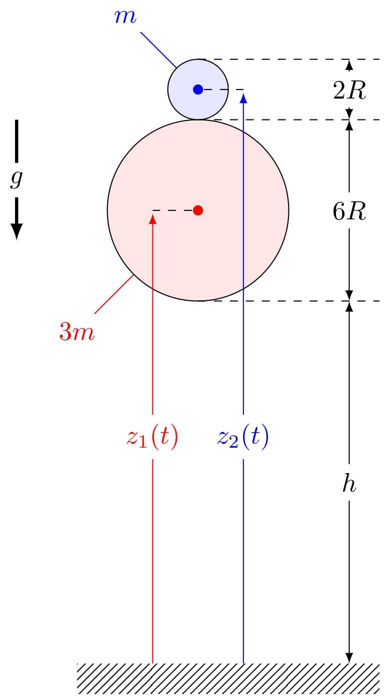


Thomas wants to bounce balls...

Thomas has two plastic balls.
The radii of the balls are $R_1 = 3R = 0.3$ m and $R_2 = R = 0.1$ m.
Their masses are $3m = 0.03$ kg and $m = 0.01$ kg.
Thomas places the small ball on top of the large ball.

At $t = 0$, Thomas releases the two balls from a height $h = 3.2$ m.
The vertical positions of the centers are denoted by $z_1(t)$ and $z_2(t)$.

At $t = t_*$, the large ball reaches the ground.
The collisions between the balls and the ground are **perfectly elastic**.
These collisions are also **instantaneous and successive**.
After the impacts, the balls have two distinct velocities $v_1^+ < v_2^+$.
Then a periodic bouncing motion sets in.
The period of this motion is T.

In the calculations, we will use $g = 10$ m/s^2.

1. Compute the time t_* corresponding to the free fall of the two balls.

   This is just a standard free-fall calculation!

$$h = g\frac{t_*^2}{2} \quad \longrightarrow \quad t_* = \sqrt{\frac{2h}{g}} = \sqrt{0.64}$$

$$v = gt_* \quad \longrightarrow \quad v = gt_* = \sqrt{2hg} = \sqrt{64}$$

Hence:

$$\boxed{\begin{array}{rcl} v &=& 8 \text{ m/s} \\ t_* &=& 0.8 \text{ s} \end{array}}$$

2. Compute the speed v before the collision, then the speeds v_1^+ and v_2^+.

   To determine the two velocities after the collision, two relations are required.
   The statement mentions two distinct, simultaneous and elastic collisions.
   First, we consider the elastic rebound of the large ball on the ground.
   Next, we consider an elastic collision between the two balls.

   Both collisions conserve kinetic energy.
   The linear momentum is conserved during the collision between the two balls.

$$\begin{cases} 3mv - mv = 3mv_1^+ + mv_2^+ \\ 4mv^2 = 3m\left(v_1^+\right)^2 + m\left(v_2^+\right)^2 \end{cases}$$

With the initial velocity $v = 8$, the system becomes:

$$\begin{cases} 16 = 3\, v_1^+ + v_2^+ \\ 256 = 3\left(v_1^+\right)^2 + \left(v_2^+\right)^2 \end{cases}$$

And by substituting one of the unknowns, we obtain a quadratic equation for v_1^+*:*

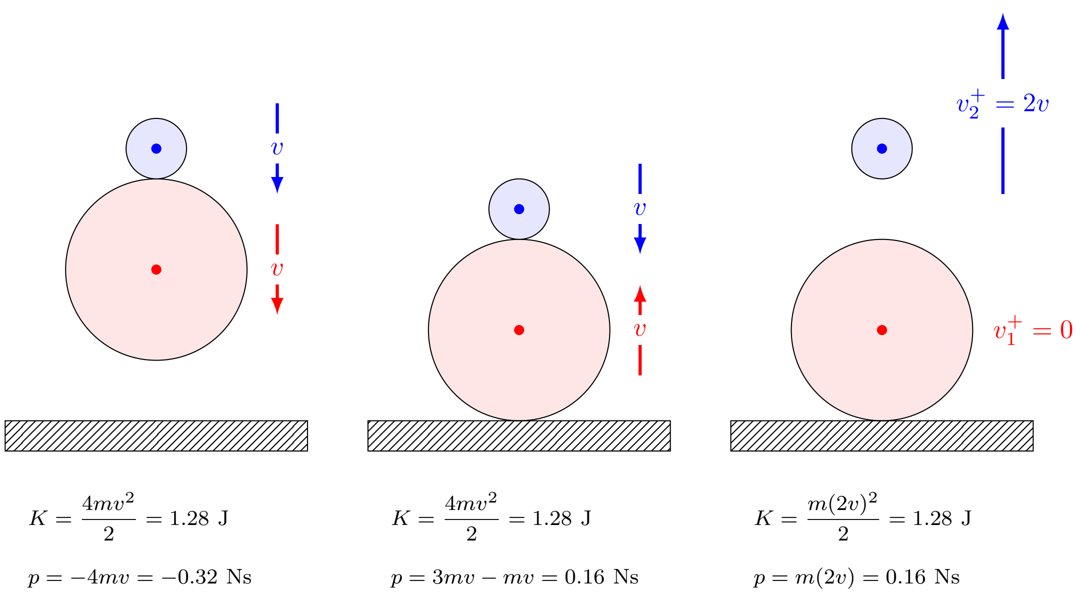
$$256 = 3\left(v_1^+\right)^2 + \left(16 - 3v_1^+\right)^2$$
$$\downarrow$$
$$256 = 3\left(v_1^+\right)^2 + 256 + 9\left(v_1^+\right)^2 - 96v_1^+$$
$$\downarrow$$
$$0 = 12\, v_1^+\left(8 - v_1^+\right)$$

There are two possible solutions: either $v_1^+ = 0$*, or* $v_1^+ = 8$.

Since the statement requires the velocity of the large ball to be strictly smaller, we reject the solution where the balls rebound with the same value and we choose the zero value $v_1^+ = 0$.

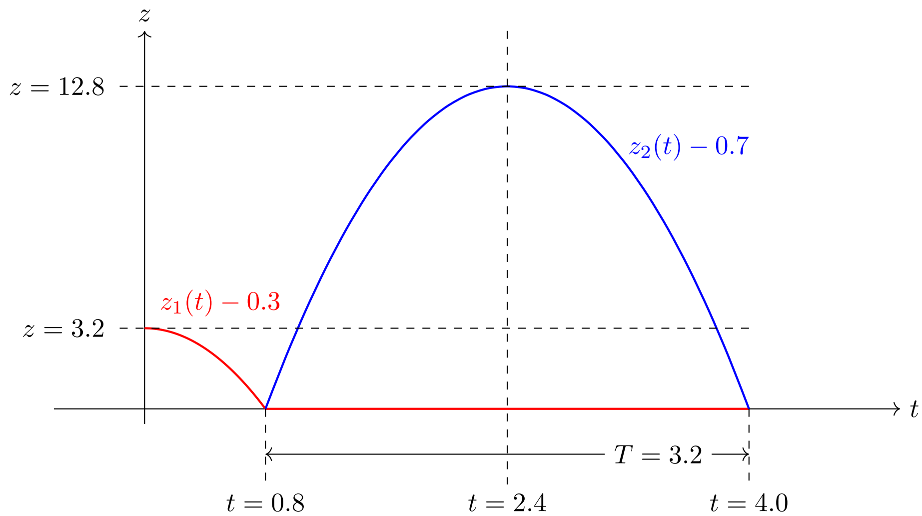
The final answer is therefore:

$$\begin{aligned} v_1^+ &= 0 \text{ m/s} \\ v_2^+ &= 16 \text{ s} \end{aligned}$$



3. Plot the curves $z_1(t) - 3R$ and $z_2(t) - 7R$ if the electric field E is never switched on.
   What is the maximum height reached by the small ball?
   What is the period T of the bouncing motion?

You just need to plot two parabolas!
The small ball is launched with an initial velocity of 16 m/s *at time* $t = 0.8$.
It will reach a maximum height of $4h = 4 \times 3.2 = 12.8$ m.
It will take a time $2t_* = 2 \times 0.8 = 1.6$ s *to reach the maximum height.*
And the period of the bouncing motion is $T = 3.2$ s.



z
t
z = 12.8
z = 3.2
z_1(t) − 0.3
z_2(t) − 0.7
T = 3.2
t = 0.8
t = 2.4
t = 4.0