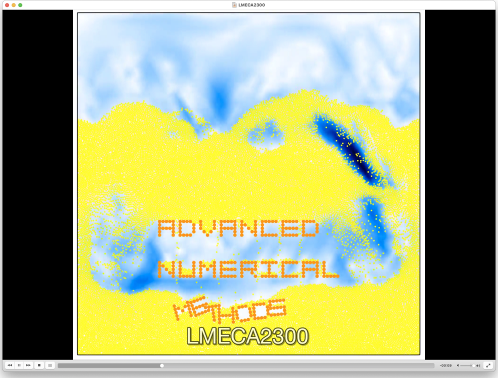
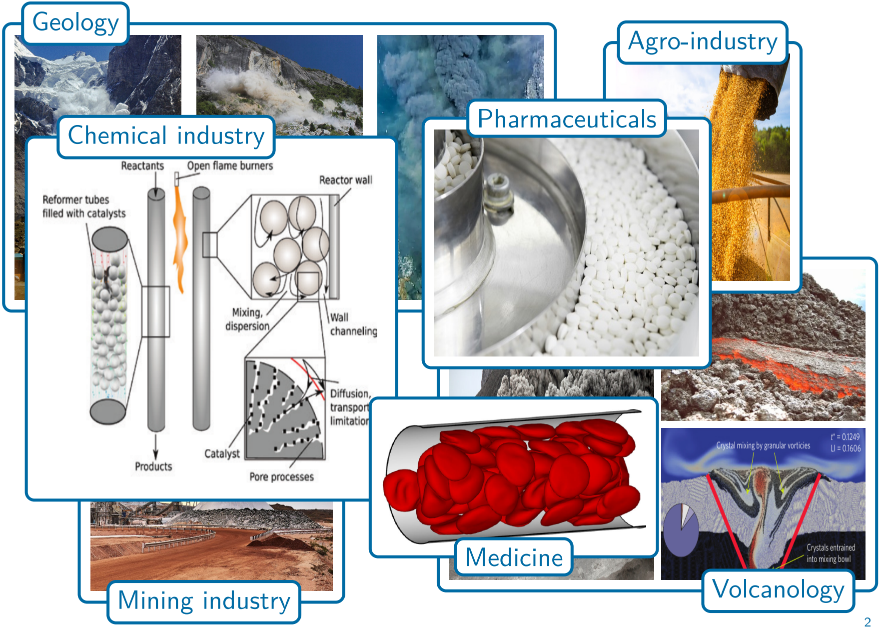

LMECA2300
ADVANCED
NUMERICAL
METHODS
LMECA2300

# Granular materials are ubiquitous



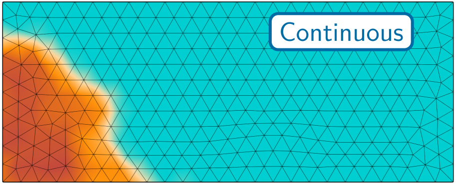
# Why numerical methods in granular materials?

- Analytical solutions can be difficult to obtain
- Experiments can be costly
- Some data are difficult to access experimentally
- Controlling and varying the physical parameters is difficult in experimental conditions

Calibration/validation

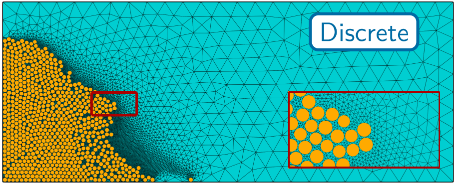
Computational power

# All models are wrong, but some are useful




- Large scale applications
- Depth-averaged equations
  $\mu(I)$ rheology

- Insights on grain physics
- Nonsmooth Contact Dynamics
  Discrete Elements Method
  Event-Driven



# Course contents

**Numerical models**

- Event Driven Method (ED)
- Discrete Element Method (DEM)
- Nonsmooth Contact Dynamics (NSCD)

- 10 Lectures
- 5 Homeworks
- 1 Project

**Numerical issues**

- Contact detection
- Temporal scheme
- Stability
- ...

**Granular physics**

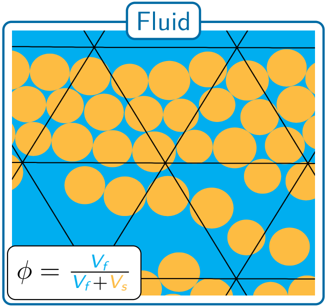
- Stress state
- Dilatancy
- Angle of repose
- ...

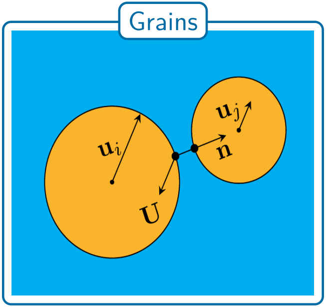
# Lectures

1. Introduction
2. Event Driven method
3. Smooth DEM
4. Friction in smooth DEM
5. Nonsmooth Contact Dynamics
6. Friction in NSCD
7. Nonspherical grains
8. Contact detection
9. Parallel computing
10. Fluid coupling

# The MigFlow software

- **M**odel for **I**mmersed **G**ranular **Flows**




$$\frac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{v} = 0$$

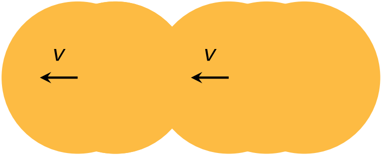
$$\rho(\frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot \frac{\mathbf{v}\mathbf{v}}{\phi}) = \nabla \cdot [2\mu\phi\mathbf{d} - p\mathbf{I}] + \mathbf{f} - \rho\phi\mathbf{g}$$

$$\mathbf{M}\frac{d\mathbf{u}}{dt} = \mathbf{M}\mathbf{g} - \mathbf{f} + \mathbf{r}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}$$

# Contacts can be classified in three different types

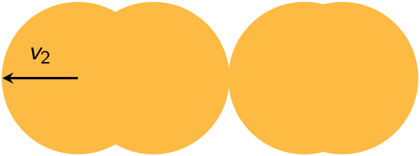
**Perfectly inelastic** | Elastic | Inelastic




- Momentum conservation
- $u = 2v$

# Contacts can be classified in three different types

Perfectly inelastic | **Elastic** | Inelastic



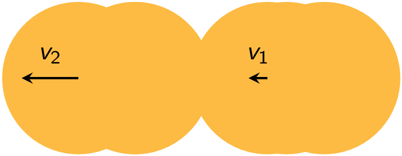

- Momentum conservation
- $u = v_1 + v_2$
- Energy conservation
- $u^2 = v_1^2 + v_2^2$

# Contacts can be classified in three different types

Perfectly inelastic

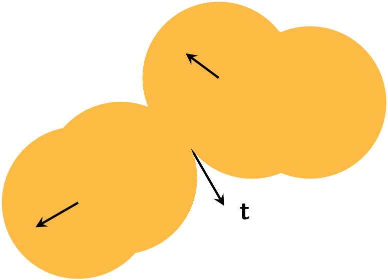
Elastic

Inelastic




- Momentum conservation
- $u = v_1 + v_2$
- Restitution coefficient
- $ru = v_2 - v_1$

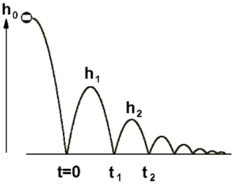
# In two dimensions, things get a bit more complicated...



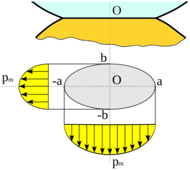

- Normal relative velocity
- $u_n = \mathbf{u} \cdot \mathbf{n}$
- Momentum conservation
- $u_n = v_{1n} + v_{2n}$
- Restitution coefficient
- $ru_n = v_{2n} - v_{1n}$

# The real physics of contacts can be tricky

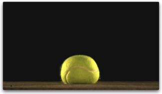
- The exact contact time may not be relevant




- Real bodies are not perfectly rigid

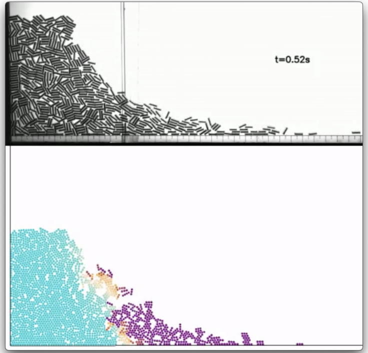



- Contacts are not really instantaneous



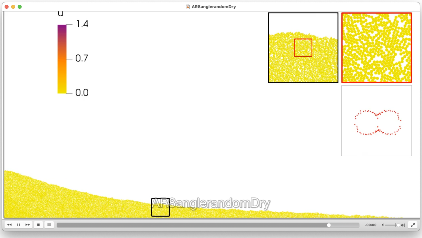
- Physical properties of surfaces are difficult to obtain

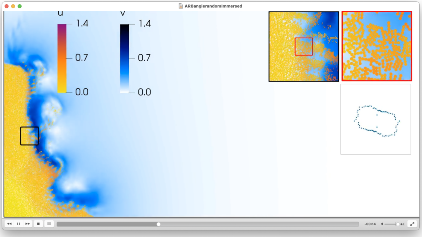
# Avalanches




- Numerical parameters were calibrated from a lab experiment
- The deposit shape and runout are reproduced
- 150 simulations
- Up to 90000 grains
- Up to 14 days of CPU time
- 670 Go of data

# Avalanches

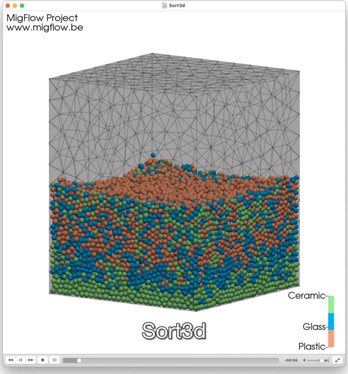



- Fluid slows the collapse down and reduces its runout
- The runout depends on the front kinetic energy
- The runout is reduced compared to circular grains

Coppin, N., Henry, M., Cabrera, M., Azéma, E., Dubois, F., Legat, V., Lambrechts, J., (2023). *Collapse dynamics of two-dimensional dry and immersed granular columns of elongated grains* . Physical Review Fluids 8, 094303.

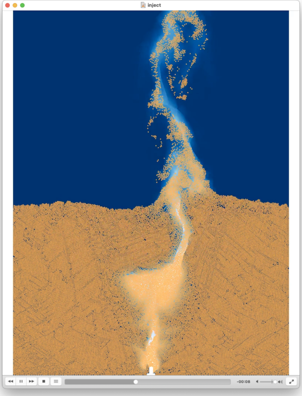
# Industrial sorting




- Density and size sorting via fluid injection
- Grain clusters accelerate the sorting
- Friction in the densest species controls the dynamics

Constant, M., Coppin, N., Dubois, F., Artoni, R., Lambrechts, J., Legat, V. (2021).
*Numerical investigation of the density sorting of grains using water jigging*.
Powder Technology 393, 705-721.

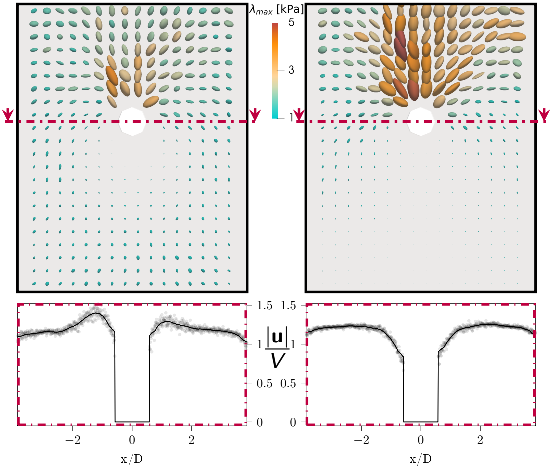
# Submarine vents




- The fluidised area grows at the top of the bed
- Deposit shape depends on flow rate
- Stress decreases over time

Constant, M., Coppin, N., Dubois, F., Vidal, V., Legat, V., Lambrechts, J. (2021).
*Simulation of air invasion in immersed granular beds with an unresolved FEM–DEM model.*
Computational Particle Mechanics 8, 535-560.

# Granular drag




Coppin, N., Constant, M., Lambrechts, J., Dubois, F., Legat, V. (2022).
*Numerical analysis of the drag on a rigid boy in an immersed granular flow*.
Computational Particle Mechanics 9, 393-406.

# Take-home messages

- Many approaches exist concerning the simulation of granular materials
- Lagrangian methods provide insights on the physics at grain scale: we will focus on the three main ones (ED, DEM and NSCD)
- Numerical simulations have been, are, and will be key tools to explore the complex behaviours exhibited by granular matter