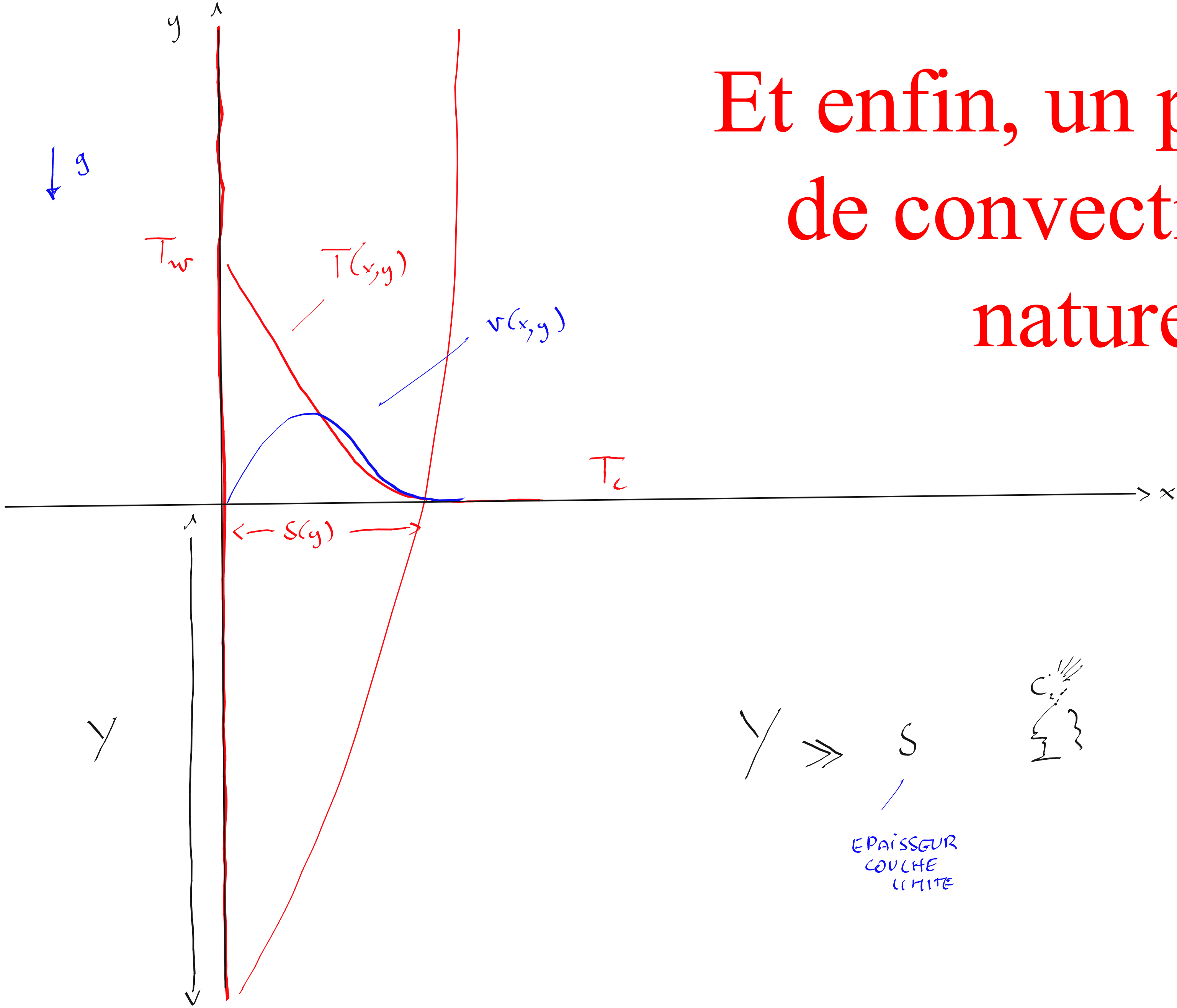
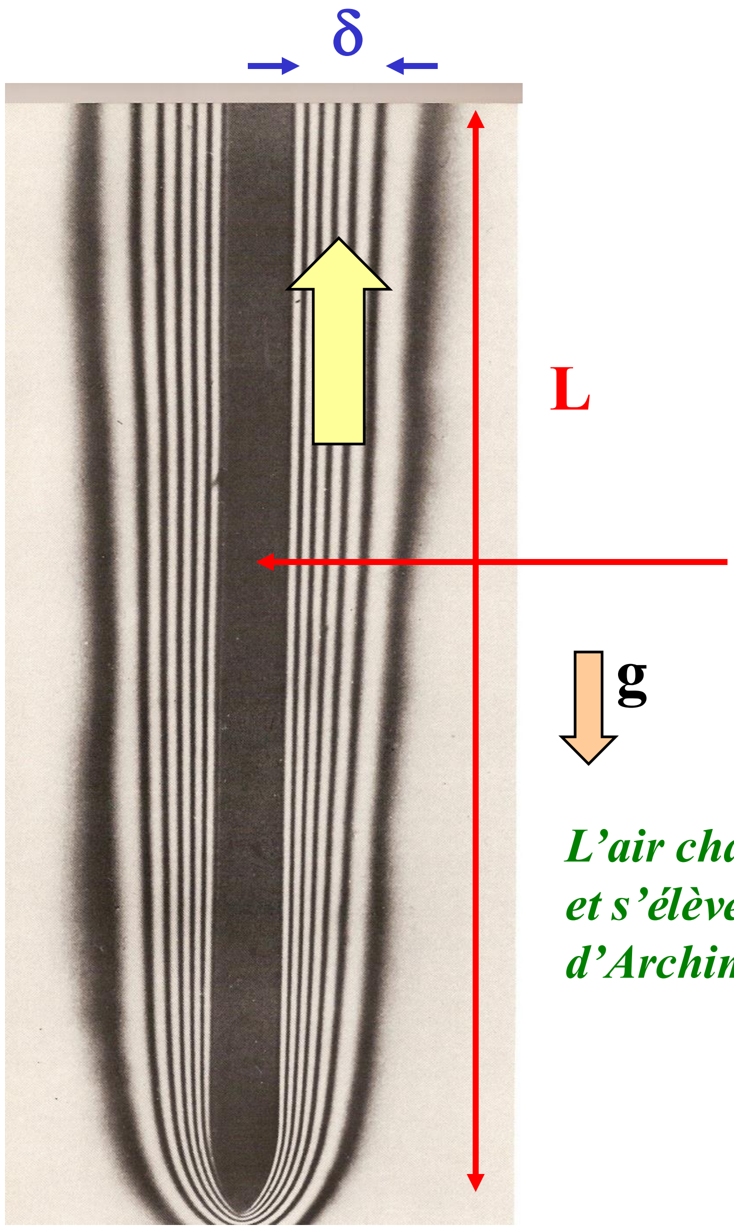
Et enfin, un peu de convection naturelle





Convection naturelle le long d'une plaque suspendue dans l'air

L'air chaud près de la plaque devient plus léger et s'élève <u>naturellement</u> sous l'effet de la force d'Archimède (flottabilité)(buoyancy)

La photo a été dilatée d'un facteur six dans le long de l'axe horizontal !
(Gebhart, University of Pennsylvania)

$$\rho_0(1 - \underbrace{\beta(T - T_0)}_{\ll 1})\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \underbrace{-\frac{\partial p}{\partial y}}_{\rho_0 g} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho_0(1 - \beta(T - T_0))g$$

$$\downarrow$$

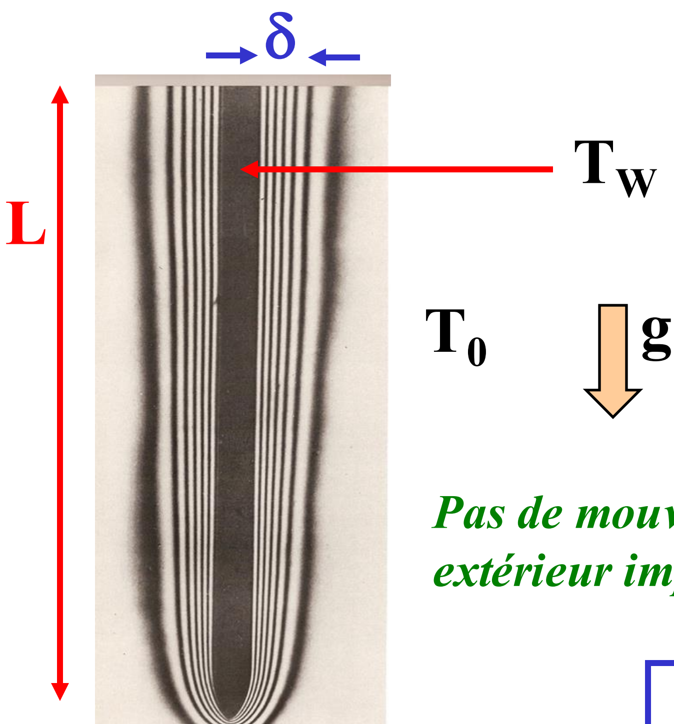
$$\rho_0\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \rho_0\beta(T - T_0)g$$

Approximation de Boussinesq

On néglige les variations de masse volumique dans tous les termes sauf dans la poussée d'Archimède…

**** Au passage, on observe que cette hypothèse n'a vraiment du sens que dans le cas où la pression est hydrostatique … En d'autres mots, on introduit soit l'approximation hydrostatique, soit l'approximation de Boussinesq, mais on doit introduire en tous cas une approximation :-)***

Reprenons le problème de convection naturelle



Pas de mouvement extérieur imposé

$$\delta \ll Y$$
$$\delta_T \ll Y$$

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} &= \beta g (T - T_0) + \nu\,\frac{\partial^2 v}{\partial x^2} \\
u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} &= \alpha\,\frac{\partial^2 T}{\partial x^2}
\end{aligned}
$$

$$\boxed{u \frac{\partial T}{\partial x}} + \boxed{v \frac{\partial T}{\partial y}} = \boxed{\alpha \frac{\partial^2 T}{\partial x^2}} + \cancel{\boxed{\alpha \frac{\partial^2 T}{\partial y^2}}}$$

$$\underbrace{\frac{U \Delta T}{\delta_T} \qquad \frac{V \Delta T}{Y}}_{\frac{V \Delta T}{Y}} \qquad \frac{\alpha \Delta T}{\delta_T^2} \qquad \cancel{\frac{\alpha \Delta T}{Y^2}}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{U}{\delta_T} \qquad \frac{V}{Y}$$

$$1 = \frac{}{} = \frac{V \Delta T}{Y} \frac{\delta_T^2}{\alpha \Delta T}$$

$$\frac{\delta_T^2}{Y^2} = \frac{\alpha}{V Y}$$

$$\frac{\delta_T}{Y} = \sqrt{\frac{\alpha}{V Y}} = \sqrt{\frac{1}{Pe}}$$

Couche limite thermique, on connaît :-)

Lieu où l'ordre de la convection et de la conduction sont identiques

$$\frac{\boxed{\text{Convection}}}{\boxed{\text{Conduction}}} = \frac{V\Delta T/Y}{\alpha\Delta T/\delta_T^2} = \underbrace{\frac{VY}{\alpha}}_{Pe_Y}\frac{\delta_T^2}{Y^2} = 1$$

Couche limite thermique, on connaît :-)

$$\frac{\delta_T}{Y} = \sqrt{\frac{1}{Pe_Y}} = \sqrt{\frac{1}{Pr\,Re_Y}}$$

Problème : on <u>ne connaît pas</u> V :-(

$$\boxed{\begin{aligned} \delta &\ll Y \\ \delta_T &\ll Y \end{aligned}}$$

$$\boxed{\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y} &= \beta g(T - T_0) + \nu\,\frac{\partial^2 v}{\partial x^2} \\ u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} &= \alpha\,\frac{\partial^2 T}{\partial x^2} \end{aligned}}$$

Longueur verticale caractéristique : Y
Longueur horizontale caractéristique : δ
Vitesse horizontale caractéristique : U = V δ / Y (incompressibilité :-)
Ecart de température caractéristique : $T_w - T_0$

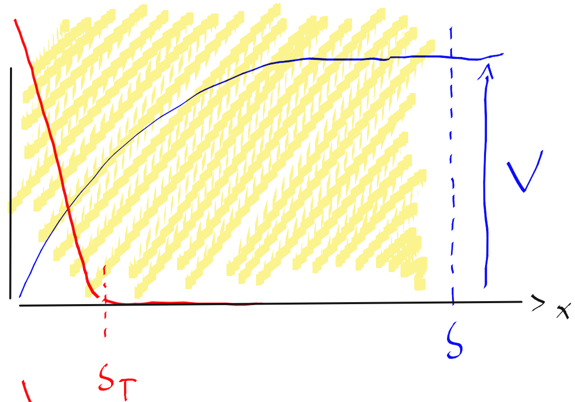
Donc, comment choisir la vitesse verticale caractéristique V ?

Buoyancy balanced by friction

$$\cancel{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}} = \boxed{\beta g (T - T_0)} + \boxed{\nu \frac{\partial^2 v}{\partial x^2}}$$

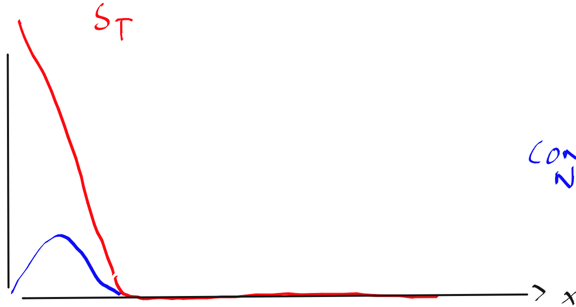
$$\beta g \Delta T \qquad \nu \frac{V}{\delta_T^2}$$

$$V = \frac{\beta g \Delta T \, \delta_T^2}{\nu}$$



CONVECTION FORCEE

$$\frac{\delta_T}{y} = \sqrt{\frac{\alpha}{V y}} = \sqrt{\frac{\nu \alpha}{\beta g \Delta T \, y^3}} \qquad \frac{y}{\delta_T}$$



CONVECTION NATURELLE

$$\frac{\delta_T}{y} = \underbrace{\left(\frac{\alpha \, \nu}{\beta g \Delta T \, y^3} \right)^{1/4}}_{Gr^{-1/4} \, Pr^{-1/4}}$$

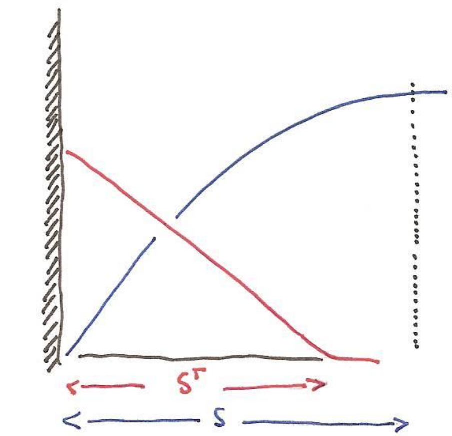
$$Gr = \frac{\beta g \Delta T \, y^3}{\nu^2}$$

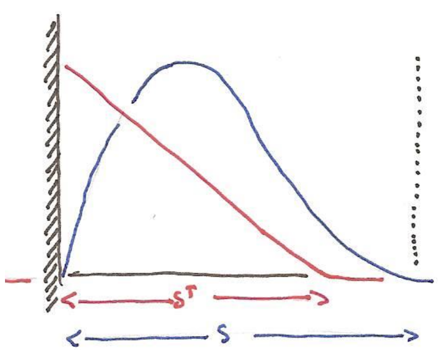
Buoyancy balanced by friction

$$\cancel{u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}} = \boxed{\beta g (T - T_0)} + \boxed{\nu\,\frac{\partial^2 v}{\partial x^2}}$$

$$\Downarrow$$

$$V = \frac{\beta g \Delta T \delta_T^2}{\nu}$$





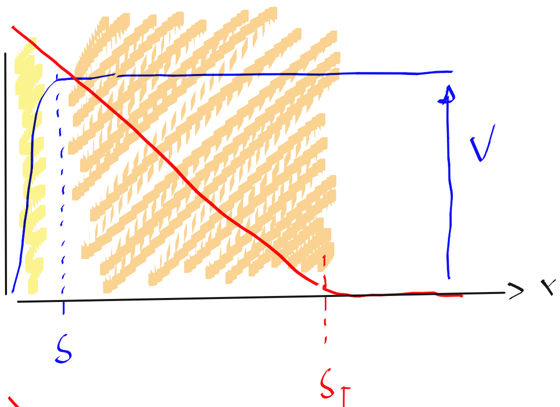
$$\boxed{\begin{aligned} \frac{\delta_T}{Y} &= \sqrt{\frac{\alpha}{VY}} = \sqrt{\frac{\nu\ \alpha}{\beta g \Delta T\ Y \delta_T^2}} = (Gr)^{-1/4}\,(Pr)^{-1/4} \\ \frac{\delta}{Y} &= (Gr)^{-1/4}\,(Pr)^{1/4} \end{aligned}}$$

Buoyancy balanced by inertia

$$\boxed{u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}} = \boxed{\beta g(T-T_0)} + \nu\cancel{\frac{\partial^2 v}{\partial x^2}}$$

$$V^2/Y \simeq \beta g \Delta T$$

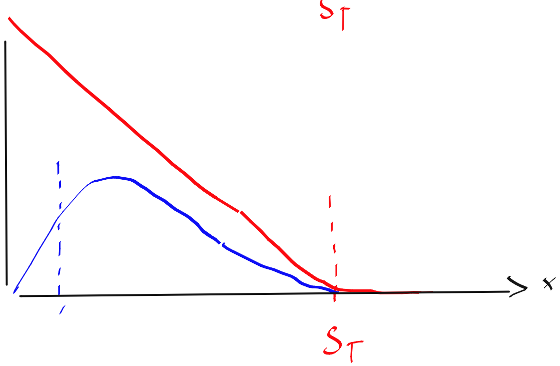
$$V = \sqrt{\beta g \Delta T\, Y}$$



CONVECTION FORCEE

$$\frac{\delta_T}{Y} = \sqrt{\frac{\alpha}{VY}} = \left(\frac{\alpha^2}{\beta g \Delta T\, Y^3}\right)^{1/4}$$

$$Gr^{-1/4} \quad Pr^{-1/2}$$



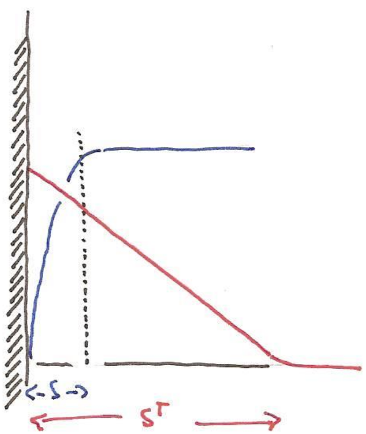
CONVECTION NATURELLE

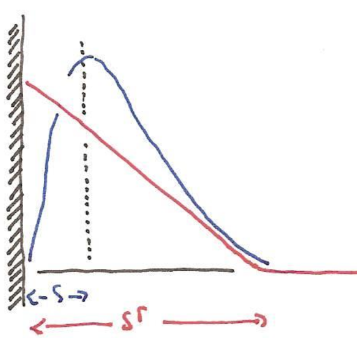
$$Pr = \frac{\alpha}{\nu} \qquad Gr = \frac{\beta g \Delta T\, Y^3}{\nu^2}$$

Buoyancy balanced by inertia

$$\boxed{u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}} = \boxed{\beta g (T - T_0)} + \nu\,\cancel{\frac{\partial^2 v}{\partial x^2}}$$

$$V = \sqrt{\beta g \Delta T\, Y}$$





$$\frac{\delta_T}{Y} = \sqrt{\frac{\alpha}{VY}} = \left(\frac{\alpha^2}{\beta g \Delta T\, Y^3}\right)^{1/4} = (Gr)^{-1/4}\,(Pr)^{-1/2}$$

$$\frac{\delta}{Y} = (Gr)^{-1/4}$$

Nombre de Grashof

$$Gr = \frac{\beta \Delta T g L^3}{\nu^2}$$

$$\frac{(\text{ARCHIMEDE})\,(\text{INERTIE})}{(\text{VISQUEUX})^2}$$

$$\underbrace{\rho \beta g \Delta T}_{\text{ARCHIMEDE}} \; \underbrace{\frac{L^4}{\nu^2 \rho^2 U^2}}_{\frac{1}{(\text{VISQUEUX})^2}} \; \underbrace{\frac{\rho U^2}{L}}_{\text{INERTIE}} = \frac{\beta g \Delta T L^3}{\nu^2}$$

$$\boxed{\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right)} = \boxed{\rho\beta(T - T_0)g} + \boxed{\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)}$$

Nombre de Grashof

$$Gr = \frac{\beta \Delta T g L^3}{\nu^2}$$

1822-1893, University of Karlsruhe, Germany
Professor of Mechanical Engineering

$$\frac{\text{(Forces d'inertie) (Forces d'Archimède)}}{\text{(Forces visqueuses)}^2}$$

Grashof

$$\boxed{\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)} = \boxed{\rho\beta(T-T_0)g} + \boxed{\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)}$$

$$\frac{\color{red}\blacksquare}{\color{green}\blacksquare} = Re \qquad \frac{\color{blue}\blacksquare}{\color{green}\blacksquare} = \frac{Gr}{Re} \qquad \frac{\color{blue}\blacksquare}{\color{red}\blacksquare} = \frac{Gr}{Re^2}$$

$$Gr = \frac{(\text{Forces d'Archimède})(\text{Forces d'inertie})}{(\text{Forces visqueuses})^2} = \frac{\beta \Delta T g L^3}{\nu^2}$$

Grashof - Reynolds

Forces visqueuses

$$\boxed{\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) \quad \mathcal{O}(\rho U^2/L)} = \boxed{\rho\beta(T-T_0)g \quad \mathcal{O}(\rho\beta\Delta T g)} + \boxed{\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad \mathcal{O}(\mu U/L^2)}$$

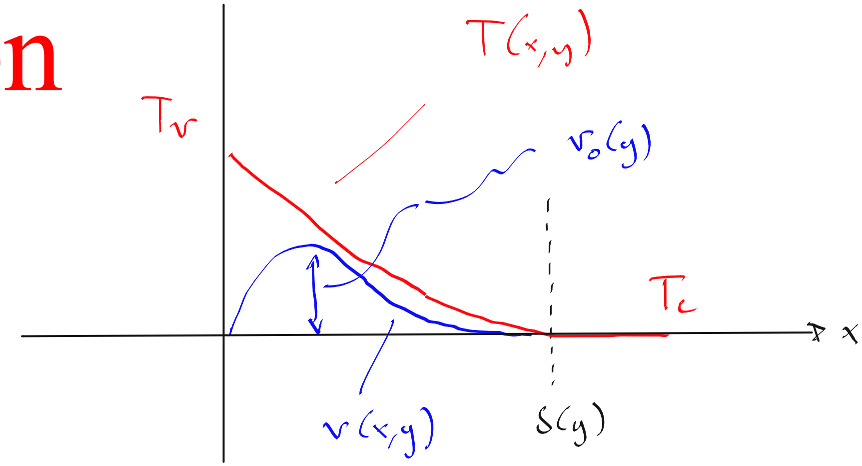
Forces d'inertie

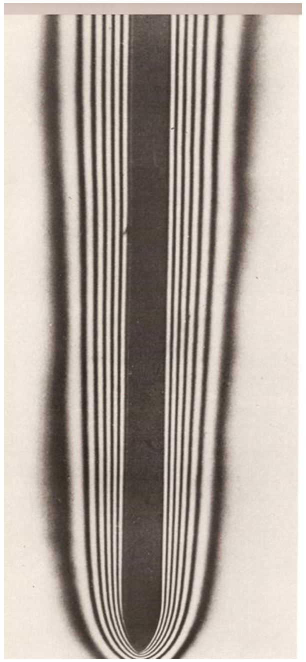
Forces d'Archimède

$$Re = \frac{\color{red}\blacksquare}{\color{green}\blacksquare} \qquad \frac{Gr}{Re} = \frac{\color{blue}\blacksquare}{\color{green}\blacksquare} \qquad \frac{Gr}{Re^2} = \frac{\color{blue}\blacksquare}{\color{red}\blacksquare}$$

$$Gr = \frac{(\text{Forces d'Archimède})(\text{Forces d'inertie})}{(\text{Forces visqueuses})^2} = \frac{\beta\Delta T g L^3}{\nu^2}$$

Une solution approchée pour la convection naturelle…





$$v(x,y) = v_0(y) \underbrace{\frac{x}{\delta(y)}}_{\eta} \left(1-\underbrace{\frac{x}{\delta(y)}}_{\eta}\right)^2$$

$$\frac{T(x,y)-T_e}{T_w-T_e} = \left(1-\frac{x}{\delta(y)}\right)^2$$

IT IS BEAUTIFUL!

$v = v_0 \, \eta (1-\eta)^2$

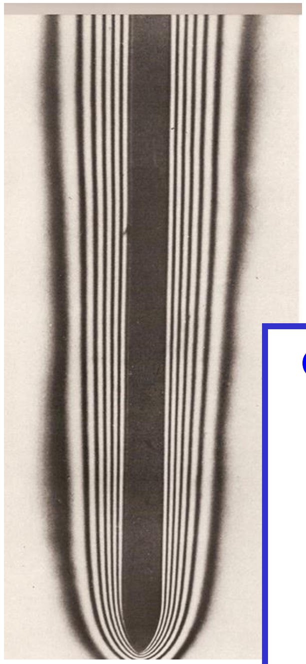
$\theta = (1-\eta)^2$

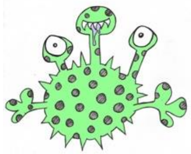
IL FAUT DETERMINER UNIQUEMENT $v_0(y)$ ET $\delta(y)$

Une solution approchée pour la convection naturelle…

$$v(x,y) = v_0(y)\,\frac{x}{\delta(y)}\left(1-\frac{x}{\delta(y)}\right)^2$$

$$\frac{T(x,y)-T_0}{T_w-T_0} = \left(1-\frac{x}{\delta(y)}\right)^2$$





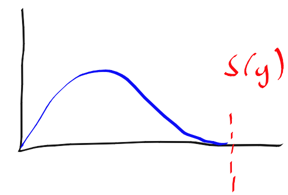
On considère des profils semblables avec conditions de raccord très sommaires à la couche limite… mais cela va nous donner une idée des ordres de grandeurs

$$\begin{aligned} v(0,y) &= 0 & v(\delta,y) &= 0 & \frac{\partial v}{\partial x}(\delta,y) &= 0 & \mu\frac{\partial^2 v}{\partial x^2}(0,y) &= -\rho\beta g(T_w-T_0) \\ T(0,y) &= T_w & T(\delta,y) &= T_0 & \frac{\partial T}{\partial x}(\delta,y) &= 0 \end{aligned}$$

$$\underbrace{\int_0^\delta u \frac{\partial v}{\partial x}}_{[uv]_0^\delta - \int_0^\delta v \frac{\partial u}{\partial x}} + \int_0^\delta v \frac{\partial v}{\partial y} = \beta g \int_0^\delta (T - T_e) + \underbrace{\nu \int_0^\delta \frac{\partial^2 v}{\partial x^2} dx}_{-\nu \left.\frac{\partial v}{\partial x}\right|_0}$$

$$\underbrace{[uv]_0^\delta}_{=0} - \int_0^\delta v \underbrace{\frac{\partial u}{\partial x}}_{= -\frac{\partial v}{\partial y}}$$

$$2 \int_0^\delta v \frac{\partial v}{\partial y} = \int_0^\delta \frac{\partial}{\partial y}\left(v^2\right)$$



$$\boxed{\frac{d}{dy} \int_0^\delta v^2 = \beta g \int_0^\delta (T - T_e) - \nu \left.\frac{\partial v}{\partial x}\right|_0}$$

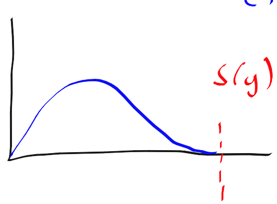
Calculons $\delta(y)$ et $v_0(y)$!

$$\frac{d}{dy}\int_0^{S(y)} v^2(x,y)\,dx \overset{?}{=} \int_0^{S(y)} \frac{\partial}{\partial y}\left(v^2(x,y)\right)\,dx$$

Oui

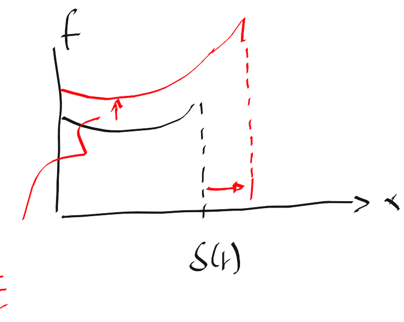
CAR $V = 0$

en $S(y)$:-)

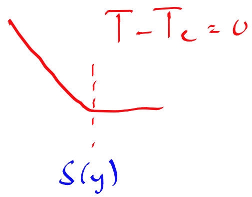


$$\frac{d}{dt}\int_0^{S(t)} f(x,t)\,dx = \int_0^{S(t)} \frac{\partial f}{\partial t}(x,t)\,dt$$

$$+ f(S(t),t)\,S'(t)$$



Calculons $\delta(y)$ et $v_0(y)$!



$$\underbrace{\int_0^\delta u\,\frac{\partial T}{\partial x}}_{\left[u\,(T-T_e)\right]_0^\delta \;-\; \int_0^\delta (T-T_e)\,\frac{\partial u}{\partial x}} + \int_0^\delta v\,\frac{\partial T}{\partial y} = \alpha \underbrace{\int_0^\delta \frac{\partial^2 T}{\partial x^2}\,dx}_{-\alpha \left.\frac{\partial T}{\partial x}\right|_0}$$

$$\underbrace{\left[u\,(T-T_e)\right]_0^\delta}_{=0} - \int_0^\delta (T-T_e)\,\underbrace{\frac{\partial u}{\partial x}}_{-\frac{\partial v}{\partial y}}$$

$$\int_0^\delta \underbrace{v\,\frac{\partial T}{\partial y} + (T-T_e)\,\frac{\partial v}{\partial y}}_{\frac{\partial}{\partial y}\left((T-T_e)\,v\right)}$$

$$\boxed{\frac{d}{dy}\int_0^\delta v\,(T-T_e)\,dx = -\alpha \left.\frac{\partial T}{\partial x}\right|_0}$$

Calculons $\delta(y)$ et $v_0(y)$!

Il faut encore estimer $\delta(y)$ et $v_0(y)$!

$$\int_0^\delta u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}dx = \beta g\int_0^\delta (T(x,y)-T_0)\,dx + \nu\int_0^\delta \frac{\partial^2 v}{\partial x^2}dx$$

$$\downarrow$$

$$[uv]_0^\delta - \int_0^\delta v\frac{\partial u}{\partial x}dx + \int_0^\delta v\frac{\partial v}{\partial y}dx = \beta g\int_0^\delta (T(x,y)-T_0)\,dx - \nu\left.\frac{\partial v}{\partial x}\right|_{x=0}$$

$$\downarrow$$

$$\boxed{\frac{d}{dy}\int_0^\delta v^2(x,y)dx = \beta g\int_0^\delta (T(x,y)-T_0)\,dx - \nu\left.\frac{\partial v}{\partial x}\right|_{x=0}}$$

$$\int_0^\delta u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}dx = \alpha\int_0^\delta \frac{\partial^2 T}{\partial x^2}dx$$

$$\downarrow$$

$$\boxed{\frac{d}{dy}\int_0^\delta v(x,y)\Big(T(x,y)-T_0\Big)dx = -\alpha\left.\frac{\partial T}{\partial x}\right|_{x=0}}$$

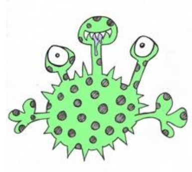
Intégrons les équations de conservation de la quantité de mouvement et de l'énergie dans la couche limite en tirant profit de l'incompressibilité…

Que deviennent ces équations intégrales ?

$$\left[\frac{T_w - T_e}{3}\,\delta\right]$$

$$\begin{cases} \dfrac{d}{dy}\displaystyle\int_0^\delta v^2 dx = \beta g \displaystyle\int_0^\delta (T - T_0)\, dx - \nu \left.\dfrac{\partial v}{\partial x}\right|_{x=0} \\ \dfrac{d}{dy}\displaystyle\int_0^\delta v(T - T_0) dx = -\alpha \left.\dfrac{\partial T}{\partial x}\right|_{x=0} \end{cases}$$

En intégrant les expressions approchées que nous avons introduites…



$$\eta = \frac{x}{\delta}$$

$$v = v_0\, \eta\, (1-\eta)^2$$

$$\theta' = (1-\eta)^2$$

$$v_0^2 \int_0^1 (1-\eta)^4 \eta^2$$

$$= \left[\frac{1}{3} - \frac{4}{4} + \frac{6}{5} - \frac{4}{6} + \frac{1}{7}\right]$$

$$= \frac{35 - 105 + 126 - 70 + 15}{105}$$

$$= \frac{1}{105}$$

$$v_0 \int_0^1 (1-\eta)^4 \eta$$

$$(1 - 2\eta + \eta^2)(1 - 2\eta + \eta^2)$$

$$1 - 2\eta + \eta^2$$

$$-2\eta + 4\eta^2 - 2\eta^3$$

$$\eta^2 - 2\eta^3 + \eta^4$$

$$(1 - 4\eta + 6\eta^2 - 4\eta^3 + \eta^4)\,\eta$$

$$\left[\frac{\eta^2}{2} - \frac{4\eta^3}{3} + \frac{6\eta^4}{4} - \frac{4\eta^5}{5} + \frac{\eta^6}{6}\right]_0^1 = \frac{15 - 40 + 45 - 24 + 5}{30} = \frac{1}{30}$$

$$\int_0^1 (1-\eta)^2$$

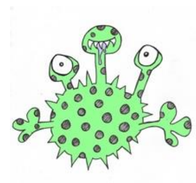
$$= \left[\eta - \eta^2 + \frac{\eta^3}{3}\right]_0^1$$

$$= 1 - 1 + \frac{1}{3} = \frac{1}{3}$$

Que deviennent ces équations intégrales

$$\left\{\begin{array}{rcl} \dfrac{d}{dy}\displaystyle\int_0^{\delta} v^2 dx &=& \beta g \displaystyle\int_0^{\delta} (T - T_0)\, dx - \nu \left.\dfrac{\partial v}{\partial x}\right|_{x=0} \\ \dfrac{d}{dy}\displaystyle\int_0^{\delta} v(T - T_0) dx &=& -\alpha \left.\dfrac{\partial T}{\partial x}\right|_{x=0} \end{array}\right.$$

En intégrant les expressions approchées que nous avons introduites…



$$\left\{\begin{array}{rcl} \dfrac{1}{105}\, \dfrac{d}{dy}\left(v_0^2(y)\delta(y) \right) &=& \dfrac{\beta g\; \Delta T \delta(y)}{3} - \dfrac{\nu\; v_0(y)}{\delta(y)} \\ \dfrac{1}{30}\, \dfrac{d}{dy}\left(v_0(y)\delta(y) \right) &=& \dfrac{2\alpha}{\delta(y)} \end{array}\right.$$

On obtient finalement de deux équations différentielles ordinaires avec deux fonctions inconnues…

Résolution des équations intégrales

$$
\left\{
\begin{aligned}
\frac{1}{105}\,\frac{d}{dy}\Big(v_0^2(y)\delta(y)\Big) &= \frac{\beta g\,\Delta T\delta(y)}{3} - \frac{\nu\, v_0(y)}{\delta(y)} \\
\frac{1}{30}\,\frac{d}{dy}\Big(v_0(y)\delta(y)\Big) &= \frac{2\alpha}{\delta(y)}
\end{aligned}
\right.
$$

Essayons une solution de la forme…

$$
\begin{aligned}
v_0(y) &= V\ y^m \\
\delta(y) &= D\ y^n
\end{aligned}
$$

$$
\left\{
\begin{aligned}
\frac{1}{105}\,V^2 D(2m+n)\ y^{2m+n-1} &= \frac{\beta g\,\Delta T\ D}{3}\ y^n - \frac{\nu\ V}{D}\ y^{m-n} \\
\frac{1}{30}\,VD(m+n)\ y^{m+n-1} &= \frac{2\alpha}{D}\ y^{-n}
\end{aligned}
\right.
$$

Et cela marche avec 2m = 1 et 4n = 1…

$m = 2n$

$$
\left\{
\begin{aligned}
2m + n - 1 &= n = m - n \\
m + n - 1 &= -n
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
4n + n - 1 &= n \\
2n + n - 1 &= -n
\end{aligned}
\right.
$$

$4n = 1$

$4n = 1$

$n = 1/4$

$m = 1/2$

Résolution des équations intégrales

$$\begin{cases} \frac{1}{105}\frac{d}{dy}\left(v_0^2(y)\delta(y)\right) = \frac{\beta g\,\Delta T\delta(y)}{3} - \frac{\nu\, v_0(y)}{\delta(y)} \\ \frac{1}{30}\frac{d}{dy}\left(v_0(y)\delta(y)\right) = \frac{2\alpha}{\delta(y)} \end{cases}$$

IDEE $v_0(y) = V y^m$
$\delta(y) = D y^n$

$$\frac{1}{105} V^2 D\,(2m+n)\, y^{2m+n-1} = \frac{\beta g \Delta T}{3} D y^n - \nu \frac{V}{D} y^{m-n}$$

$$\frac{1}{30} V D\,(m+n)\, y^{m+n-1} = \frac{2\alpha}{D} y^{-n}$$

$$v'(0) = v_0\left[\underbrace{(1-n)^2 - 2n(1-n)}_{1}\right] = 0$$

$$\theta'(0) = \left[\underbrace{-2n(1-n)}_{-2}\right] = 0$$

C'est presque fini !

$$\begin{cases} \dfrac{V^2 D}{84} = \dfrac{\beta g\, \Delta T\, D}{3} - \dfrac{\nu\, V}{D} \\[2ex] \dfrac{V D}{40} = \dfrac{2\alpha}{D} \end{cases}$$

$$\boxed{V = \frac{80\,\alpha}{D^2}}$$

$1 + \frac{1}{4} = \frac{5}{4}$

$$\frac{1}{105} V^2 D \,(2m+n)\, y^{2m+n-1} = \frac{\beta g\, \Delta T}{3} D\, y^{n} - \nu \frac{V}{D} y^{m-n}$$

$$\frac{1}{30} V D \,(m+n)\, y^{m+n-1} = \frac{2\alpha}{D} y^{-n}$$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$\frac{\cancel{5}}{4}\ \frac{1}{21 \times \cancel{5}}$

$m = 1/2$

$n = 1/4$

$$\frac{80^2 \alpha^2}{84\, D^3} = \frac{\beta g\, \Delta T}{3} D - \nu \frac{80\,\alpha}{D^3}$$

$$\boxed{\alpha^2 \frac{80^2}{84} + \nu\, 80\,\alpha = \frac{\beta g\, \Delta T}{3} D^4}$$

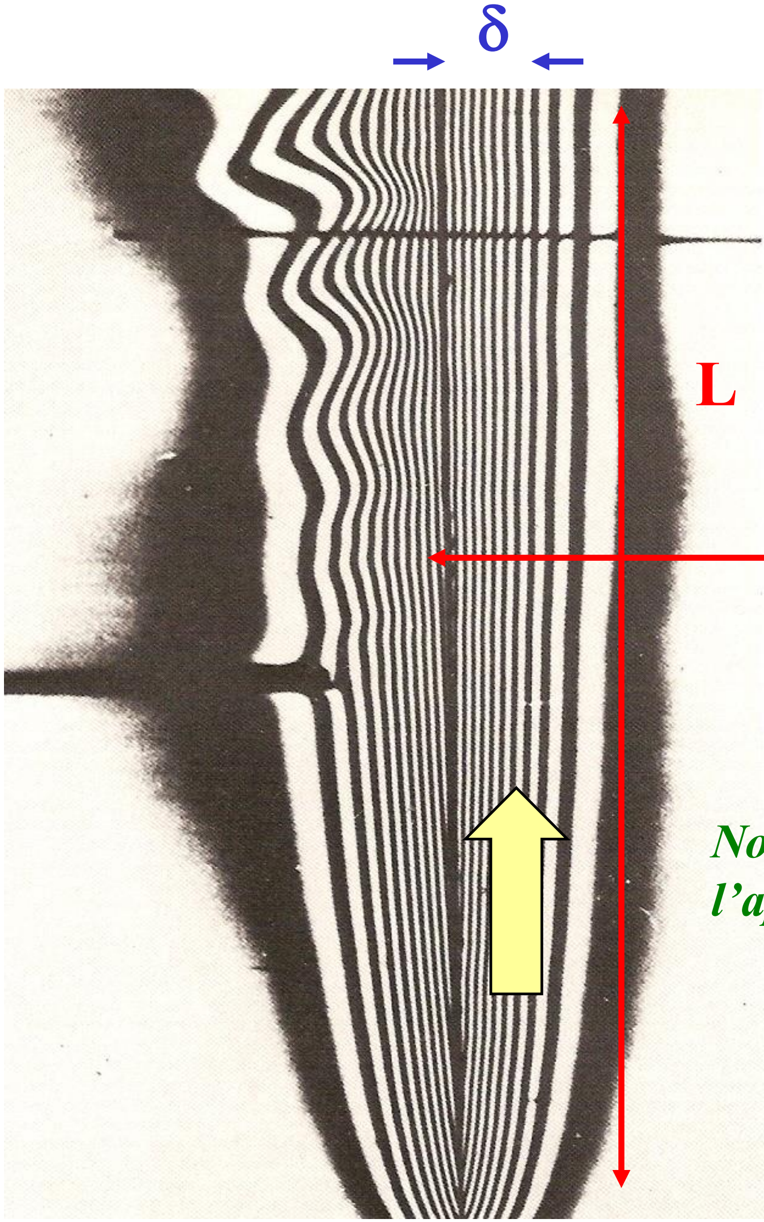
$$\left\{ \begin{aligned} \frac{V^2 D}{84} &= \frac{\beta g \ \Delta T \ D}{3} - \frac{\nu \ V}{D} \\ \frac{V D}{40} &= \frac{2\alpha}{D} \end{aligned} \right.$$

$$D^4 = 240 L^3 \left(\frac{20}{21} + \frac{\nu}{\alpha} \right) \frac{\nu^2}{\beta g \Delta T L^3} \frac{\alpha^2}{\nu^2}$$

Solution …

$$\frac{\delta_T(y)}{y} = 3,936 \ \left(\ Pr \ \right)^{-1/2} \left(Gr(y) \right)^{-1/4} \left(\frac{20}{21} + Pr \right)^{1/4}$$

On peut ensuite calculer le profil de vitesse et de température, le transfert de chaleur et des nombres de Nusselt locaux et moyens.



Est-ce que cette solution est stable ?

Plaque chaude

Non, après une certaine distance, on constate l'apparition d'instabilités : c'est la turbulence !

Cela, c'est très très très très compliqué.

Grégoire est bientôt de retour!