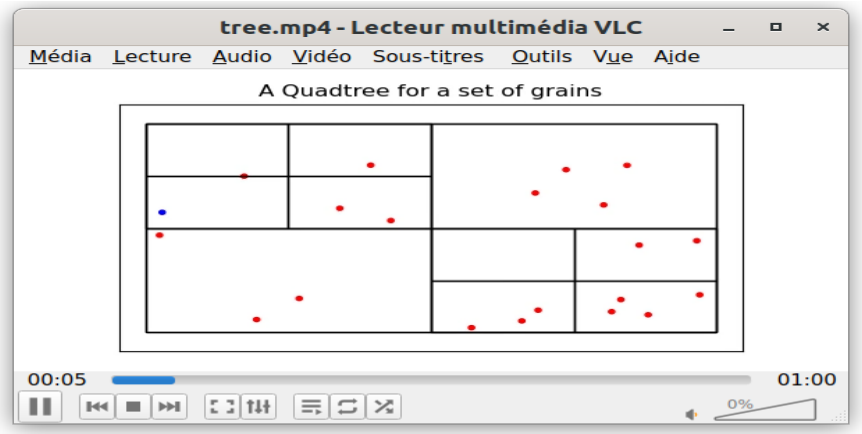
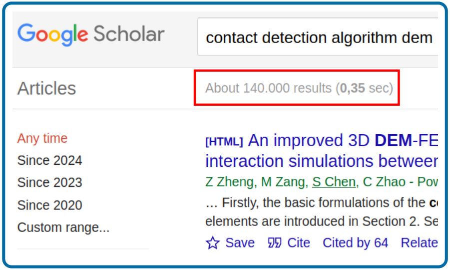
Lecture 8: Where are my contacts?



Broad phase

Narrow phase

Contact detection is a crucial step



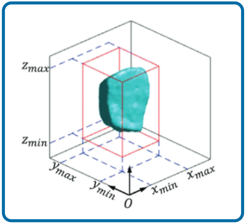
- Contact detection is performed at each time step
- Up to 80% of the computational cost in DEM
- 10% in NSCD but higher for granular gases
- Also useful in games, CAD, VR, FEM, Robotics,...

Detecting contacts

Broad phase

Spatial sorting

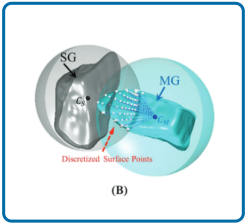
- Sweep and Prune
- Uniform grid
- Quadtree
- BSP, k-d trees,...



Pair testing

- Position difference
- Surface functions
- Node to surface
- GJK, Common plane, Voronoï,...

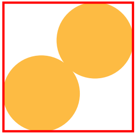
Narrow phase



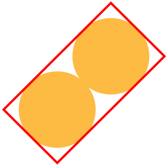
Wang, Xiang, et al. "A spherical-harmonic-based approach to discrete element modeling of 3D irregular particles." International Journal for Numerical Methods in Engineering 122.20 (2021): 5626-5655.

Broad phase: bounding volumes

- **Axis-Aligned Bounding Box**:
  - Easy to compute
  - Easy to detect overlap
  - Not tight

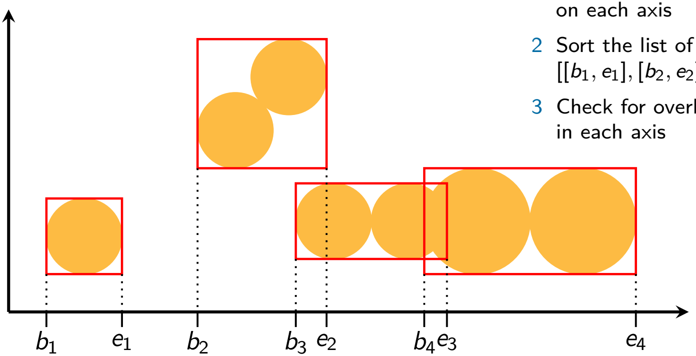


- **Object-Oriented Bounding Box**:
  - Tighter
  - Easy to rotate
  - More costly
  - Overlap more difficult to detect



- **Many other shapes**:
  - Spheres
  - Triangles
  - Ellipses
  - Polygons

Broad phase: Sweep and Prune

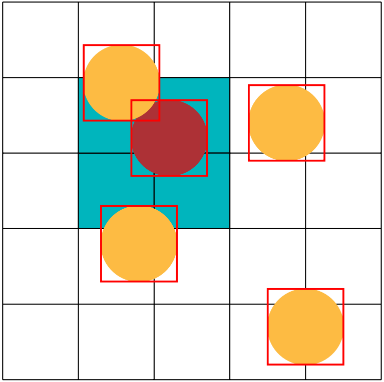


1. Projet the AABBs on each axis
2. Sort the list of projections $[[b_1, e_1], [b_2, e_2], ...]$
3. Check for overlaps in each axis

- Temporal coherence: insertion sort in $O(n)$
- Meshless: no need to optimise cell size

Cohen, Jonathan D., et al. "I-collide: An interactive and exact collision detection system for large-scale environments." Proceedings of the 1995 symposium on Interactive 3D graphics. 1995.

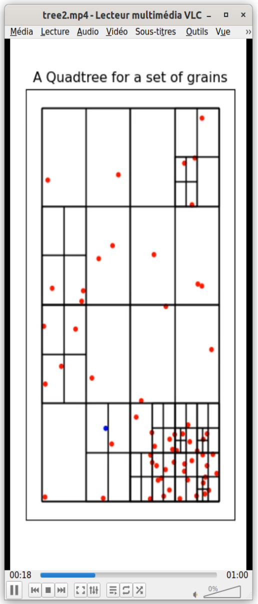
Broad phase: Uniform grid



1. Divide the domain in equallly sized cells
2. Insert the grains in the grid noting the intersecting cells
3. Perform the narrow phase on relevant grains

- Cell size is crucial to the efficiency
- Weak for uneven and highly polydisperse samples
- Hierarchical grids for different grain shapes/sizes

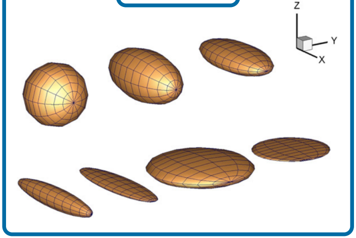
Broad phase: Quadtrees



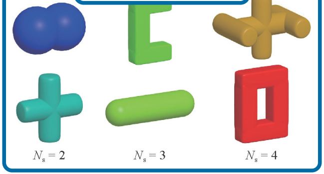
- Each node represents a region of space
- A node is a leaf with $N_g < N_{max}$ grains **or** a parent with 4 children (8 in 3D)
- The tree is built incrementally by inserting grains
- To insert a grain, browse the tree checking for intersection between the nodes and the grain until a leaf is reached
- If $N_g + 1 > N_{max}$, split the leaf
- The narrow phase is performed with grains in neighboring leaves
- $O(Nlog(N))$

Narrow phase: Analytic surfaces

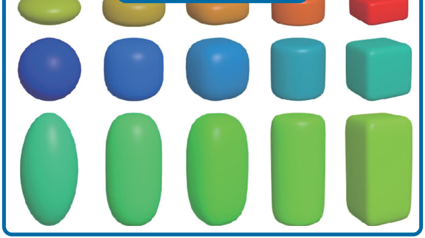
Ellipsoids



Superquadrics



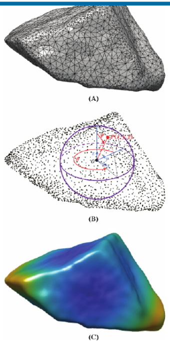
Super-ellipsoids



- Solve $f_i^S(\mathbf{x}) = f_j^S(\mathbf{x})$
- Nonlinear equations: iterative scheme
- Sensitive to first guess
- Other approach: discretisation

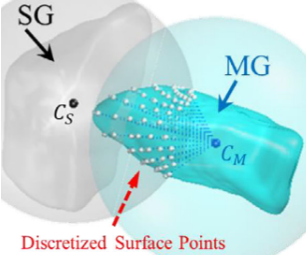
Narrow phase: Parametric surfaces

Spherical harmonics



$$r(\theta, \phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} a_n^m Y_n^m(\theta, \phi)$$

- Discretise the surface of the MG with points



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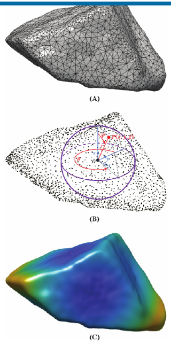
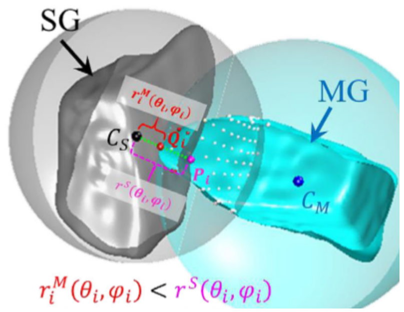
Narrow phase: Parametric surfaces

Spherical harmonics



$$r(\theta, \phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} a_n^m Y_n^m(\theta, \phi)$$

- Compare distances between the SG center and the MG and SG points



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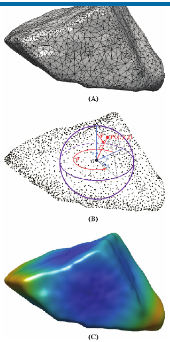
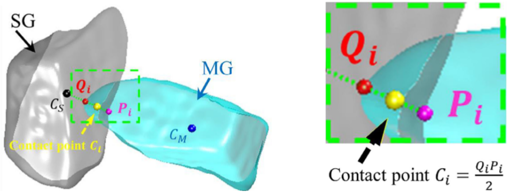
Narrow phase: Parametric surfaces

Spherical harmonics



$$r(\theta, \phi) = \sum_{n=0}^{N} \sum_{m=-n}^{n} a_n^m Y_n^m(\theta, \phi)$$

- The contact point is between the MG and the SG points



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Narrow phase: Polyhedras

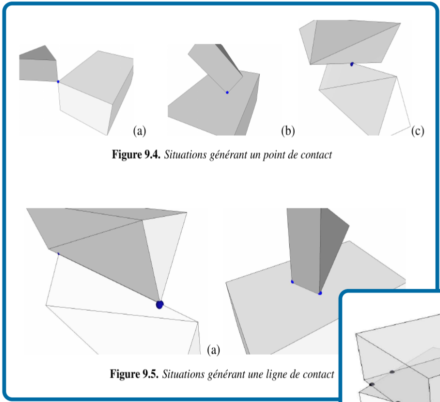
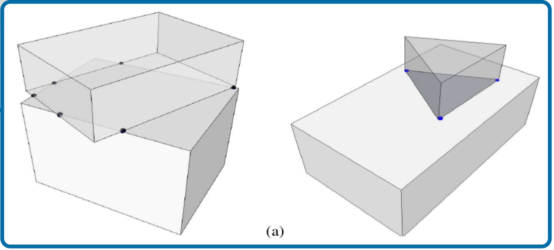


Figure 9.4. *Situations générant un point de contact*

Figure 9.5. *Situations générant une ligne de contact*

- Minkowski difference: $M = A\text{-}B := \{a\text{-}b | a \in A, b \in B\}$
- Intersection if $(0,0)$ is inside M
- Distance between $(0,0)$ and M gives (interpenetration) distance between A and B
- Normal/contact point choice not trivial



Radjaï, F., Dubois, F. (2010). Modélisation numérique discrète des matériaux granulaires. Hermes science publications.

Take-home messages

- Contact detection is performed in two phases
- Many options exist for the broad phase
- The complexity of the narrow phase depends on grain shape
- The relevance of an algorithm depends on the implementing language