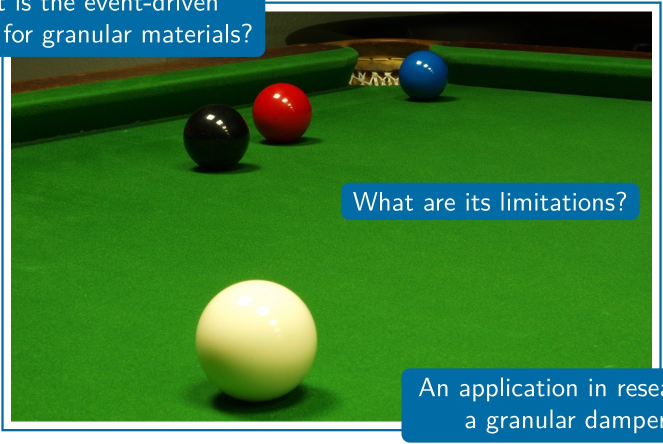
Lecture 2: from event to event



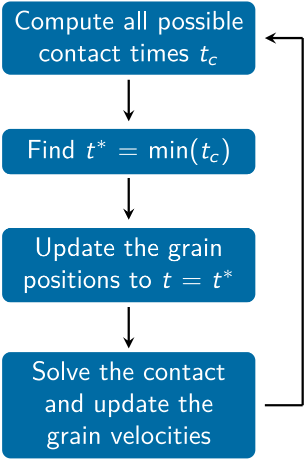
What is the event-driven method for granular materials?

What are its limitations?

An application in research: a granular damper

The event-driven method

- The motion of grains is deduced from U(A)RM equations
- There is no fixed time step: the method jumps from one contact to another
- Only the next contact is guaranteed to happen
- Contacts are binary and instantaneous
- A finite number of contacts occurs in a finite time



Compute all possible contact times t_c

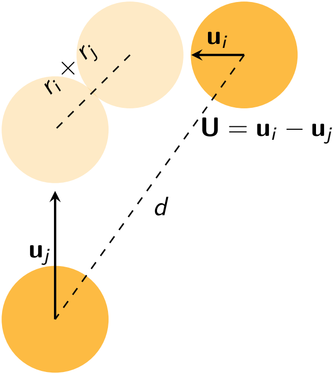
Find $t^* = \min(t_c)$

Update the grain positions to $t = t^*$

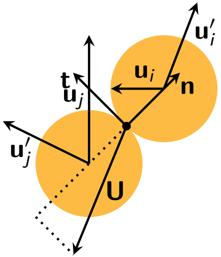
Solve the contact and update the grain velocities

Finding the time of a contact

- $\mathbf{x}_k(t) = \mathbf{x}_{k0} + \mathbf{u}_k t + \mathbf{a}_k \frac{t^2}{2}$
- $\mathbf{d} = \mathbf{x}_i(t) - \mathbf{x}_j(t)$
- Contact when $d = r_i + r_j$
- If $\mathbf{a}_i = \mathbf{a}_j$:
  $U^2 t_c^2 + 2\mathbf{d} \cdot \mathbf{U} t_c + d^2 - (r_i + r_j)^2 = 0$
  Otherwise, quartic equation!



Solving a contact: the normal component



Momentum conservation

$$(m_i\mathbf{u}_i + m_j\mathbf{u}_j)\cdot\mathbf{n} = (m_i\mathbf{u}_i' + m_j\mathbf{u}_j')\cdot\mathbf{n}$$

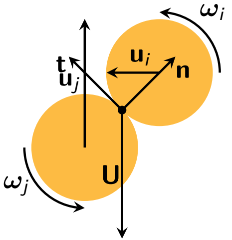
Restitution coefficient

$$\mathbf{U}'\cdot\mathbf{n} = -r\mathbf{U}\cdot\mathbf{n}$$

Update velocities

$$\mathbf{u}_i' = \mathbf{u}_i + \Delta u_n\mathbf{n}$$
$$\mathbf{u}_j' = \mathbf{u}_j - \Delta u_n\mathbf{n}$$

Solving a contact: what about the tangential part?



$$\mathbf{U} = \mathbf{u}_i - \mathbf{u}_j + \omega_i \times \mathbf{r}_i - \omega_j \times \mathbf{r_j}$$

$$I(\omega_i' - \omega_i) = m\mathbf{r}_i \times (\mathbf{u}_i' - \mathbf{u}_i)$$
$$I(\omega_j' - \omega_j) = m\mathbf{r}_i \times (\mathbf{u}_j' - \mathbf{u}_j)$$

Momentum conservation

$$(\mathbf{u}_i + \mathbf{u}_j) \cdot \mathbf{t} = (\mathbf{u}_i' + \mathbf{u}_j') \cdot \mathbf{t}$$

Restitution coefficient

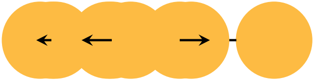
$$\mathbf{U}' \cdot \mathbf{t} = -s\mathbf{U} \cdot \mathbf{t}$$

Event-driven methods are fast and easy...

- Contacts are analytically solved
- No stability constraint on the time step
- Trajectories are easy to compute...but with what accuracy?
- Only one collision per time step...but how many time steps?
- Enduring contacts cannot be represented
- Parallelisation is difficult

...but not perfect :-)

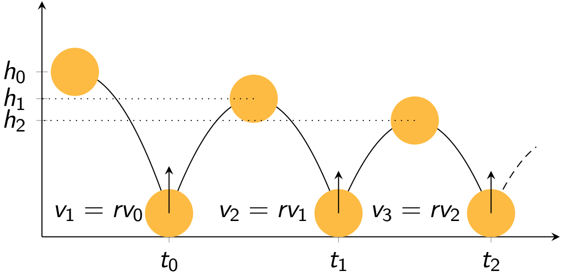
Finite machine precision can lead to unwanted configurations



- Check the past (expensive)
- Modify the diameters (changes dynamics)
- Consider a contact when overlapped and approaching

Bannerman, M. N., Strobl, S., Formella, A., Pöschel, T. (2014). Stable algorithm for event detection in event-driven particle dynamics. Computational Particle Mechanics, 1, 191-198.

The bouncing ball: beware of the inelastic collapse



$$t = \sqrt{\frac{2h_0}{g}}\frac{1+r}{1-r}$$

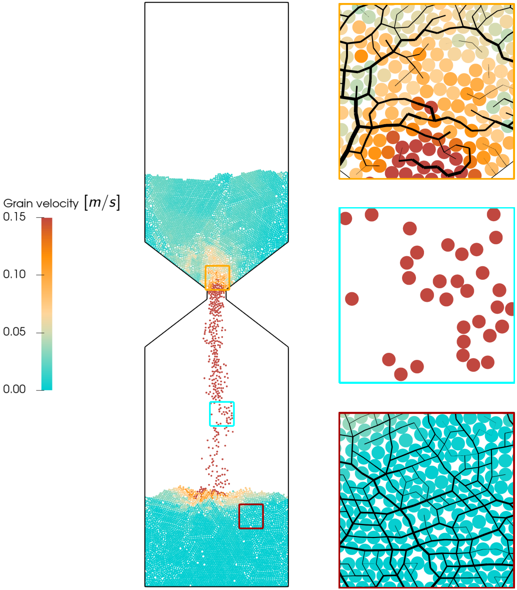
Is inelastic collapse inevitable?

$$r < 7 - 4\sqrt{3}$$

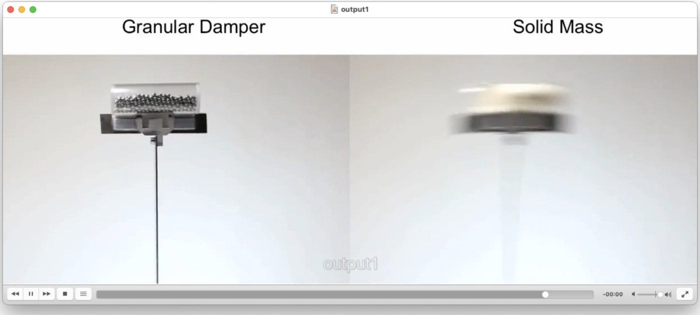
- In 1D, $N_{min} \sim ln(\frac{4}{1-r})/(1-r) \sim 40$ for $r = 0.9$
- Less likely but still possible in 2D/3D
- Solution: dynamic adaptation of r

McNamara, S., Young, W. R. (1992). Inelastic collapse and clumping in a one-dimensional granular medium. Physics of Fluids A: Fluid Dynamics, 4(3), 496-504.

Granular materials can span three states of matter



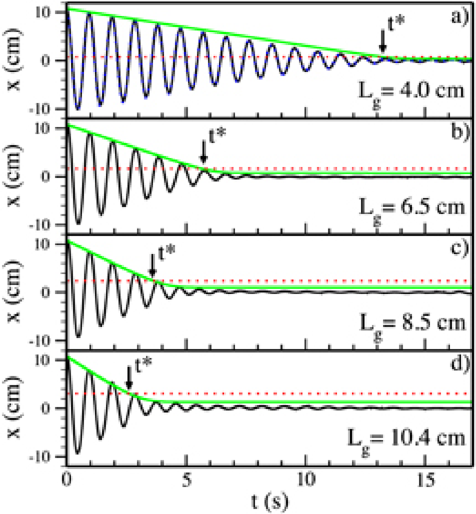
A granular damper



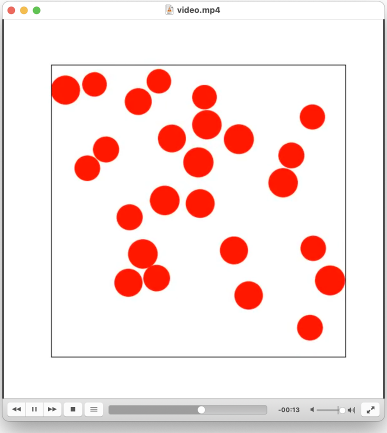
- Large frequency range
- Ages better than rubber and oil
- Unsensitive to temperature
- No need for an anchor

Kollmer, J. E., Sack, A., Heckel, M., Pöschel, T. (2013).
Relaxation of a spring with an attached granular damper. New Journal of Physics, 15(9).

Two damping regimes can be identified in microgravity



First homework : a granular gas



- Implement the Event Driven method
- Square domain
- No gravity
- No tangential interaction

Take-home messages

- The event-driven method is fast and simple
- It is best suited to granular gases
- Global minimization makes it difficult to parallelise
- Inelastic collapse is its major drawback,
  though it can be mitigated with computational tricks