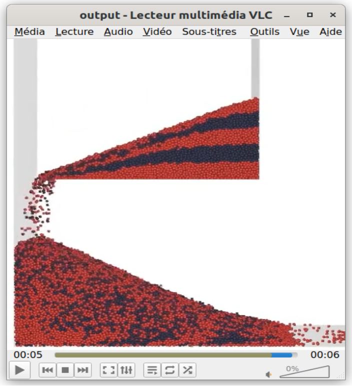
Lecture 4: let there be friction

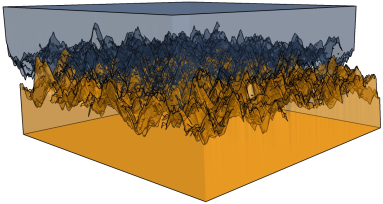


What is friction?

How is it modelled?

The stress tensor
and
The Janssen effect

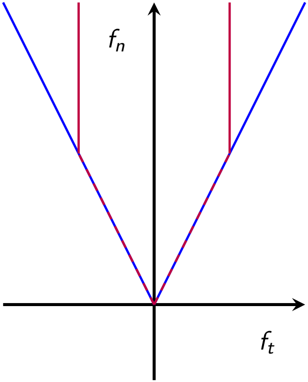
Complex, you said?



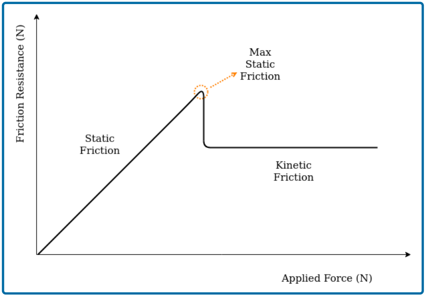
- Friction is a macroscopic phenomenon arising from microscopic ones
  - Geometrical entanglements
  - Chemical bonds
- It is almost independent of the sliding velocity
- It is almost independent of the contact area

Hanaor, D. A., Gan, Y., Einav, I. (2016). Static friction at fractal interfaces. Tribology International, 93, 229-238.

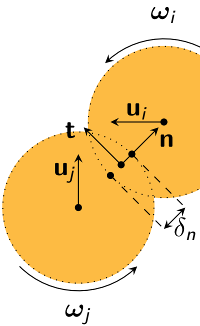
The classical Coulomb model



- The friction force is bounded and lies within the Coulomb cone: $f_t \leq \mu f_n$
- Coulomb-Orowan model: Plasticity further bounds f_t



Friction in DEM models



$$u_n = (\mathbf{u}_i - \mathbf{u}_j + \boldsymbol{\omega}_i \times \mathbf{r}_i - \boldsymbol{\omega}_j \times \mathbf{r}_j) \cdot \mathbf{n}$$
$$u_t = (\mathbf{u}_i - \mathbf{u}_j + \boldsymbol{\omega}_i \times \mathbf{r}_i - \boldsymbol{\omega}_j \times \mathbf{r}_j) \cdot \mathbf{t}$$

- $m_k \frac{d\mathbf{u}_k}{dt} = \sum_\beta \mathbf{f}_k^\beta - m_k \mathbf{g}$

  $\mathbf{I}_k \frac{d\boldsymbol{\omega}_k}{dt} = \sum_\beta \mathbf{r}^\beta \times \mathbf{f}_k^\beta$

- $\mathbf{f}_k^\beta = \mathbf{f}_n + \mathbf{f}_t$
  $\mathbf{f}_n = (k_n |\delta_n|^{3/2} - \gamma_n u_n)\mathbf{n}$
  $\mathbf{f}_t = -k_t \boldsymbol{\delta}_t - \gamma_t u_t \mathbf{t}$

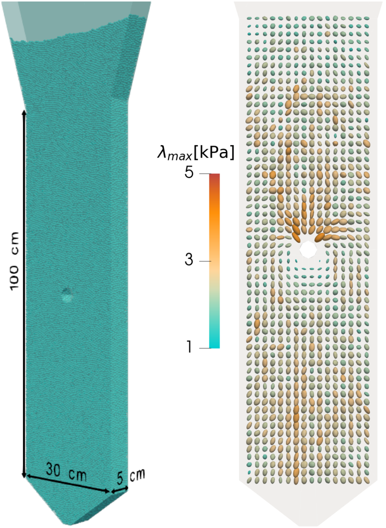
- $\boldsymbol{\delta}_t = u_t \mathbf{t} \Delta t$

  $\boldsymbol{\delta}_t' = \boldsymbol{\delta}_t + u_t' \mathbf{t}' \Delta t - \mathbf{n}'(\boldsymbol{\delta}_t \cdot \mathbf{n}')$

- $\mathbf{f}_t \leftarrow \min(\mu f_n, f_t)\mathbf{f}_t / f_t$

  $\boldsymbol{\delta}_t \leftarrow -\frac{1}{k_t}(\mathbf{f}_t + \gamma_t u_t \mathbf{t})$

The stress tensor in granular materials



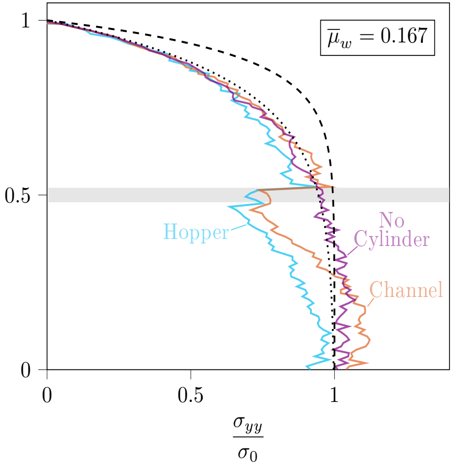
- The stress inside the grains affects the drag force on a rigid body in a granular flow

- The stress tensor can be defined from the contact forces

  $\sigma_{ij} = \frac{1}{V} \sum_{\beta \in V} f_i^{\beta} r_j^{\beta}$

- The principal stresses and directions are the eigen values and vectors of $\boldsymbol{\sigma}$

The Janssen effect



- Pressure saturates with depth
- Hypotheses of Janssen

  $\sigma_r = K\sigma_{yy}$

  $f_t = \mu f_n$

- In a rectangular container

  $\sigma_{yy} = \sigma_0(1 - e^{(y-H)/y_0})$

  $\sigma_0 = \phi\rho g y_0$

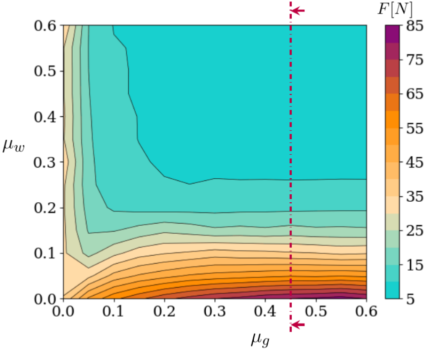
  $y_0 = \frac{1}{2\mu K}\frac{(L-2r)(W-2r)}{L+W-4r}$

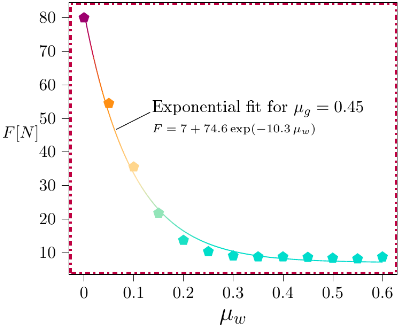
- Added load

  $Q_0 e^{(y-H)/y_0}$

Janssen, H.A. (1895). Versuche uber getreidedruck in silozellen. Zeitschr. d. Vereines deutscher Ingenieure 39, 1045.

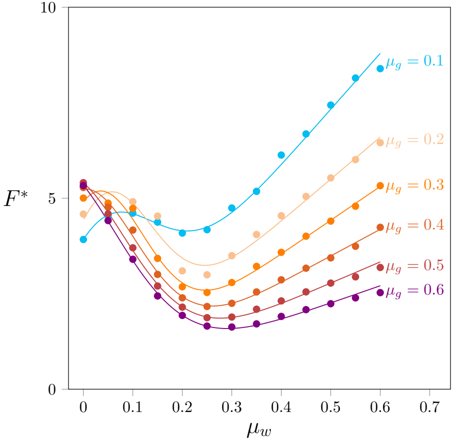
Friction and the drag force





- Wall friction μ_w, Grain friction μ_g
- Increasing friction decreases the drag force

Friction and the drag force



- $F^* = F/(\sigma_0 DW)$

  $$F^* = A\mu_w/(B + \mu_g) + C\exp(-D(\mu_w - E)^2)$$

- Wall friction increases F^*, but as μ_g increases, the grain network reorganisation dominates

Take-home messages

- Friction can be simply represented with a Coulomb law
- Friction further complexifies DEM
  because of the memory requirements
- A stress tensor can be computed
  from the contacts between the grains
- Friction strongly affects the stress state
- Friction is a key property for numerical models of granular materials