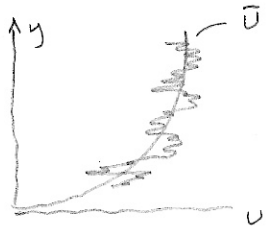
EX 20



1

$$\bar{\sigma}^t_{xx} = -\rho \overline{u'u'}$$
$$\bar{\sigma}^t_{yy} = -\rho \overline{v'v'}$$
$$\bar{\sigma}^t_{xy} = -\rho \overline{u'v'}$$

$$\boxed{\begin{aligned} \bar{k} &= \frac{1}{2}\left[\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right] \\ \bar{\tau}^t_{xy} &= \bar{\sigma}^t_{xy} \\ \bar{\tau}^t_{xx} &= \underbrace{\bar{\sigma}_{xx} + \frac{2}{3}\rho\bar{k}}_{\neq 0} \end{aligned}}$$

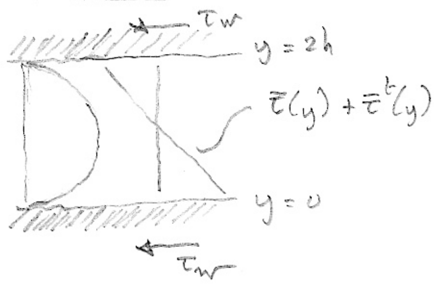
CONTRAIREMENT A $\bar{\tau}_{xx}$:-)

2

$$\boxed{\underbrace{\frac{dp}{dx}}_{=CST} = \frac{d\bar{\tau}}{dy} + \frac{d\bar{\tau}^t}{dy}}$$

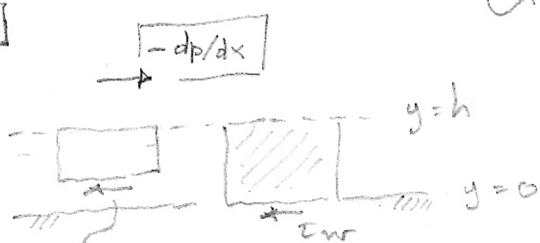
AVEC $\bar{\tau} = \bar{\tau}_{xy}$

$$\underbrace{\bar{\tau}(y) + \bar{\tau}^t(y) = \bar{\tau}_w\left(1 - \frac{y}{h}\right)}$$



LA CONTRAINTE EST POSITIVE POUR $y = 0$
ET NEGATIVE POUR $y = 2h$
..... CAR LES NORMALES DES FRONTIERES SONT OPPOSÉES :-)

3



$$-\frac{dp}{dx}(y-h) = \bar{\tau}(y) + \bar{\tau}^t(y)$$
$$-\frac{dp}{dx}h = \bar{\tau}_w$$

$$\boxed{\bar{\tau}(y) + \bar{\tau}^t(y) = \frac{\bar{\tau}_w}{h}(y-h)} \quad \square$$

NOTE $\bar{\tau}^t(y)$ N'EST PAS VRAIMENT UNE FORCE APPLIQUEE MAIS EST UN TRANSFERT DE QUANTITE DE MVT DU AUX FLUCTUATIONS DE VITESSE .... si ! si !

4

$$-\frac{dp}{dx}h = \bar{\tau}_w$$

$$\underbrace{\frac{-\frac{dp}{dx}h}{\frac{1}{2}\rho\bar{U}_m^2}}_{2\lambda} = \underbrace{\frac{\bar{\tau}_w}{\frac{1}{2}\rho\bar{U}_m^2}}_{C_f}$$

$$\boxed{\lambda = 2C_f}$$

COMME EN LAMINAIRE :-)

MEME RAISONNEMENT

$$\boxed{\frac{\bar{U}_\tau^2}{\bar{U}_m^2} = \frac{\bar{\tau}_w}{\rho\bar{U}_m^2} = \frac{C_f}{2} = \frac{\lambda}{4}} \quad \square$$

$\bar{U}_\tau$ EST BIEN UNE VITESSE REPRESENTATIVE DES CONTRAINTES OBSERVEES :-)

5

$$\nu_T(y) \triangleq \frac{-\overline{u'v'}}{d\bar{U}/dy}$$

DEFINITION DE LA VISCOSITE TURBULENTE

$$= \frac{\bar{\tau}^t(y)}{d\bar{U}/dy(y)}$$

$$\boxed{\nu \frac{d\bar{U}}{dy}(y) + \nu_T(y)\frac{d\bar{U}}{dy}(y) = \underbrace{\frac{\tau_w}{\rho}}_{\bar{U}_\tau^2}\left(1-\frac{y}{h}\right)}$$

6

$$1-\frac{y}{h} \simeq 1$$

LORSQUE $\frac{y}{h} < 0.15$

UN PEU BRUTAL COMME HYPOTHESE....

$$\nu \frac{d\bar{U}(y)}{dy} + \nu^T(y)\frac{d\bar{U}(y)}{dy} \simeq \bar{U}_\tau^2$$

NEGLIGEABLE DANS LA ZONE TURBULENTE

ZONE 3a

NEGLIGEABLE DANS LA SOUS-COUCHE LAMINAIRE

ZONE 1

7

$$\nu \frac{d\bar{U}}{dy}(y) = \bar{U}_\tau^2$$

$$\nu\, \bar{U}(y) = \bar{U}_\tau^2\, y$$

$$\underbrace{\frac{\bar{U}(y)}{\bar{U}_\tau}}_{\bar{U}^+} = \underbrace{\frac{y\,\bar{U}_\tau}{\nu}}_{y^+}$$

$$\boxed{\bar{U}^+(y) = \bar{y}^+}$$

PROFIL LINEAIRE
ZONE I

$\boxed{8}$

VON KARMAN

$$\underbrace{\underbrace{\varkappa^2 \frac{\left(\frac{d\bar U}{dy}\right)^2}{\left(\frac{d^2\bar U}{dy^2}\right)^2}}_{\ell^2} \frac{d\bar U}{d\bar y}}_{\nu^T} \frac{d\bar U}{d\bar y} = \bar U_\tau^2$$

EN ADIMENSIONNALISANT TOUT EN $\bar U^+$ ET EN $\bar y^+$

$$\varkappa \frac{\left(\frac{d\bar U^+}{d\bar y^+}\right)^2}{\left(\frac{d^2\bar U^+}{d\bar y^{+2}}\right)} = \pm 1$$

$$\varkappa = \pm \frac{\frac{d}{dy^+}\left(\overset{c'''}{\bigcirc}\right)}{\left(\overset{c'''}{\bigcirc}\right)^2}$$

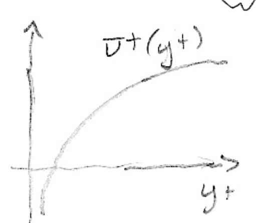
SEUL LE SIGNE MOINS EST OPERATIONNEL !

$= 0$ CAR ON SUPPOSE $d\bar U/d\bar y \to \infty$ EN $\bar y^+ = 0$:)

$$\varkappa y + \cancel{A} = \frac{1}{d\bar U^+/d\bar y^+}$$

$$\frac{d\bar U^+}{d\bar y^+} = \frac{1}{\varkappa}\frac{1}{\bar y^+}$$

$$\bar U^+(\bar y^+) = \frac{1}{\varkappa}\log(\bar y^+) + C$$



PRANDTL

$$\varkappa^2 y^2 \left(\frac{d\bar U}{d\bar y}\right)^2 = \bar U_\tau^2$$

$$\varkappa \bar y^+ \frac{d\bar U^+}{d\bar y^+} = 1$$

$$\frac{d\bar U^+}{d\bar y^+} = \frac{1}{\varkappa}\frac{1}{y^+}$$ IDEM :-)

VISCOSITE LINEAIRE

$$\varkappa y \bar U_\tau \frac{d\bar U}{d\bar y} = \bar U_\tau^2$$

$$\varkappa y^+ \frac{d\bar U^+}{dy^+} = 1$$

IDEM :-)

VISCOSITE QUADRATIQUE

ON NE FAIT PLUS L'HYPOTHESE DE PRANDTL - VK !

$$\varkappa y \left(1-\cancel{\frac{y}{h}}\right) \bar U_\tau \frac{d\bar U}{d\bar y} = \bar U_\tau^2 \cancel{\left(1-\frac{y}{h}\right)}$$

$$\varkappa \bar y^+ \frac{d\bar U^+}{d\bar y^+} = 1$$

IDEM :-)