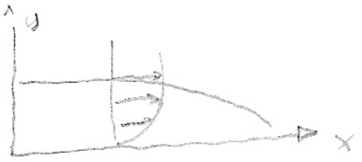
EX 11

1-3



ECOULEMENT RAMPANT HYDROSTATIQUE

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$$\boxed{p(x,y,t) = p_0 + \rho g\,(h(x,t) - y)}$$

PRESSION HYDROSTATIQUE

$$\frac{\partial p}{\partial y} = -\rho g$$

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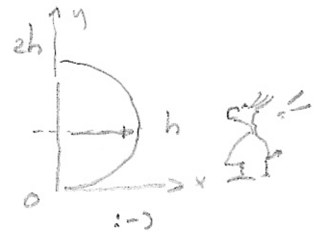
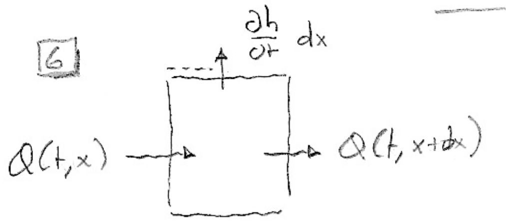
$$\boxed{\underbrace{\frac{\partial p}{\partial x}}_{\rho g \frac{\partial h}{\partial x}} = \mu \frac{\partial^2 u}{\partial y^2}} \qquad \boxed{\begin{array}{c} u(0) = 0 \\ \left.\frac{\partial u}{\partial y}\right|_h = 0 \end{array}}$$

$$\boxed{u(x,y,t) = \overbrace{-\frac{\rho g}{2\mu}\underbrace{\frac{\partial h}{\partial x}}_{<0}}^{>0}\,(2h - y)\,y}$$



6



$$\boxed{\frac{\partial Q}{\partial x} = -\frac{\partial h}{\partial t}} \qquad \boxed{A = -1}$$

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$$Q(x,t) = -\frac{\rho g}{2\mu}\frac{\partial h}{\partial x}\underbrace{\int_0^h (2h - y)\,y\,dy}_{\left[2h\frac{y^2}{2} - \frac{y^3}{3}\right]_0^h = h^3 - \frac{h^3}{3} = \frac{2h^3}{3}}$$

$$= -\frac{\rho g}{3\mu}\frac{\partial h}{\partial x}h^3$$

$$\boxed{-\frac{\rho g}{3\mu}\frac{\partial}{\partial x}\left(h^3\frac{\partial h}{\partial x}\right) = -\frac{\partial h}{\partial t}} \qquad \boxed{B = \frac{3\mu}{\rho g}}$$

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$$\boxed{\begin{array}{l} Q_0 = -\frac{\rho g}{3\mu}\,h^3(0,t)\,\frac{\partial h}{\partial x}(0,t) \\ h(L(x,t)) = 0 \end{array}}$$

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BILAN GLOBAL DE MASSE

$$\boxed{Q_0\,t = \int_0^{L(t)} h(x,t)\,dx}$$

ON SUPPOSE EVIDEMMENT QUE $L(0) = 0$:-)

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$$\boxed{h(x,t) = t^{\alpha} f(\eta) \qquad \eta = t^{\beta} x}$$

$$\frac{\partial h}{\partial x} = t^{\alpha} f'(\eta) \underbrace{\frac{\partial \eta}{\partial x}}_{t^{\beta}} = t^{\alpha+\beta} f'$$

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial h}{\partial x}\right) = \left(t^{3\alpha} f^3\, t^{\alpha+\beta} f'\right)' \underbrace{\frac{\partial \eta}{\partial x}}_{t^{\beta}} = t^{4\alpha+2\beta} (f^3 f')'$$

$$\frac{\partial h}{\partial t} = \alpha t^{\alpha-1} f + t^{\alpha} f' \underbrace{\frac{\partial \eta}{\partial t}}_{\beta t^{\beta-1} x} = \alpha t^{\alpha-1} f + \underbrace{t^{\alpha+\beta-1} \beta x}_{\eta \beta t^{\alpha-1}} f'$$

$$\boxed{\frac{\partial}{\partial x}\left(h^3 \frac{\partial h}{\partial x}\right) = \frac{3\mu}{\rho g} \frac{\partial h}{\partial t}}$$

$$\boxed{-\frac{\rho g}{3\mu} \frac{\partial h}{\partial x}(0,t)\, h^3(0,t) = Q_0}$$

$$t^{4\alpha+2\beta} (f^3 f')' = \frac{3\mu}{\rho g} (\alpha f + \beta \eta f') t^{\alpha-1}$$

$$-\frac{\rho g}{3\mu} t^{4\alpha+\beta} \underbrace{f'(0) f^3(0)}_{\text{ct}} = \underbrace{Q_0}_{\text{ct}}$$

SOLUTION
DE SIMILITUDE

$$4\alpha + 2\beta = \alpha - 1$$
$$4\alpha + \beta = 0$$

$$\boxed{\alpha = 1/5} \quad \boxed{\beta = -4/5}$$

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$$B(\alpha f + \beta \eta f') = (f^3 f')'$$

$$-B_0 Q_0 = f^3(0) f'(0)$$

$$f(\eta_L) = 0 \qquad f(0) = 1$$

$$Q_0 = \int_0^{\eta_L} f(\eta)\, d\eta$$

METHODE
DU TIR SUR Q_0

$$Q_0 t = \overbrace{t^{\alpha-\beta}}^{=1} \int_0^{\eta_L} f \, d\eta \qquad dx = t^{-\beta} d\eta$$

C'EST OK

$$\boxed{Q_0 t = \int_0^{L(t)} h(x,t)\, dx}$$