$Pr = 1$ $Ec \ll 1$
$Pr \neq 1$ $Ec \ll 1$	$Pr \neq 1$ $Ec \ll 1$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nu \frac{\partial^2 T}{\partial x^2}$$

$$Pr = 1$$

$$Ec \ll 1$$

Les équations d'énergie et de quantité de mouvement expriment les mêmes opérateurs différentiels pour la température et la vitesse verticale !

$$T = Av + B$$

Si, si, c'est aussi simple !

Relation de Crocco

$Pr = 1$ $Ec \ll 1$	$Pr = 1$ $Ec \ll 1$
$Pr \neq 1$ $Ec \ll 1$	$Pr \neq 1$ $Ec \ll 1$

$$cT + \frac{v^2}{2} = Av + B$$

On a les mêmes opérateurs différentiels pour l'énergie interne totale et la vitesse verticale !

$$Pr = 1$$

$$Ec \ll 1$$

$$v \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = v \left[\nu \frac{\partial^2 v}{\partial x^2} \right]$$

$$c \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = c \left[\frac{\nu}{c} \left(\frac{\partial v}{\partial x} \right)^2 + \nu \frac{\partial^2 T}{\partial x^2} \right]$$

$$\frac{\partial}{\partial x} \left(\frac{v^2}{2} \right) = \frac{1}{2} \cancel{2} v \frac{\partial v}{\partial x}$$

Relation de Crocco

$$cT + \frac{v^2}{2} = Av + B$$

$$v \frac{\partial}{\partial x} \left[cT + \frac{v^2}{2} \right] + v \frac{\partial}{\partial y} \left[cT + \frac{v^2}{2} \right] = v \frac{\partial}{\partial x^2} \left[cT + \frac{v^2}{2} \right]$$

$$v \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = v \left[\nu \frac{\partial^2 v}{\partial x^2} \right]$$

$$\begin{aligned} & \frac{\partial^2}{\partial x^2} \left[\frac{v^2}{2} \right] \\ &= \frac{\partial}{\partial x} \left[v \frac{\partial v}{\partial x} \right] \\ &= \left[\frac{\partial v}{\partial x} \right]^2 + v \frac{\partial^2 v}{\partial x^2} \end{aligned}$$

$$c \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = c \left[\frac{\nu}{c} \left(\frac{\partial v}{\partial x} \right)^2 + \cancel{\nu \frac{\partial^2 T}{\partial x^2}} \right]$$

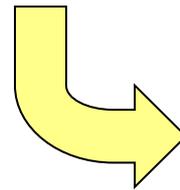
$Pr = 1$	$Pr = 1$
$Ec \ll 1$	$Ec \ll 1$
$Pr \neq 1$	$Pr \neq 1$
$Ec \ll 1$	$Ec \ll 1$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\theta(\eta) = \frac{T - T_w}{T_e - T_w}$$

On procède comme pour Blasius !



$$Pr f \theta' + \theta'' = 0$$

Solution de similitude

$Pr \neq 1$
$Ec \ll 1$

$$\eta = \sqrt{\frac{x}{2y_{13}}} \frac{1}{V}$$

$$\Theta(\eta) = \frac{T - T_w}{T_e - T_w}$$

$$f f'' + f''' = 0$$

$$Pr f \theta' + \theta'' = 0$$

$$\theta(0) = 0$$

$$\theta(\eta \rightarrow \infty) = 1$$

$$\exp(g) \underbrace{Pr f}_{g'} \theta' + \exp(g) \theta'' = 0$$

$$\left(\exp(g) \theta' \right)' = 0$$

$$\exp(g) \theta' = A$$

$$\theta' = A \exp(-g)$$

$$\theta = A \int_0^\eta \exp(-g) + B$$

$$Pr f \theta' + \theta'' = 0$$

Calcul de la solution de similitude

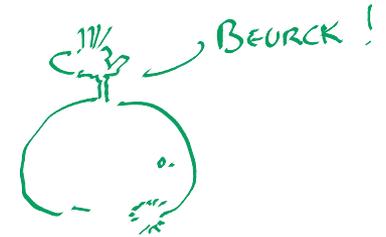
$$\Theta = A \int_0^{\eta} \exp(-g) + B$$

$$\Theta(\eta) = A \int_0^{\eta} \exp(-g(\xi)) d\xi + B$$

$$g' = Pr f$$

$$g(x) = Pr \int_0^x f(\xi) d\xi$$

$$\Theta(\eta) = \frac{\int_0^{\eta} \exp \left[-Pr \int_0^{\xi} f(\zeta) d\zeta \right] d\xi}{\int_0^{\infty} \dots}$$



$$Pr f \theta' + \theta'' = 0$$

Calcul des deux constantes

$$\begin{array}{l} Pr = 1 \\ Ec \ll 1 \end{array}$$

$$\begin{array}{l} Pr = 1 \\ Ec \ll 1 \end{array}$$

$$\begin{array}{l} Pr \neq 1 \\ Ec \ll 1 \end{array}$$

$$\begin{array}{l} Pr \neq 1 \\ Ec \ll 1 \end{array}$$

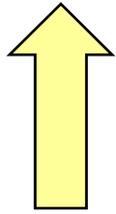
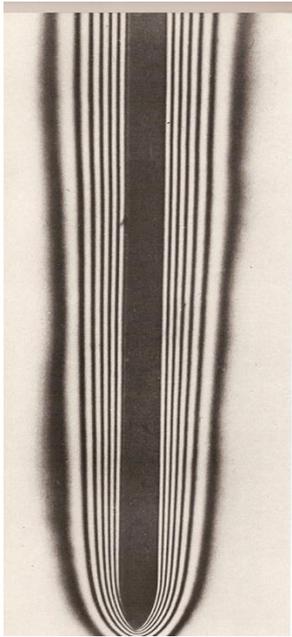
$$\begin{array}{l} Pr \neq 1 \\ Ec \ll 1 \end{array}$$

**Le cas le plus général
(et donc aussi le plus probable !)**

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{c} \left(\frac{\partial v}{\partial x} \right)^2 + \alpha \frac{\partial^2 T}{\partial x^2}$$

Pas de solution analytique...



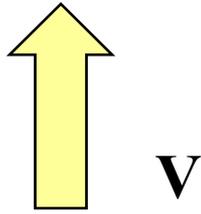
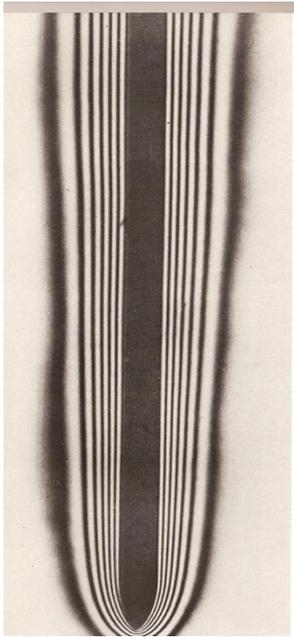
V

$$\begin{aligned}
 \tau_w &= \mu \left. \frac{\partial v}{\partial x} \right|_{x=0} && \sqrt{\frac{\nu}{2uy}} \\
 &= \mu V \underbrace{f''|_{\eta=0}}_{0.4696} \frac{1}{\delta} \\
 &= 0.332 \sqrt{\frac{\mu^2 V^3}{\rho \nu}} \left. \right\} \sqrt{\frac{1.32 \rho^2 V^3}{\rho \nu}} \\
 &= \frac{\rho V^2}{2} \underbrace{0.664}_{\sqrt{\frac{11}{\nu y}}} \underbrace{\left. \right\} Re^{-1/2}}
 \end{aligned}$$

$$\begin{aligned}
 C_f &= \frac{\tau_w}{\rho V^2 / 2} \\
 &= 0.664 Re^{-1/2}
 \end{aligned}$$



Calcul de la force de trainée



$$\begin{aligned}
 q_w &= -k \left. \frac{\partial T}{\partial x} \right|_{x=0} \\
 &= -k (T_e - T_w) \underbrace{\Theta' \Big|_{\eta=0}}_{\approx 0.332 \sqrt{2}} \frac{1}{\delta} \sqrt{\frac{V}{2\nu y}} \\
 &= -k (T_e - T_w) 0.332 \sqrt{\frac{V}{\nu y}} Pr^{1/3} \quad \text{VALABLE SI } Pr > 0.01 \\
 &= -\rho c V (T_e - T_w) 0.332 \underbrace{\frac{k}{\rho c}}_{\alpha} \underbrace{\sqrt{\frac{V}{V^2 \nu y}}}_{\frac{1}{\nu} \sqrt{\frac{\nu}{Vy}}} Pr^{1/3} \\
 &= -\rho c V (T_e - T_w) 0.332 Pr^{-1} Re^{-1/2} Pr^{1/3}
 \end{aligned}$$

$$\begin{aligned}
 St &= \frac{-q_w}{\rho c V (T_e - T_w)} \\
 &= 0.332 Pr^{-2/3} Re^{-1/2}
 \end{aligned}$$



Calcul du flux de chaleur

$$C_f = 2 Pr^{2/3} St$$

FORCE DE FROTTEMENT ADIMENSIONNELLE
 FLUX DE CHALEUR ADIMENSIONNEL

$$C_f = \frac{\tau_w}{\rho V^2/2}$$

$$= 0.664 Re^{-1/2}$$

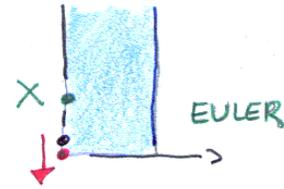
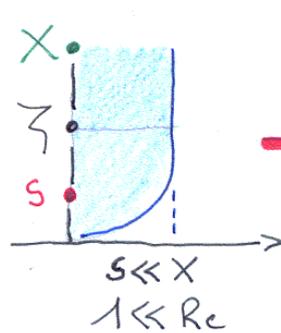
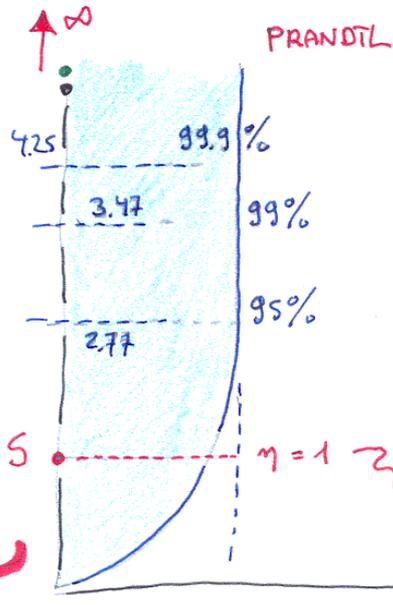


$$St = \frac{q_w}{\rho c V (T_c - T_w)}$$

$$= 0.332 Pr^{-2/3} Re^{-1/2}$$



Analogie de Reynolds



$$u_e(\underbrace{\eta = 3.47}_{\eta_{0.99}}) = 0.99 u_e$$

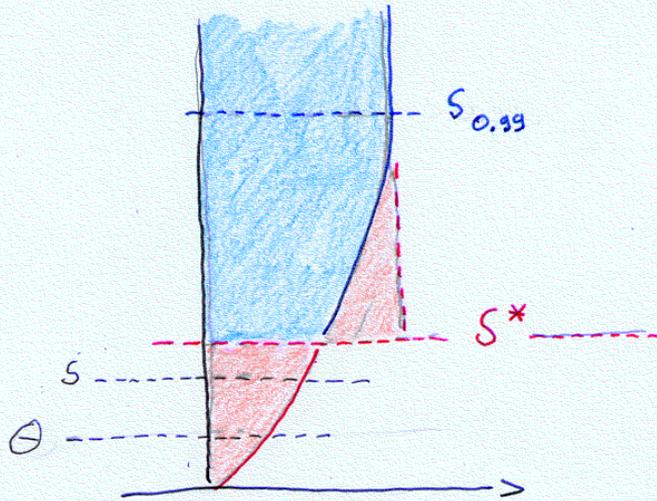
S
CE N'EST QU'UN ORDRE

DE GRANDEUR ADEQUAT !

$$\frac{S}{X} \triangleq \sqrt{\frac{2.11}{u_e X}}$$

$\frac{S_{0.95}}{X}$	=	2.77	$\sqrt{2}$	$\frac{1}{\sqrt{Re}}$	=	3.92	$\frac{1}{\sqrt{Re}}$
$\frac{S_{0.99}}{X}$	=	3.47	$\sqrt{2}$	$\frac{1}{\sqrt{Re}}$	=	4.91	$\frac{1}{\sqrt{Re}}$
$\frac{S_{0.999}}{X}$	=	4.25	$\sqrt{2}$	$\frac{1}{\sqrt{Re}}$	=	6.02	$\frac{1}{\sqrt{Re}}$

EPAISSEUR DE DEPLACEMENT δ^*



$$+ \quad \zeta \gg \delta^*$$

$$(\zeta - \delta^*) u_e = \int_0^{\zeta} u \, dy$$

$$\int_0^{\zeta} u_e \, dy - \delta^* u_e$$

$$=$$

$$\delta^* \triangleq \int_0^{\zeta} \left(1 - \frac{u}{u_e}\right) dy$$

CONVERGE si $\zeta \rightarrow \infty$

EPAISSEUR DE LA QUANTITÉ DE MOUVEMENT

$$\Theta \triangleq \int_0^{\infty} \frac{u}{u_e} \left(1 - \frac{u}{u_e}\right) dy$$

$u_e = \text{cst}$

$$\frac{\Theta}{X} = 0.664 \frac{1}{\sqrt{Re}}$$

$C_f = \frac{C_{f,m}}{2} = \frac{D}{\rho u_e^2 X / 2}$



$$\Theta = D / \rho u_e^2$$

DENSITÉ DE FORCE DE TRAÎNÉE NORMALISÉE

EXERCÉE PAR LA PLAQUE SUR LE FLUIDE ENTRE 0 ET X

