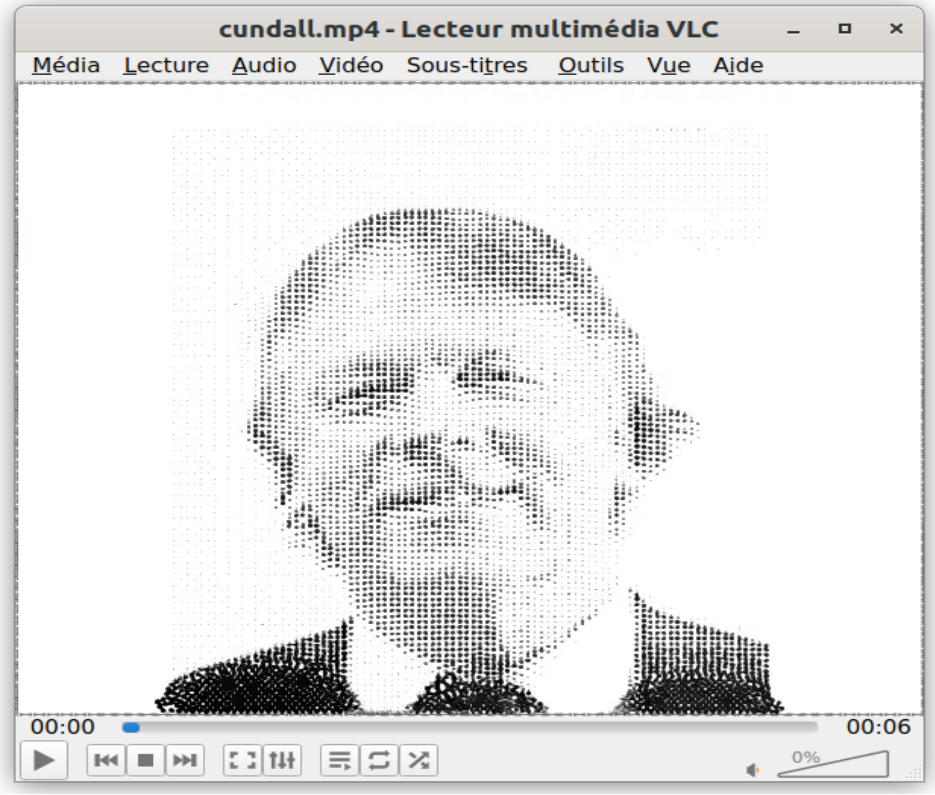
Lecture 3: make it smooth

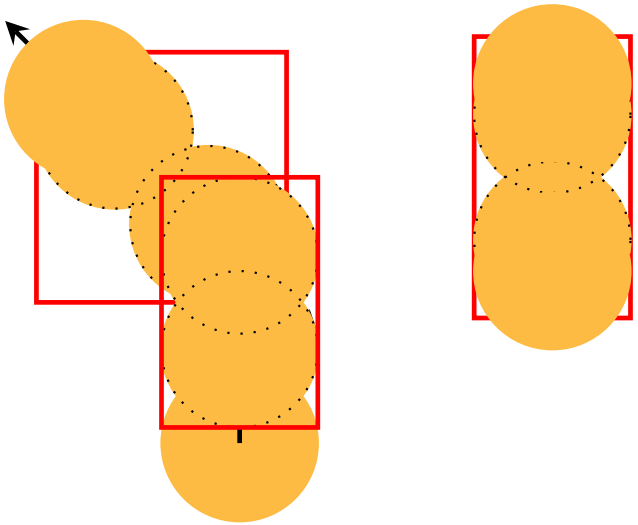


What is the Discrete Elements Method?

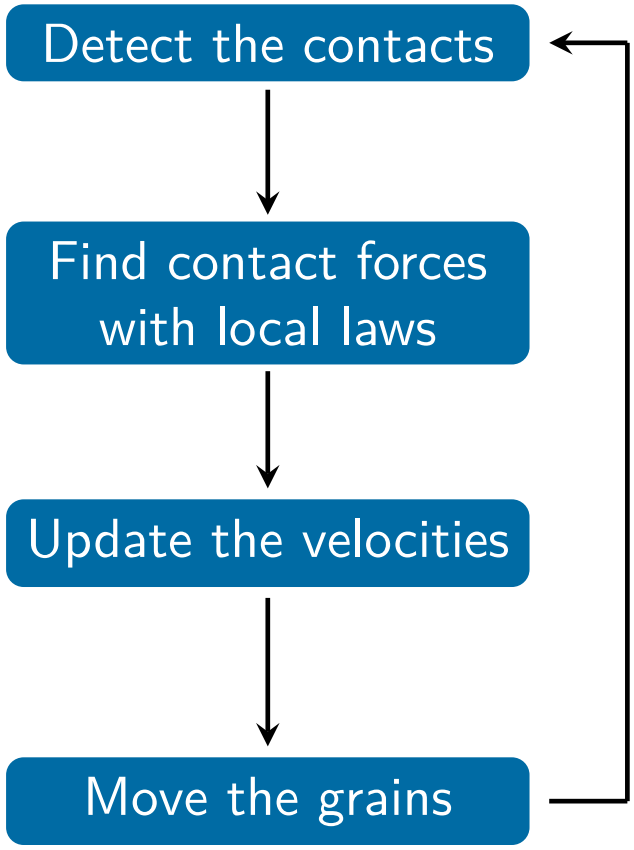
What are its strengths and weaknesses?

Granular jamming and robotics

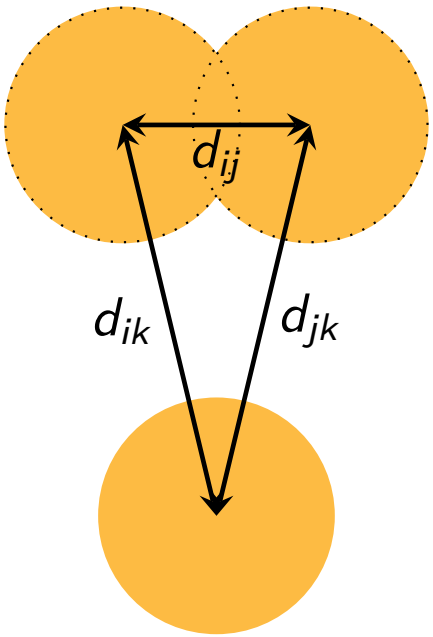
The Discrete Element Method



- The physics of the contacts is modelled at small scale
- The method is explicit with a fixed time step



Contact detection is only based on position



- Contact if $d_{ij} < \overbrace{r_i + r_j}^{\text{Interaction diameter}}$
- The naive approach is extremely costly $\rightarrow O(n^2)$
- Using tree structures improves performance $\rightarrow O(n \cdot log(n))$
- Cohesive contacts can be included by increasing the interaction diameter

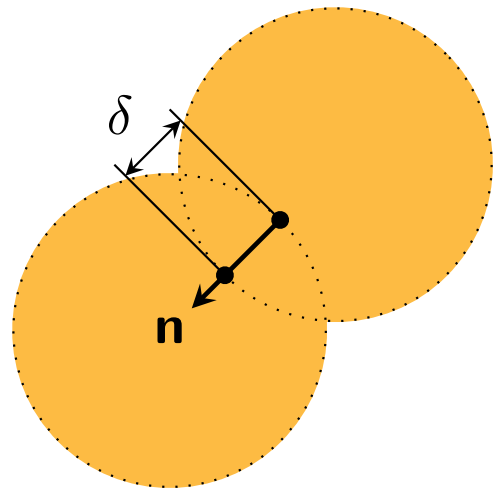
Many models exist to compute the contact force...

- Hooke's law

$$f_n = \begin{cases} -k_n\delta, & \delta < 0 \\ 0, & \delta \geq 0 \end{cases} \quad k_n \simeq 2Er$$

- Hertz law

$$f_n = \begin{cases} k_n|\delta|^{\frac{3}{2}}, & \delta < 0 \\ 0, & \delta \geq 0 \end{cases} \quad k_n = \frac{2}{3}\frac{E}{1-\nu^2}\sqrt{\frac{r}{2}}$$



- Kelvin-Voigt

$$f_n = \begin{cases} \max(0, -k_n\delta - \gamma u_n), & \delta < 0 \\ 0, & \delta \geq 0 \end{cases}$$

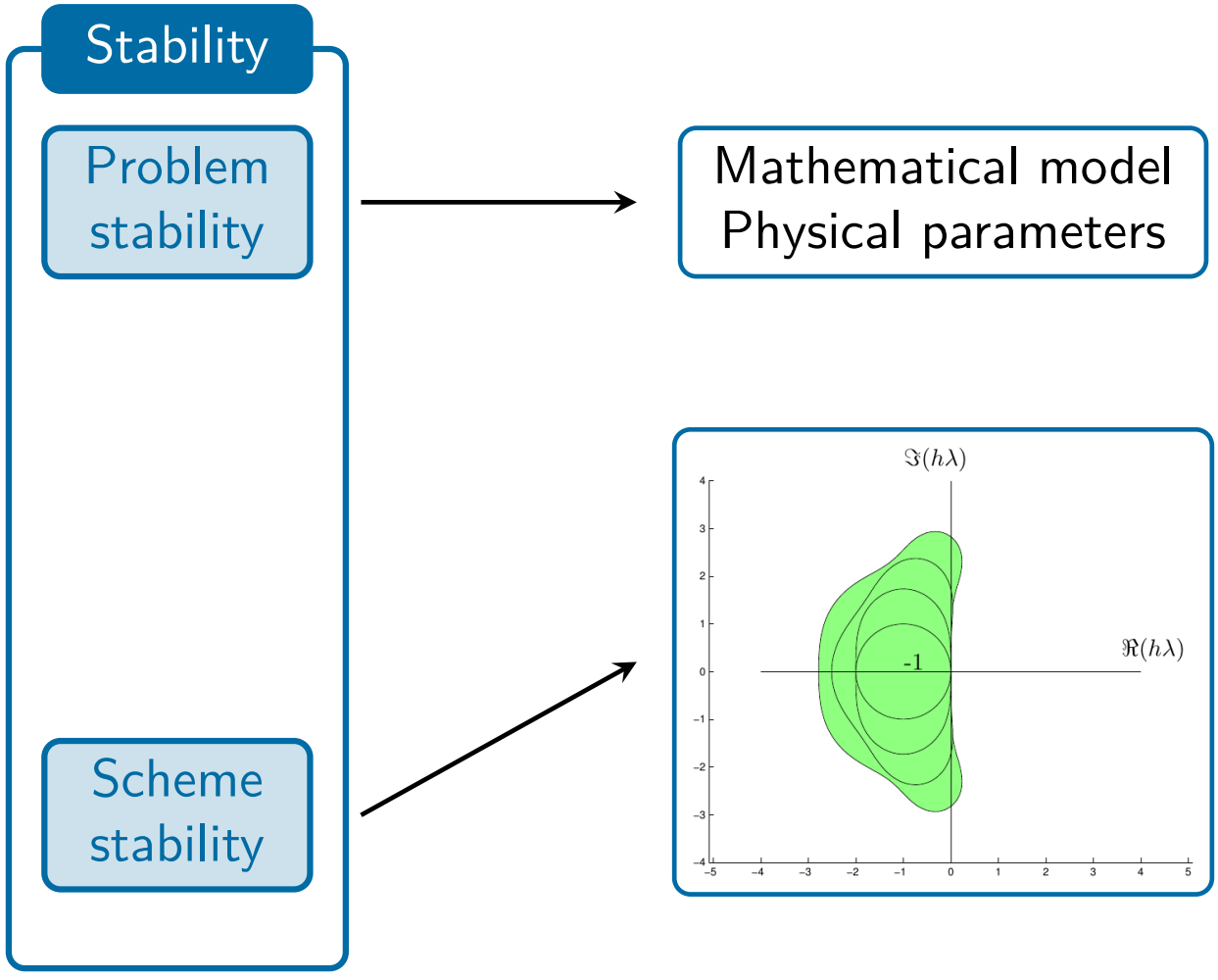
...with many parameters!

The discrete element method is powerful...

- The physics of contacts is represented... but calibration is required
- Any force can be included... but with a lot of parameters
- Solid, liquid and gaseous states can be simulated
- Parallelisation is achievable
- An explicit method is easy to implement... but can be unstable
- The computational cost can be prohibitive

...but not perfect :-)

Warning, explicit content!



When a numerical problem is unstable, the error can grow unbounded!

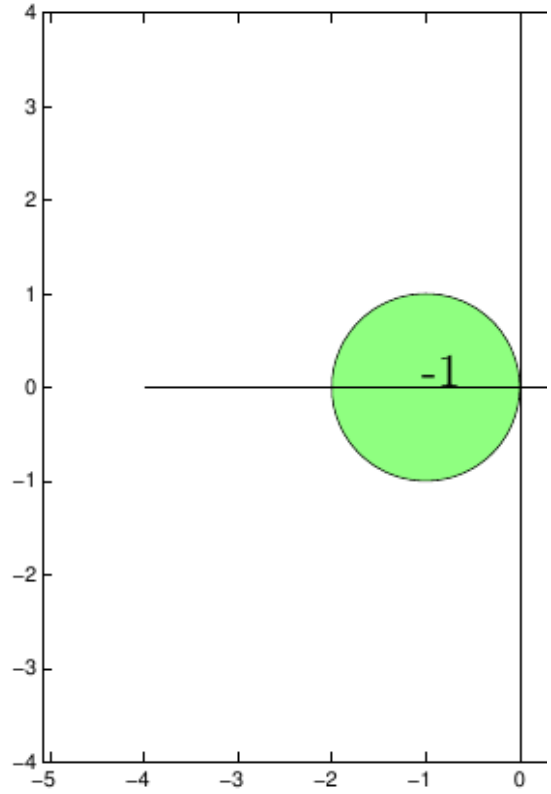
Warning, explicit content!

- Example with explicit Euler

  Hooke's law : always unstable

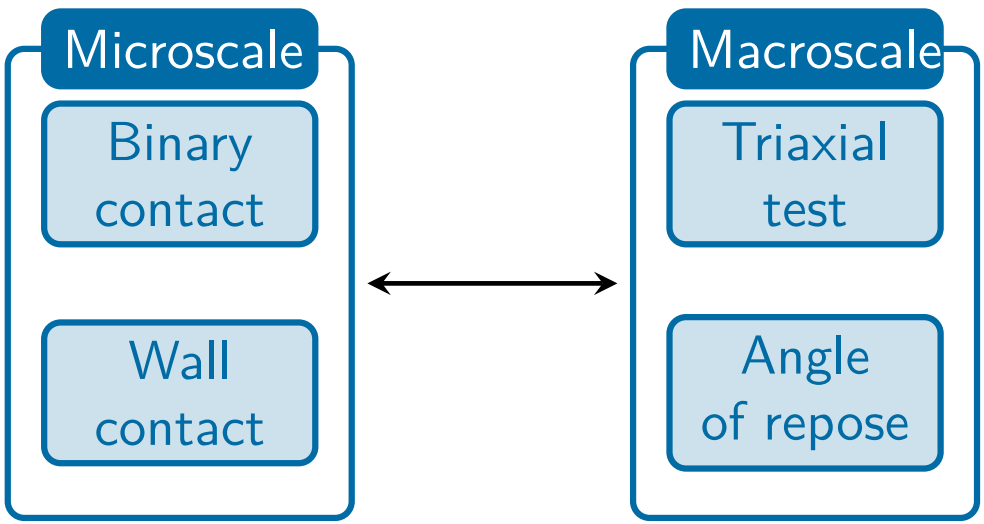
- Not only are usual schemes more complex, but so are the contact models!

- Different estimation techniques can be used

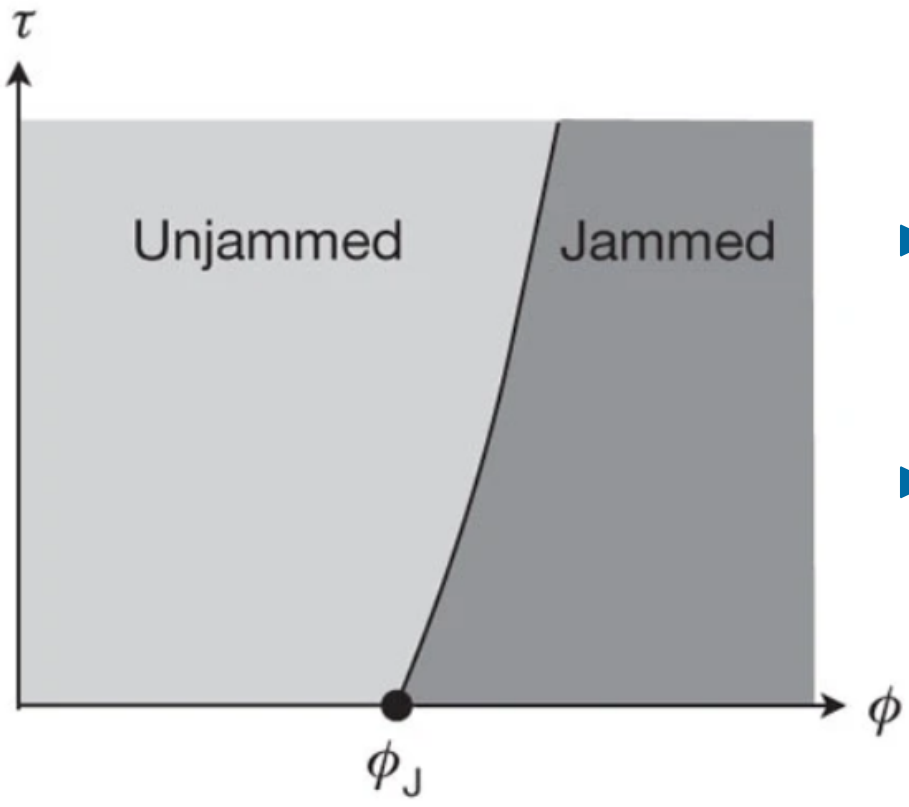


What about the physical parameters?

- The values of physical parameters are often far from theory
- They require calibration to obtain trustworthy results



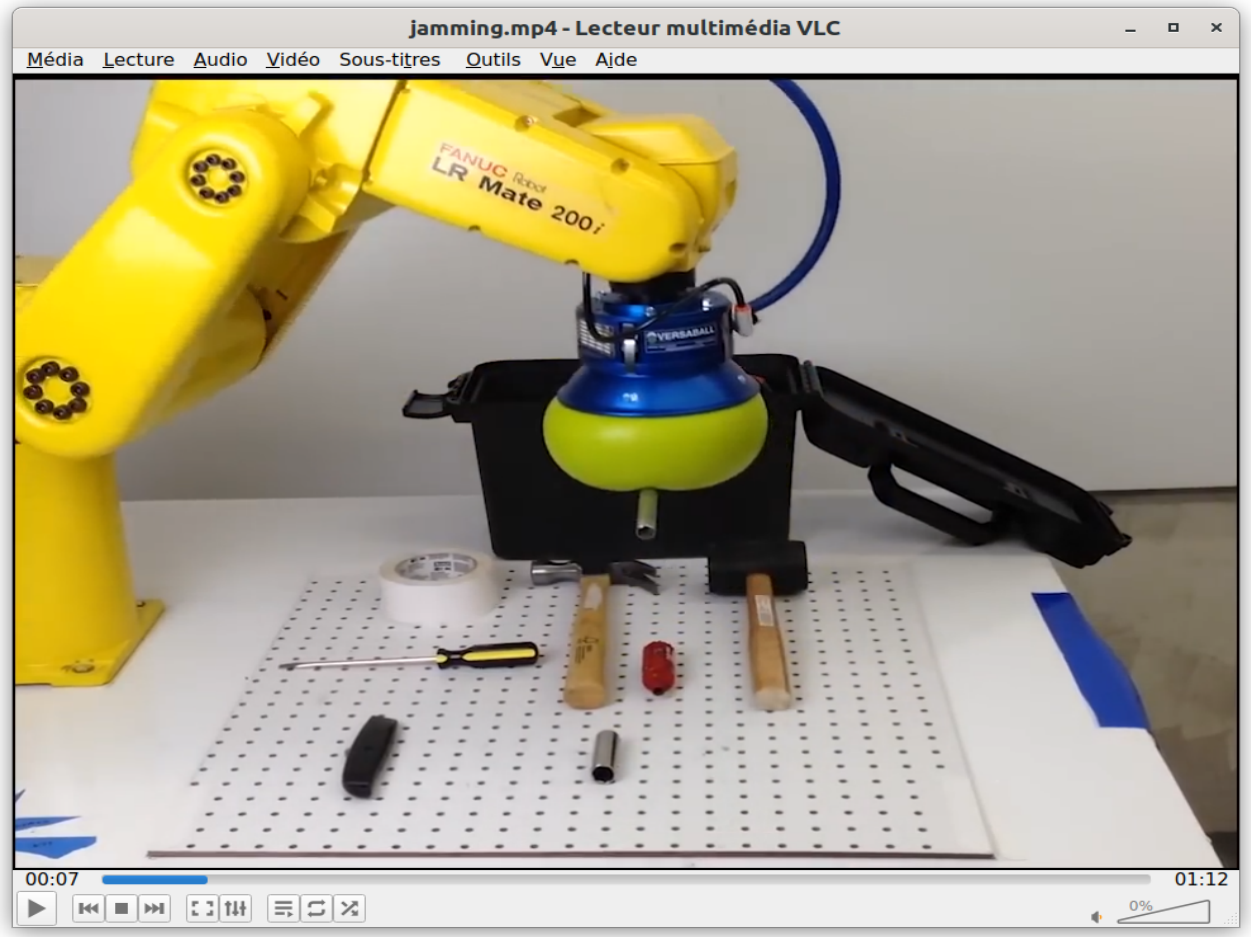
The jamming of granular materials



- Geometric constraints prevent the grains from moving
- Applied shear can move them up to a certain limit

Bi, D., Zhang, J., Chakraborty, B., Behringer, R. P. (2011). Jamming by shear. Nature, 480(7377), 355-358.

Jamming is very useful in robotics



Brown, E., Rodenberg, N., Amend, J., Mozeika, A., Steltz, E., Zakin, M. R., Jaeger, H. M. (2010). Universal robotic gripper based on the jamming of granular material. Proceedings of the National Academy of Sciences, 107(44), 18809-18814.
Video: empire robotics - versaball

Take-home messages

- The Discrete Element Method uses a smooth-sphere approach
- It is an explicit method that requires small time steps
- All states of granular materials can be simulated
- Any kind of grain interaction can be included