EX 17

**1**

$$\boxed{\begin{aligned} 0 &= -\frac{\partial p}{\partial x} + \mu \frac{\partial v}{\partial y^2} \\ 0 &= -\frac{\partial p}{\partial y} - \rho\,(1-\beta(T-T_0))\,g \\ v\frac{\partial T}{\partial x} &= \alpha\left[\frac{\partial T^2}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \end{aligned}}$$

**2**

$$T(x,y) = ax + b$$

$$\frac{\partial T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

SATISFAIT L'EDP
SATISFAIT LES CF en $y = \pm S$
LES CF en $x = \pm L$

**3**

$$\underbrace{\frac{\partial p}{\partial y}}_{\frac{a}{S} + \frac{b}{SL}x} = -\rho g - \rho g \beta \frac{\Delta T x}{2L}$$

$$\boxed{\begin{aligned} a &= -\rho g S \\ b &= -\rho g \beta \Delta T S/2 \end{aligned}}$$

**4**

$$\boxed{\begin{aligned} & v(-S) = v(S) = 0 \\ & \int_{-S}^{S} v(y)\, dy = 0 \end{aligned}}$$

**5**

EXTREMES
$(\eta-\eta^3)' = 0$
$3\eta^2 = 1$

$$\boxed{\eta = \pm 1/\sqrt{3}}$$

$$\underbrace{\frac{\partial p}{\partial x}}_{= \frac{b}{LS} y \; = -\frac{\rho g \beta \Delta T}{2L}} = -\rho \nu \frac{6}{S^3} U y$$

$$\boxed{\begin{aligned} U &= \frac{-bS^2}{6L\rho\nu} \\ &= \frac{\beta g \Delta T S^3}{12\nu L} \end{aligned}}$$

**6**

$$\boxed{Gr = \frac{g\beta\Delta T\, 8 S^3}{\nu^2} = \frac{(\text{ARCHIMEDE})(\text{INERTIE})}{(\text{VISQUEUX})^2}}$$

$$\boxed{U = Gr \, \frac{\overset{[m^2/s]}{\nu}}{\underset{[m]}{96 L}}}$$

**7**

$$v\frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$U(\eta^3-\eta)\frac{\Delta T}{2L} = \alpha \frac{1}{S^2} \tilde{T}''(\eta)$$

$\tilde{T}(y)$

$$\tilde{T}''(\eta) = \frac{U\Delta T S^2}{2\alpha L}(\eta^3-\eta)$$

$$\tilde{T}(\eta) = \ldots \left(\frac{\eta^5}{20} - \frac{\eta^3}{6} + a\eta + b\right)$$

$$\tilde{T}'(\pm 1) = 0$$

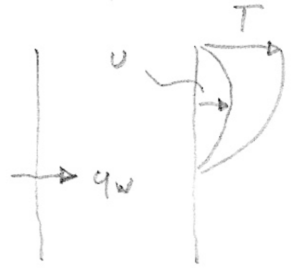
$$\frac{1}{4} - \frac{1}{2} + a = 0 \;\rightarrow\; a = 1/4$$

$b = 0$ car $\tilde{T}(0) = 0$

$$\boxed{\tilde{T}(\eta) = \frac{U\Delta T S^2}{2\alpha L}\left(\frac{\eta^5}{20} - \frac{\eta^3}{6} + \frac{\eta}{4}\right)}$$

$$\boxed{\begin{aligned}\tilde{T}(\eta) &= Gr\,\frac{\nu}{96L}\,\frac{\Delta T}{2\alpha}\,\frac{S^2}{L}\left(\frac{\eta^5}{20}-\frac{\eta^3}{6}+\frac{\eta}{4}\right)\\ &= \underbrace{Gr\,Pr\,\frac{S^2}{L^2}\,\Delta T}_{T^*}\,\frac{1}{384}\left(\frac{\eta^5}{10}-\frac{\eta^3}{3}-\frac{\eta}{2}\right)\end{aligned}}$$

$\boxed{8}$




$$\int_{-S}^{S} \rho c\, u(y)\, \tilde{T}(y)\, dy$$

FLUX CONVECTIF

$$Q = \underbrace{\frac{k\Delta T}{2L}}_{\substack{q_w \\ \text{DIFFUSIF}}} 2S + \overbrace{\frac{\rho c U T^*}{384} S \underbrace{\int_{-1}^{1}\left(\frac{\eta^5}{10}-\frac{\eta^3}{3}-\frac{\eta}{2}\right)\left(\eta-\eta^3\right) d\eta}}$$

$$= 2\int_{-1}^{1} \frac{\eta^6}{10}-\frac{\eta^4}{3}-\frac{\eta^2}{2}-\frac{\eta^8}{10}+\frac{\eta^6}{3}-\frac{\eta^4}{2}$$

$$= \int_{0}^{1} -\frac{\eta^8}{5}+\frac{13}{15}\eta^6-\frac{5}{3}\eta^4+\eta^2$$

$$= \left[-\frac{\eta^9}{45}+\frac{13\,\eta^7}{105}-\frac{5}{15}\eta^5+\frac{\eta^3}{3}\right]_0^1$$

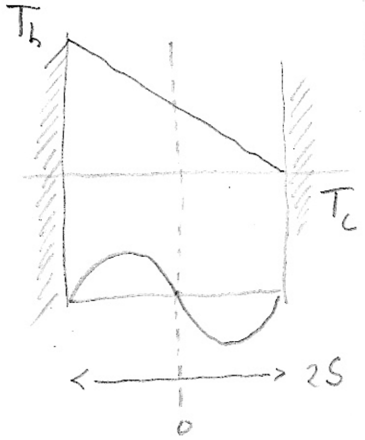
$$= -\frac{1}{45}+\frac{13}{105}-\frac{1}{3}+\frac{1}{3}$$

$$= \frac{1}{15}\underbrace{\left(-\frac{1}{3}+\frac{13}{7}\right)}_{\frac{-7+39}{21}} = \frac{32}{315}$$

$$\boxed{Q = \frac{k\Delta T S}{L} + \rho c S U T^*\,\frac{1}{384}\,\frac{32}{315}}$$

$16\times 32$ $\qquad \dfrac{1}{10080}$

$\boxed{\text{EXO 18}}$




$\boxed{1}$

$T_0 = \frac{T_h + T_c}{2}$

$\Delta T = T_h - T_c$

$$\boxed{\frac{T(x) - T_0}{\Delta T} = \frac{x}{2S}}$$

$\boxed{2}$ $\frac{\partial^2 T}{\partial x^2} = 0$

$\boxed{3}$

$$-\underbrace{\frac{dp}{dy}}_{\rho_0 g} - \rho_0 g \left(1 - \beta (T - T_0)\right) + \mu \frac{d^2 v}{dx^2} = 0$$

PRESSION HYDROSTATIQUE

$$\frac{\beta \rho_0 g}{\mu} \underbrace{(T - T_0)}_{\frac{x \Delta T}{2S}} = \frac{d^2 v}{dx^2}$$

$$\boxed{v(x) = \frac{\beta \rho_0 g \Delta T S^2}{12 \mu S} \left[ \left(\frac{x^3}{S}\right) - \left(\frac{x}{S}\right) \right]}$$

$\frac{dv}{dx} = 0 \rightarrow x = \pm \frac{S}{\sqrt{3}}$

MAXIMA MINIMA

3 RACINES :-)

LA SOLUTION EST IMMEDIATE !

$$\boxed{v(x) = \underbrace{\frac{\beta g \Delta T S^3}{\nu^2}}_{Gr} \; \underbrace{\overset{[m^2/s]}{\frac{\nu}{12 S}}}_{[m]} \left[ \left(\frac{x}{S}\right)^3 - \left(\frac{x}{S}\right) \right]}$$

C'EST BIEN UNE VITESSE

$\boxed{4}$

DISSIPATION VISQUEUSE NEGLIGEABLE

$$\boxed{P_r \, E_c \ll 1}$$

OUI CAR UNE SEULE LONGUEUR CARACTERISTIQUE $S$ !

$\frac{\nu}{\alpha}$ $\qquad \frac{V^2}{c \Delta T}$ $\qquad$ OR $V^2 = Gr^2 \frac{\nu^2}{144 S^2}$

$$\boxed{\frac{\nu^3}{\alpha c \Delta T} \frac{\beta^2 g^2 \Delta T^2 S^6}{\nu^4} \frac{\nu^2}{144 S^2} = \frac{\beta^2 g^2 \Delta T S^4}{144 \nu c \alpha} \ll 1}$$

$\boxed{\text{A CALCULER}}$

$\beta = 1/T$ GAZ PARFAITS

$\left.\begin{array}{l} c \\ \nu \\ \alpha \end{array}\right\}$ AIR :-)