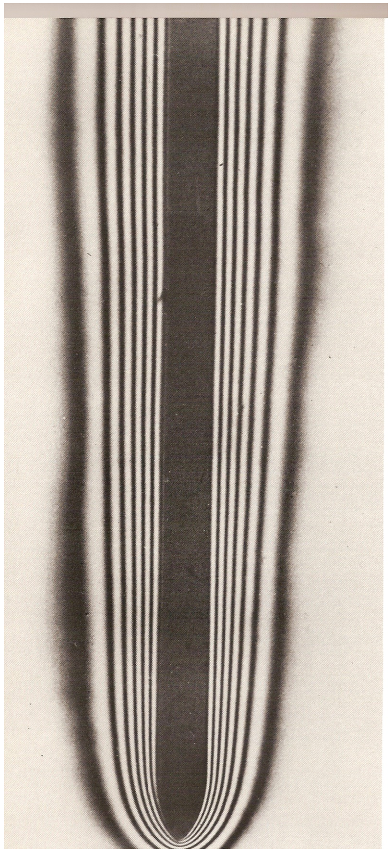
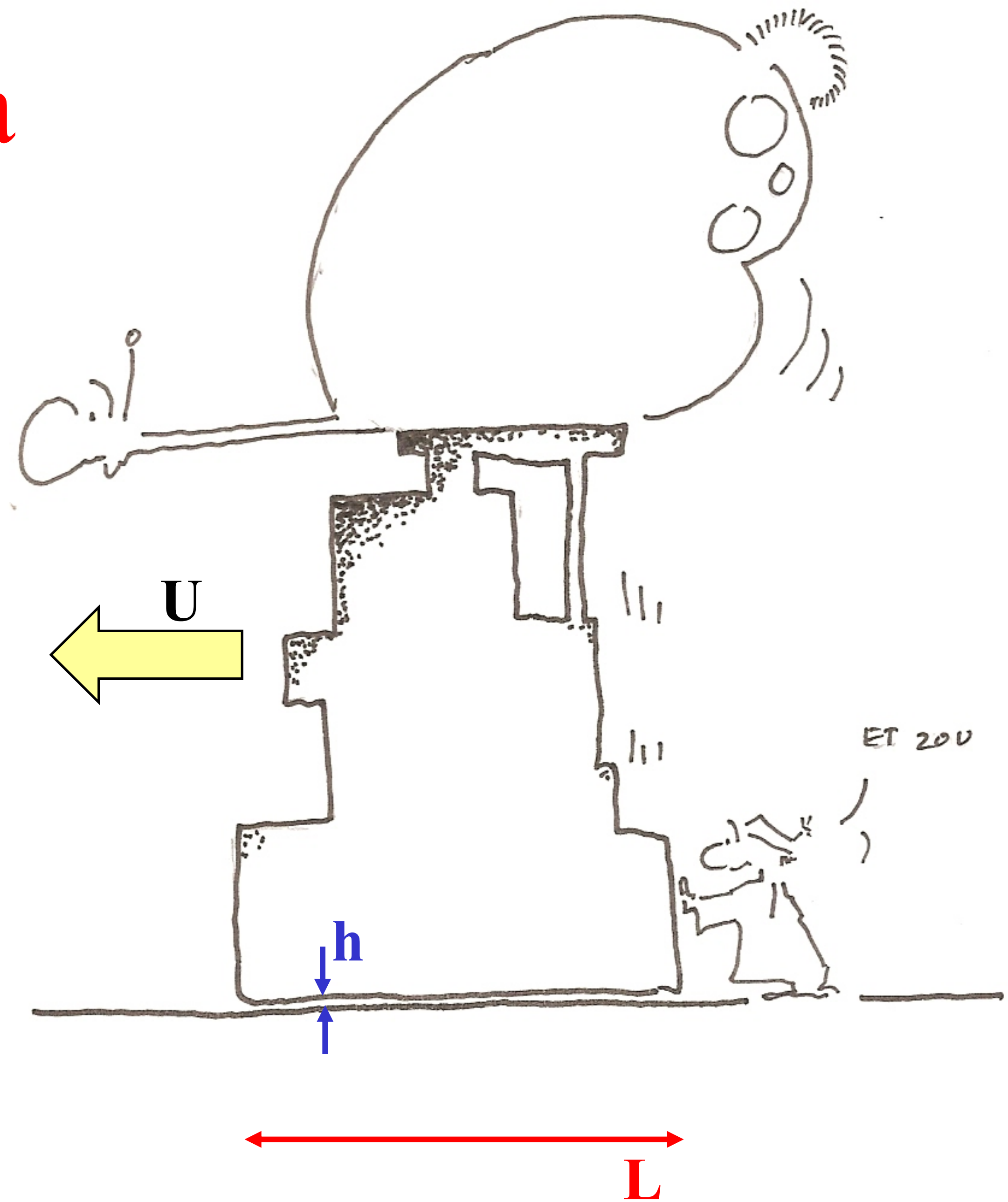
Mais que faire pour des écoulements avec deux échelles spatiales ?



Convection naturelle le long d'une plaque verticale : écoulement laminaire permanent

Lubrification et convoyage hydraulique : butée Michell

Théorie de la lubrification

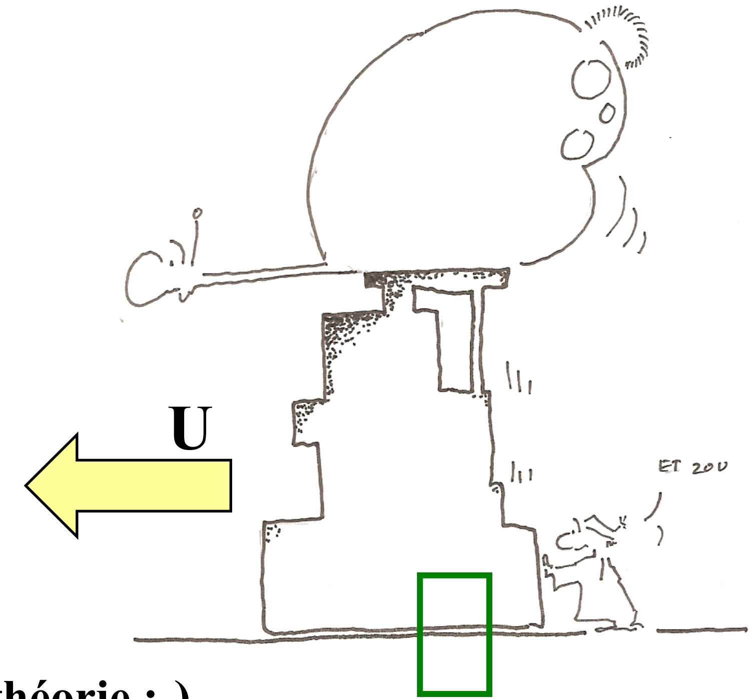


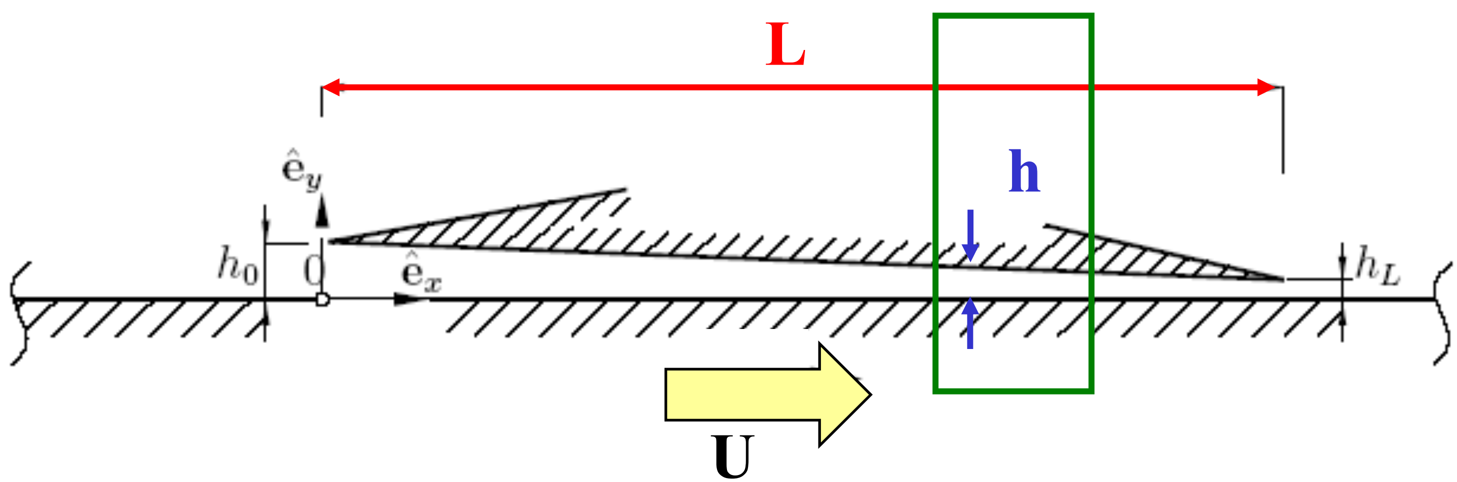
Convoyage hydraulique de charges très importantes :
- turbines hydroélectriques
- applications marines
- butées hydrauliques

Théorie de la lubrification

$$\boxed{h \ll L}$$

Hypothèse géométrique de base
Valable dans la zone centrale uniquement en théorie :-)



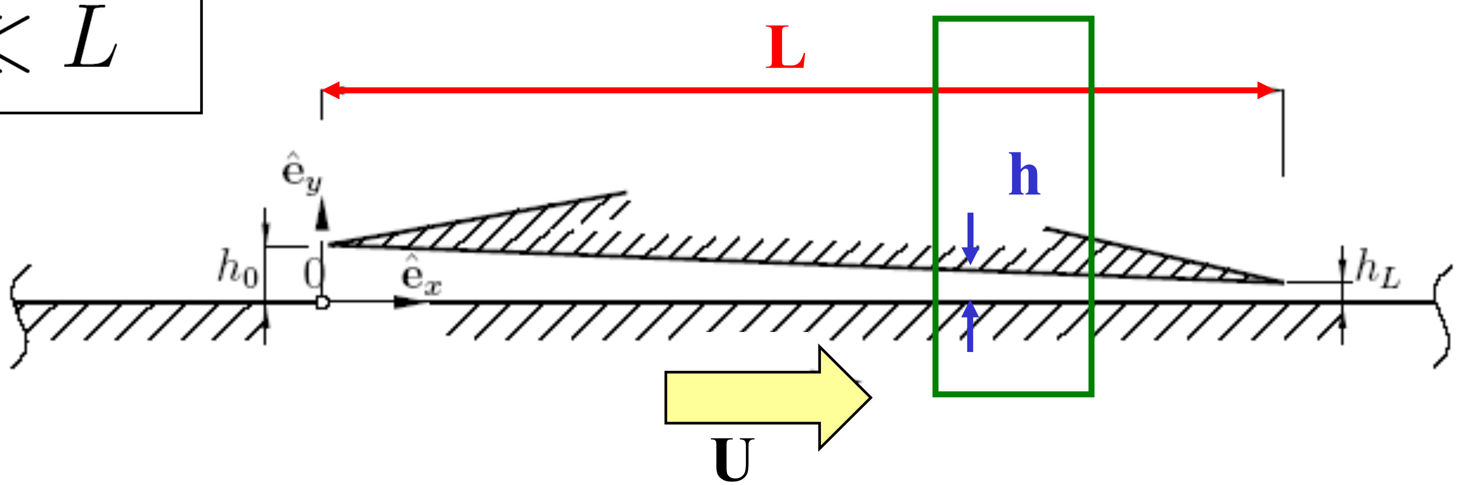


Ecoulements incompressibles plans stationnaires

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho\, u\, \frac{\partial u}{\partial x} + \rho\, v\, \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \mu\, \frac{\partial^2 u}{\partial x^2} + \mu\, \frac{\partial^2 u}{\partial y^2} \\
\rho\, u\, \frac{\partial v}{\partial x} + \rho\, v\, \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu\, \frac{\partial^2 v}{\partial x^2} + \mu\, \frac{\partial^2 v}{\partial y^2}
\end{aligned}
$$

Que deviennent ces équations ?

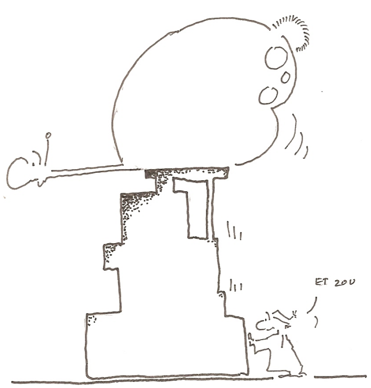
$$\boxed{h \ll L}$$



$$\boxed{h \ll L}$$

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} \\
\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}
\end{aligned}$$

Longueur horizontale caractéristique : L
Longueur verticale caractéristique : h
Vitesse horizontale caractéristique : U



Comment choisir une vitesse verticale caractéristique ?

$$\boxed{\frac{\partial u}{\partial x} \quad \mathcal{O}(U/L)} + \boxed{\frac{\partial v}{\partial y} \quad \mathcal{O}(V/h)} = 0$$

Il ne faut pas définir de vitesse caractéristique verticale !

$$\boxed{V = \frac{Uh}{L} \ll U}$$

Quand peut-on négliger les termes d'inertie ?

$$\overset{\mathcal{O}(\rho U^2/L)}{\boxed{\rho\, u\, \frac{\partial u}{\partial x}}} + \underset{\mathcal{O}(\rho V U/h)}{\overset{\mathcal{O}(\rho U^2/L)}{\boxed{\rho\, v\, \frac{\partial u}{\partial y}}}} = -\frac{\partial p}{\partial x} + \boxed{\cancel{\mu\, \frac{\partial^2 u}{\partial x^2}}} + \boxed{\mu\, \frac{\partial^2 u}{\partial y^2}}$$

$$\mathcal{O}(\mu U/L^2) \ll \mathcal{O}(\mu U/h^2)$$

Hypothèse de lubrification :
Écoulements rampants

$$\frac{\boxed{\text{Forces d'inertie}}}{\boxed{\text{Forces visqueuses}}} = \frac{\rho U^2/L}{\mu U/h^2} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L}\,\frac{h^2}{L^2} \ll 1$$

Et l'autre équation ?

$$\underbrace{\boxed{\rho\, u \frac{\partial v}{\partial x}}}_{\mathcal{O}(\rho U^2 h/L^2)} + \underbrace{\boxed{\rho\, v \frac{\partial v}{\partial y}}}_{\mathcal{O}(\rho U^2 h/L^2)} = -\frac{\partial p}{\partial y} + \boxed{\cancel{\mu \frac{\partial^2 v}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 v}{\partial y^2}}$$

$$\mathcal{O}(\mu U h/L^3) \ll \mathcal{O}(\mu U/Lh)$$

On obtient la [même condition]…

$$\frac{\boxed{\text{Forces d'inertie}}}{\boxed{\text{Forces visqueuses}}} = \frac{\rho U^2 h/L^2}{\mu U/Lh} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\boxed{\cancel{\rho\, u \frac{\partial v}{\partial x}}} + \boxed{\cancel{\rho\, v \frac{\partial v}{\partial y}}} = -\frac{\partial p}{\partial y} + \boxed{\cancel{\mu \frac{\partial^2 v}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 v}{\partial y^2}}$$

$$\mathcal{O}(\mu U/Lh)$$

Et la pression ?

$$p(x,y) - p_0 = \boxed{p(x,0) - p_0} + \boxed{y \left.\cancel{\frac{\partial p}{\partial y}}\right|_{y=0}}$$

$$\mathcal{O}(\mu UL/h^2) \gg \mathcal{O}(\mu UL/L^2)$$

$$\mathcal{O}(\mu U/h^2)$$

$$\boxed{\cancel{\rho\, u \frac{\partial u}{\partial x}}} + \boxed{\cancel{\rho\, v \frac{\partial u}{\partial y}}} = -\frac{\partial p}{\partial x} + \boxed{\cancel{\mu \frac{\partial^2 u}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 u}{\partial y^2}}$$

Equations de Reynolds (1889)

Théorie de la lubrification

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$$

Film fluide mince

$$h \ll L$$

Hypothèse de lubrification :
Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Est-ce que l'hypothèse de lubrification est réaliste ?

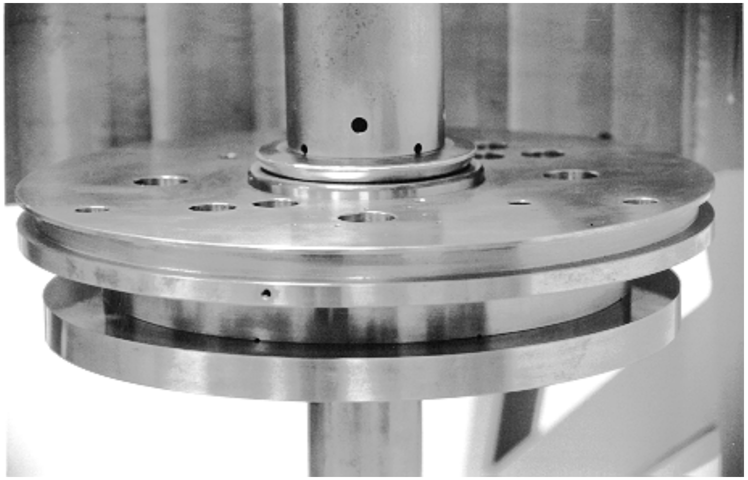
0.0375

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\begin{aligned} L &= 10\ cm \\ h &= 0.5\ mm \\ U &= 1\ m/s \end{aligned}$$

$$\begin{aligned} \rho &= 900\ kg/m^3 \\ \mu &= 60\ 10^{-3}\ Ns/m^2 \end{aligned}$$

Huile SAE50 à 60 degrés



Huile SAE 50
C’est quoi ?

Transport maritime

Marine LCX

Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

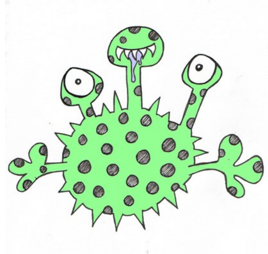
Grades offerts :
SAE 50

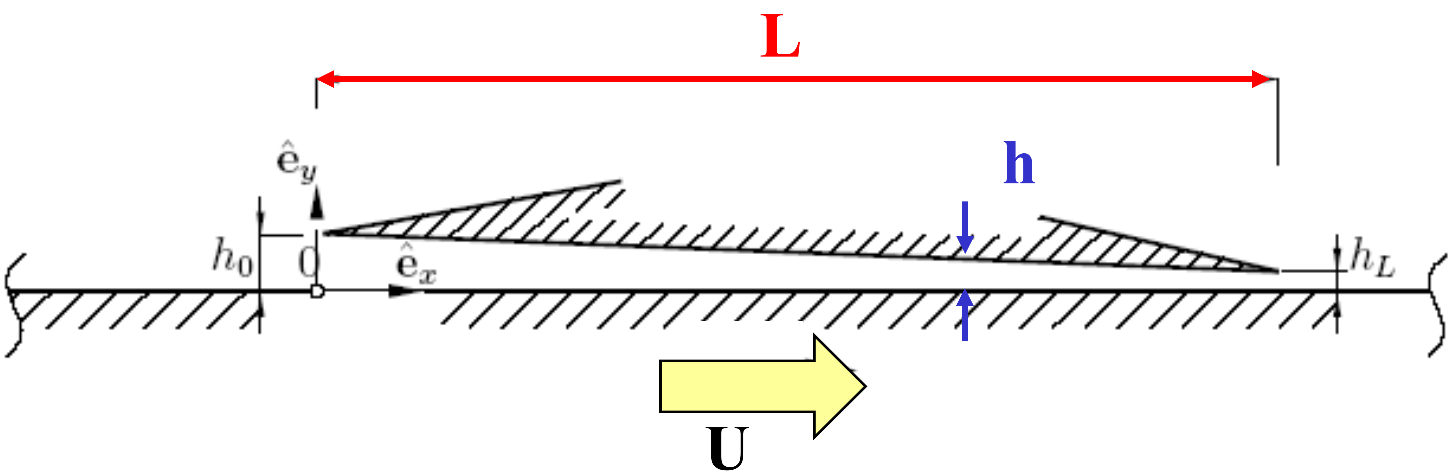
Fiche technique
Fiche signalétique

$$\left\{\begin{array}{rcl}\boxed{\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y} \;=\; 0} \\ \boxed{-\dfrac{dp}{dx}+\mu\,\dfrac{\partial^2 u}{\partial y^2} \;=\; 0}\end{array}\right.$$

-i- calcul de u(x,y)

$$\boxed{u(x,y) = -\frac{dp}{dx}\,\frac{h^2}{2\mu}\,\frac{y}{h}\left(1-\frac{y}{h}\right) + U\left(1-\frac{y}{h}\right)}$$





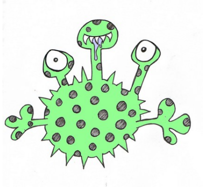
-ii- calcul de p(x)

$$\left\{\begin{array}{rcl} \boxed{-\dfrac{dp}{dx} + \mu \dfrac{\partial^2 u}{\partial y^2} = 0} \\ \boxed{\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0} \end{array}\right.$$

$$0 = \int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} dy$$

$$0 = \frac{d}{dx} \overbrace{\int_0^h u(x,y)dy}^{Q(x)} + \cancel{\Big[v(x,y) \Big]_0^h}$$

En utilisant l'expression de u(x,y)



Equation classique de Reynolds (1889)

$$\boxed{0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2} \right)}$$

$$0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2}\right)$$

$$\frac{d}{dx}\left(h^3(x)\,\frac{dp}{dx}(x)\right) = 6\mu U \boxed{\frac{dh}{dx}}$$

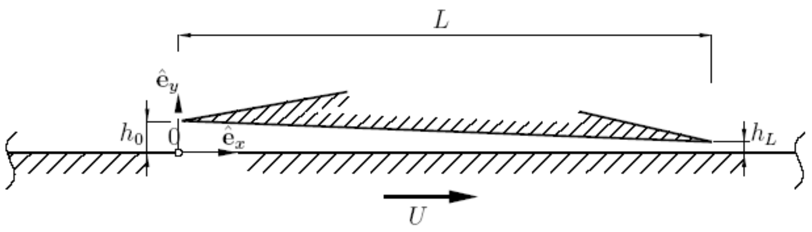
$$-\frac{d}{dh}\left(h^3\frac{dp}{dh}(h)\right) = \frac{6\mu UL}{h_0 - h_L}$$

$$-h^3\frac{dp}{dh}(h) = \frac{6\mu UL}{h_0 - h_L}(h + A)$$

$$-\frac{dp}{dh}(h) = \frac{6\mu UL}{h_0 - h_L}\left(\frac{1}{h^2} + \frac{A}{h^3}\right)$$

$$p(h) = \frac{6\mu UL}{h_0 - h_L}\left(B + \frac{1}{h} + \frac{A}{2h^2}\right)$$

Palier plat

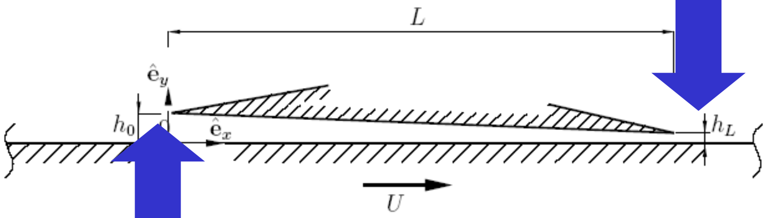


$$\frac{x}{L} = \frac{h_0 - h(x)}{h_0 - h_L}$$

$$\frac{dh}{dx} = -\frac{h_0 - h_L}{L}$$

$$p(h_L) = p_0$$

Deux conditions aux limites



$$p(h_0) = p_0$$

$$p(h) = \frac{6\mu U L}{h_0 - h_L}\left(\boxed{B} + \frac{1}{h} + \frac{\boxed{A}}{2h^2}\right)$$

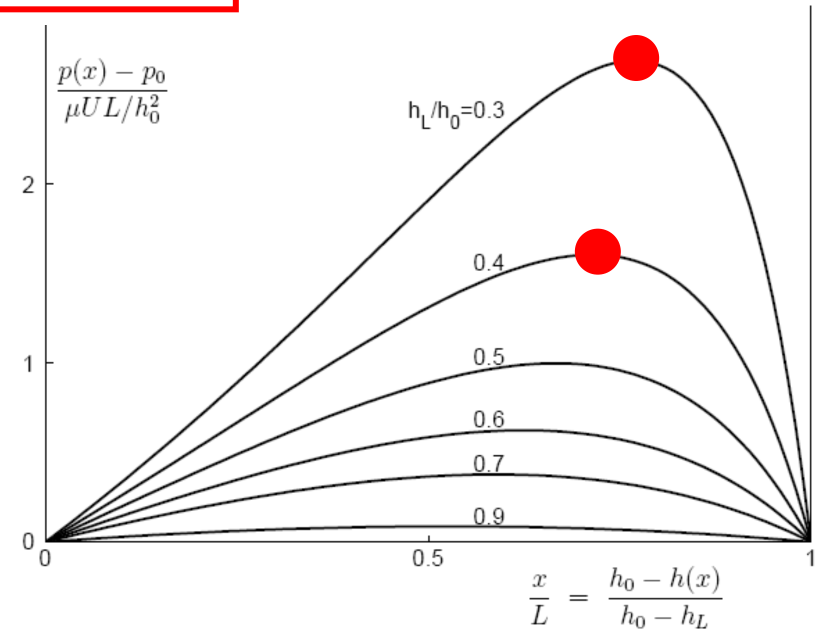
Deux constantes

$$\downarrow$$

$$p(h) - p_0 = \frac{6\mu U L (h_0 - h)(h - h_L)}{(h_0^2 - h_L^2)h^2}$$

Où la pression est-elle maximale ?

$$\boxed{h = \frac{2\,h_0\,h_L}{(h_0 + h_L)}}$$

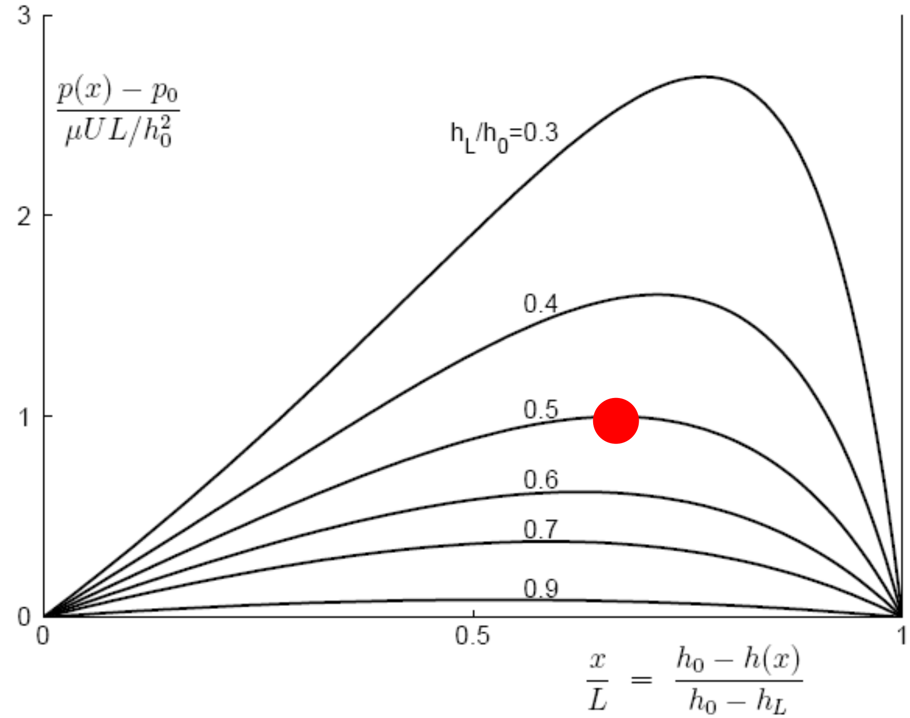


Cette pression peut être énorme !

$$
\begin{aligned}
L &= 10\ cm \\
h_0 &= 0.1\ mm \\
h_L &= 0.05\ mm \\
U &= 10\ m/s
\end{aligned}
$$

$$
\begin{aligned}
\rho &= 900\ kg/m^3 \\
\mu &= 0.1\ Ns/m^2
\end{aligned}
$$

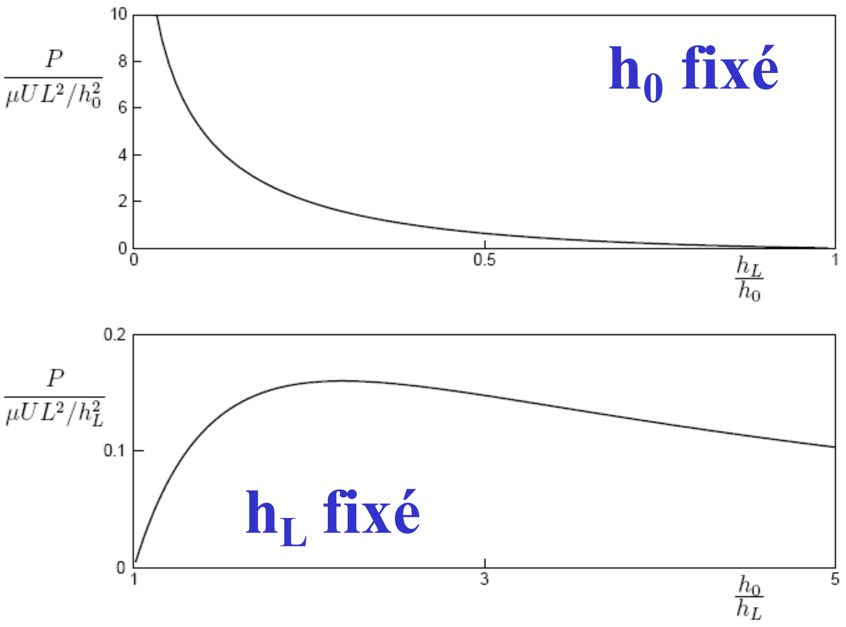
Huile SAE50 à 50 degrés



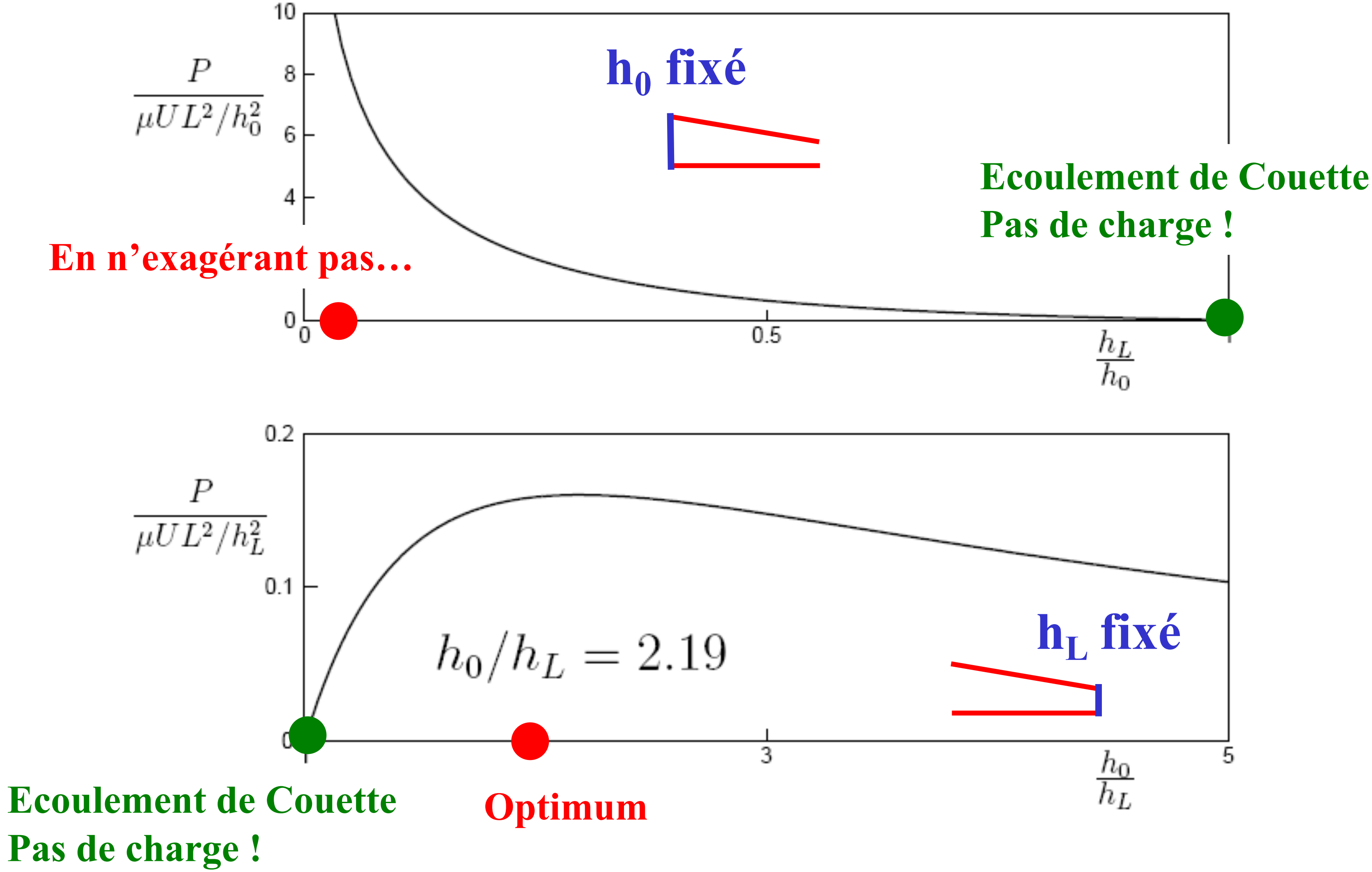
$$
p_{\max} - p_0 = \boxed{\frac{3}{2}\frac{\mu\, U\, L}{h_0\, h_L}\frac{(h_0 - h_L)}{(h_0 + h_L)}}
$$

10^7 Pascal

Charge utile

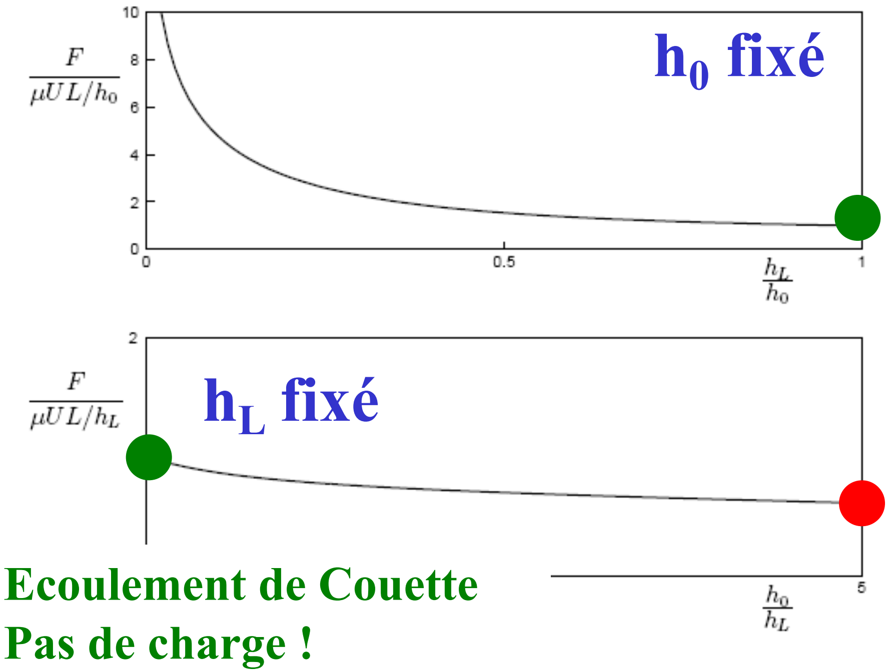


$$
\begin{aligned}
P &= \int_0^L \left(p(x) - p_0\right)\, dx \\
&= -\frac{L}{(h_0 - h_L)} \int_{h_0}^{h_L} \left(p(h) - p_0\right)\, dh \\
&= -\frac{L}{(h_0 - h_L)} \frac{6\,\mu\, U\, L}{(h_0^2 - h_L^2)} \int_{h_0}^{h_L} \left[(h_0 + h_L)\, \frac{1}{h} - \frac{h_0\, h_L}{h^2} - 1 \right] dh \\
&= -6\,\mu\, U\, L^2 \left[\frac{1}{(h_0 - h_L)^2} \log\left(\frac{h_L}{h_0}\right) + \frac{2}{(h_0^2 - h_L^2)} \right]
\end{aligned}
$$



Rapport optimal…

Force exercée par le fluide sur la partie mobile



$$
\begin{aligned}
F &= -\int_0^L \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} dx \\
&= \frac{\mu U L}{(h_0 - h_L)} \int_{h_0}^{h_L} \left[\frac{6}{h^2}\frac{h_0 h_L}{(h_0 + h_L)} - \frac{4}{h}\right] dh \\
&= -\mu U L \left[\frac{6}{(h_0 + h_L)} + \frac{4}{(h_0 - h_L)} \log\left(\frac{h_L}{h_0}\right)\right]
\end{aligned}
$$

La force diminue de façon monotone lorsque le rapport augmente…

La puissance consommée est dissipée…

$$F\,U = -\frac{\mu\,U^2\,L}{h_0}\left[\frac{6}{(1+h_L/h_0)} + \frac{4}{(1-h_L/h_0)}\log\left(\frac{h_L}{h_0}\right)\right]$$

Embêtant…

S’assurer que l’huile est bien refroidie car la viscosité (et donc la charge utile) décroît rapidement avec la température…

…en chaleur !

A propos de la viscosité de notre huile SAE 50

Transport maritime

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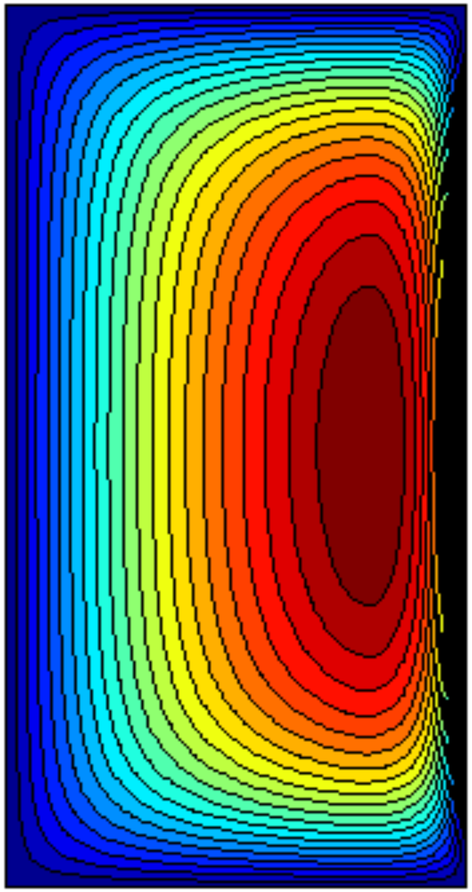
Grades offerts :
SAE 50

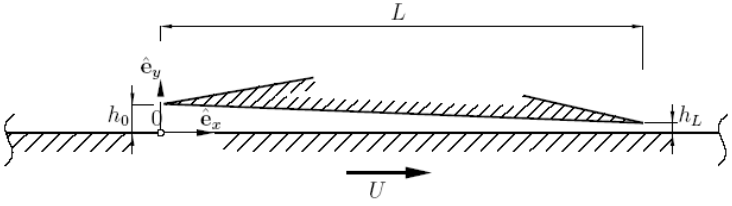
Fiche techniqu
Fiche signalét

$$
\begin{aligned}
T &= 20^0 C & \mu &= 1.100\ Ns/m^2 \\
T &= 40^0 C & \mu &= 0.210\ Ns/m^2 \\
T &= 50^0 C & \mu &= 0.100\ Ns/m^2 \\
T &= 60^0 C & \mu &= 0.060\ Ns/m^2 \\
T &= 80^0 C & \mu &= 0.025\ Ns/m^2 \\
T &= 100^0 C & \mu &= 0.013\ Ns/m^2
\end{aligned}
$$

Analyse « tridimensionnelle » du palier plat



pression sous un palier dont la largeur vaut le double de la longueur



Lubrification 2D ½

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial p}{\partial z} = 0$$

Théorie de la lubrification

Film fluide mince

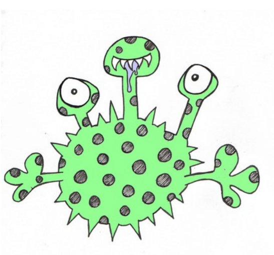
$$h \ll L$$

Hypothèse de lubrification : Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\left\{\begin{array}{rcl} \boxed{\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}+\dfrac{\partial w}{\partial z} = 0} \\ -\dfrac{\partial p}{\partial x}+\mu\dfrac{\partial^2 u}{\partial z^2} &=& 0 \\ -\dfrac{\partial p}{\partial y}+\mu\dfrac{\partial^2 v}{\partial z^2} &=& 0 \end{array}\right.$$

-i- calcul de u(x,y,z) et de v(x,y,z)



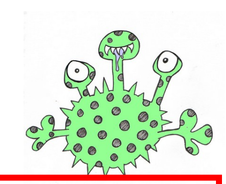
$$\boxed{\begin{array}{rcl} u(x,y,z) &=& -\dfrac{\partial p}{\partial x}\dfrac{h^2}{2\mu}\dfrac{z}{h}\left(1-\dfrac{z}{h}\right)+U\left(1-\dfrac{z}{h}\right) \\ v(x,y,z) &=& -\dfrac{\partial p}{\partial y}\dfrac{h^2}{2\mu}\dfrac{z}{h}\left(1-\dfrac{z}{h}\right) \end{array}}$$

-ii- calcul de p(x,y)

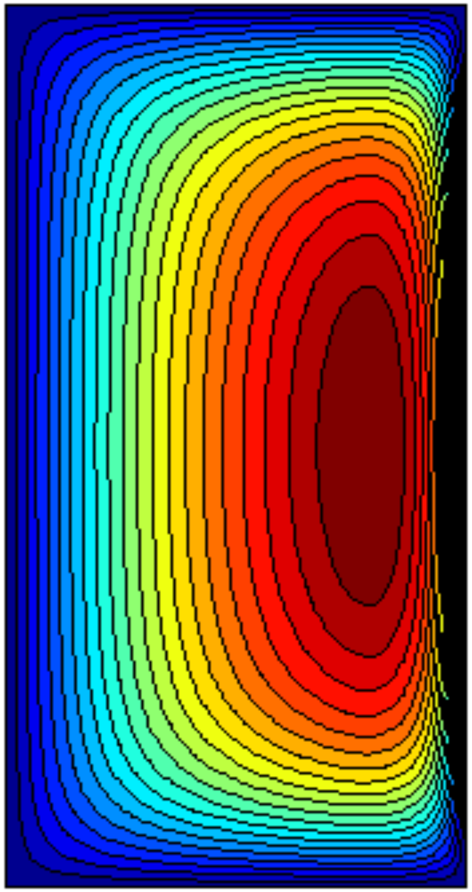
$$
\left\{
\begin{aligned}
-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2} &= 0 \\
-\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial z^2} &= 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0
\end{aligned}
\right.
$$

$$
\int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\, dz = 0
$$

$$
\frac{\partial}{\partial x}\int_0^h u(x,y,z)dz + \frac{\partial}{\partial y}\int_0^h v(x,y,z)dz + \Big[\, \cancel{w(x,y,z)}\, \Big]_0^h = 0
$$



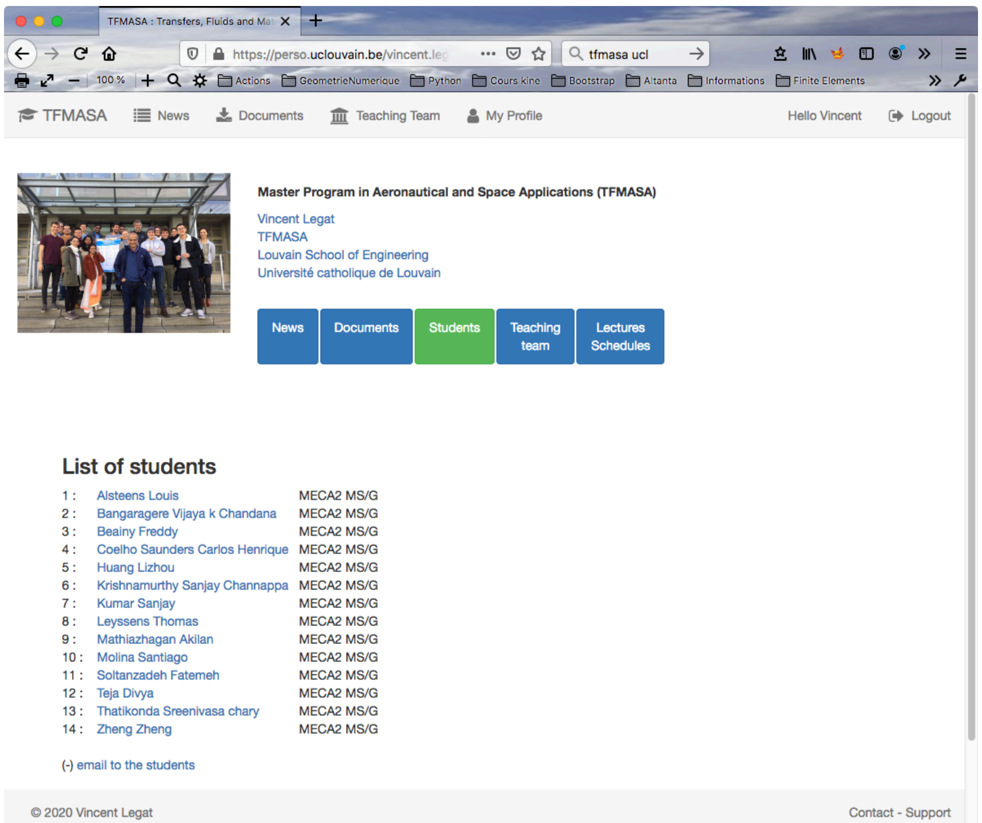
$$
\boxed{\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu U \frac{dh}{dx}}
$$



-iiii- calcul numérique par différences finies de p(x,y)

$$h^3\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+3h^2\left(\frac{h_L-h_0}{L}\right)\frac{\partial p}{\partial x} = 6\mu U\left(\frac{h_L-h_0}{L}\right)$$

Un triple diplôme belgo-germano-français ?



https://www.tfmasa.vchazallet.fr/

Et tout cela
en deux années !

1st semester : Université de Bordeaux (UBx), Bordeaux

- Simulation and Design of Structures (9 ECTS)
- Continuum Mechanics and finite element method applied to solid mechanics (6 ECTS)
- Fatigue and Fractures (3 ECTS)
- Materials and aeronautical structures (6 ECTS)
- Non-destructive evaluation for aerospace applications (3 ECTS)
- Assembly and bonding (3 ECTS)

2nd semester : UCL, Louvain-la-Neuve (4 mandatory courses out of 6)

- Internal Combustion Engines (5 ECTS)
- Aerodynamics of external flow (5 ECTS)
- Fluid compressors (5 ECTS)
- Numerical methods in fluids mechanics (5 ECTS)
- Quality management and control (5 ECTS)
- Gas dynamics and reacting flows (5 ECTS)

Elective courses :

- Advanced Numerical Methods (5 ECTS)
- Calculation of Planar Structures (5 ECTS)
- Aerodynamics of External Flows (5 ECTS)
- Thermodynamics of Irreversible Phenomena (5 ECTS)
- Plasticity and Metal Forming (5 ECTS)

3rd semester : BTU, Cottbus-Senftenberg (5 courses)

Compulsory Elective Modules I (3 out of 5) :

- Computational fluid dynamics (6 ECTS)
- Engineering Acoustics - Sounds Fields (6 ECTS)
- Turbulence Modelling (6 ECTS)
- Thermodynamics and Heat Transfer (6 ECTS)
- Flow measurements (6 ECTS)

Compulsory Elective Modules II (2 modules are to be determined in consultation with the local mentor) :

- One module from Mechanical Engineering, Aeronautical Engineering, Materials Science, or Aerospace (6 ECTS)
- One module from Physics, Mathematics or Computer Science (6 ECTS)

Master in Transferts, Fluids and Materials for Aeronautical and Space Applications

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