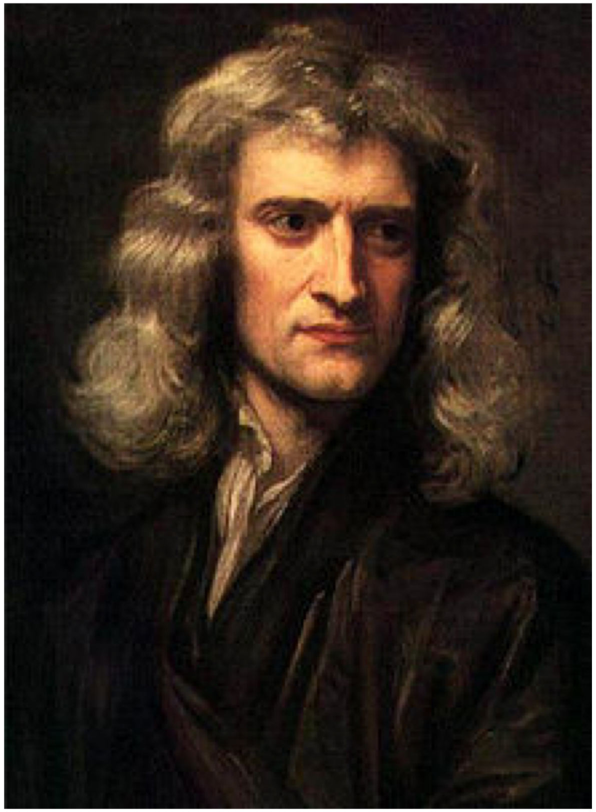
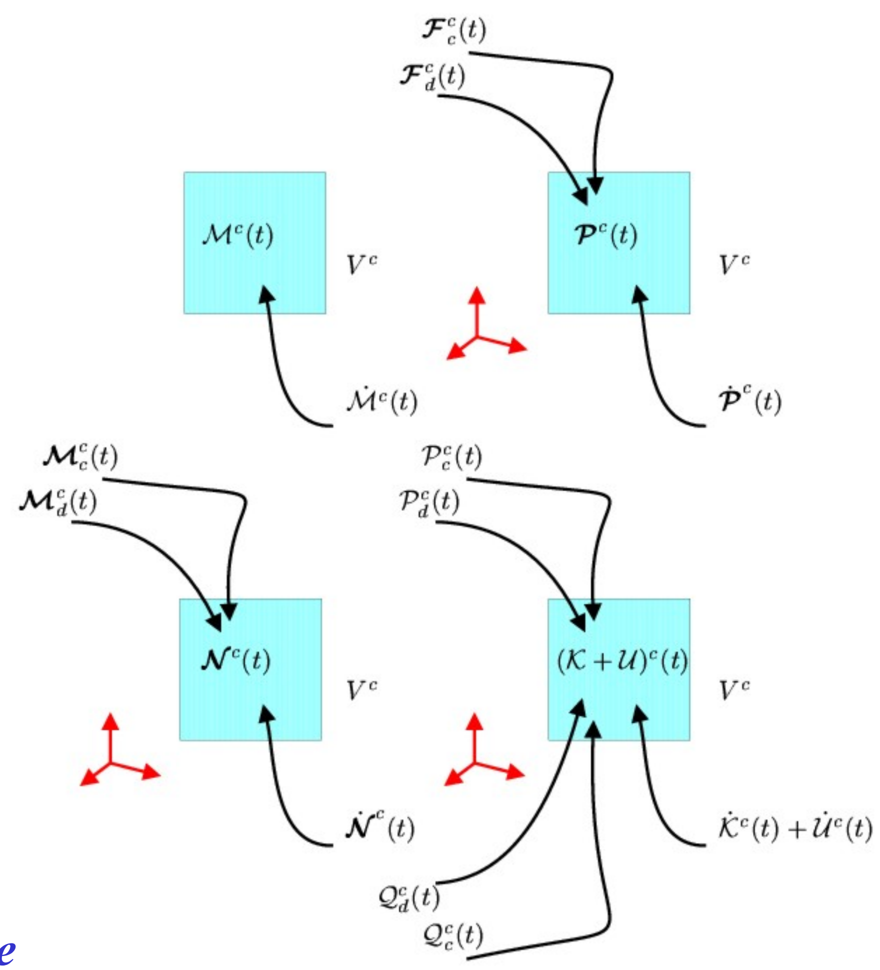
Quelques mots sur les écoulements compressibles...



ou l'erreur de notre ami Newton :-)

Sous un autre angle,
ces lois de conservation…



Forme globale
pour des volumes de controle

…dont on peut déduire des formes locales

$$\begin{aligned}\mathbf{t}(\mathbf{n}) &= \boldsymbol{\sigma}^T \cdot \mathbf{n}\\ \boldsymbol{\sigma} &= \boldsymbol{\sigma}^T\\ q(\mathbf{n}) &= -\mathbf{q}\cdot\mathbf{n}\end{aligned}
\qquad
\begin{aligned}\frac{D\rho}{Dt} + \rho\boldsymbol{\nabla}\cdot\mathbf{v} &= 0,\\ \rho\frac{D\mathbf{v}}{Dt} &= \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} + \rho\mathbf{g}\\ \rho\frac{DU}{Dt} &= \boldsymbol{\sigma}:\mathbf{d} + r - \boldsymbol{\nabla}\cdot\mathbf{q}\end{aligned}$$

Forme locale dite non-conservative

$$\begin{aligned}\mathbf{t}(\mathbf{n}) &= \boldsymbol{\sigma}^T \cdot \mathbf{n}\\ \boldsymbol{\sigma} &= \boldsymbol{\sigma}^T\\ q(\mathbf{n}) &= -\mathbf{q}\cdot\mathbf{n}\end{aligned}
\qquad
\begin{aligned}\frac{\partial\rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho\mathbf{v}) &= 0\\ \frac{\partial(\rho\mathbf{v})}{\partial t} + \boldsymbol{\nabla}\cdot(\rho\mathbf{v}\mathbf{v}) &= \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} + \rho\mathbf{g}\\ \frac{\partial(\rho U)}{\partial T} + \boldsymbol{\nabla}\cdot(\rho\mathbf{v}U) &= \boldsymbol{\sigma}:\mathbf{d} + r - \boldsymbol{\nabla}\cdot\mathbf{q}\end{aligned}$$

Forme locale dite conservative

Viscosité de volume

Viscosité de cisaillement

$$
\begin{aligned}
\boldsymbol{\sigma} &= -p\boldsymbol{\delta} + 3\boxed{\hat{\kappa}(p,T)}\mathbf{d}^s + 2\boxed{\hat{\mu}(p,T)}\mathbf{d}^d,\\
\mathbf{q} &= -\boxed{\hat{k}(p,T)}\boldsymbol{\nabla}T,\\
\rho &= \hat{\rho}(p,T),\\
H &= \hat{H}(p,T),\\
S &= \hat{S}(p,T).
\end{aligned}
$$

Conductibilité thermique

L'équation de comportement pour l'entropie n'est utile que pour vérifier que le second principe est bien satisfait !

$$
TdS = dH - \frac{dp}{\rho} = dU - \frac{p\,d\rho}{\rho^2},
$$

$$
\begin{aligned}
k &\geq 0,\\
\kappa &\geq 0,\\
\mu &\geq 0.
\end{aligned}
$$

Contraintes à respecter pour satisfaire Clausius-Duhem

Modèle du fluide visqueux newtonien

$$
\begin{aligned}
\boldsymbol{\sigma} &= -p\boldsymbol{\delta} + 3\widehat{\kappa}(p,T)\mathbf{d}^s + 2\widehat{\mu}(p,T)\mathbf{d}^d, \\
\mathbf{q} &= -\widehat{k}(p,T)\boldsymbol{\nabla}T, \\
\rho &= \widehat{\rho}(p,T), \\
H &= \widehat{H}(p,T), \\
S &= \widehat{S}(p,T).
\end{aligned}
$$

Le compte est bon !

conservation locale de la masse	ρ	1
conservation locale de la quantité de mouvement	$\mathbf{v}$	3
conservation locale de l'énergie	T	1
constitution pour les contraintes	$\boldsymbol{\sigma}$	6
constitution pour le flux calorifique	$\mathbf{q}$	3
constitution pour la masse volumique	p	1
constitution pour l'enthalpie	H	1
constitution pour l'entropie	S	1

Remarque : si une équation de comportement pour l'enthalpie est donnée… on en déduit automatique l'énergie interne et vice-versa.

$$U = -\frac{p}{\rho} + H$$

Equations d'état pour des écoulements compressibles…

Lois de conservation	**Lois de comportement**
Conditions aux limites	

$$
\begin{aligned}
\boldsymbol{\sigma} &= -p\boldsymbol{\delta} + 3\hat{\kappa}(p,T)\mathbf{d}^s + 2\hat{\mu}(p,T)\mathbf{d}^d,\\
\mathbf{q} &= -\hat{k}(p,T)\boldsymbol{\nabla} T,\\
\rho &= \hat{\rho}(p,T),\\
H &= \hat{H}(p,T),\\
S &= \hat{S}(p,T).
\end{aligned}
$$

Equations d'état ?

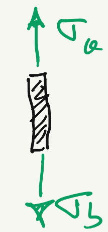
Modèle du fluide visqueux Newtonien

1 CONSERVATION DE LA QUANTITÉ DE MOUVEMENT

$$\underbrace{\mathfrak{p}(\underset{\sim}{x},t)}_{\text{DENSITÉ DE QUANTITÉ DE MVT}} = \rho(\underset{\sim}{x},t)\, \underset{\sim}{v}(\underset{\sim}{x},t)$$

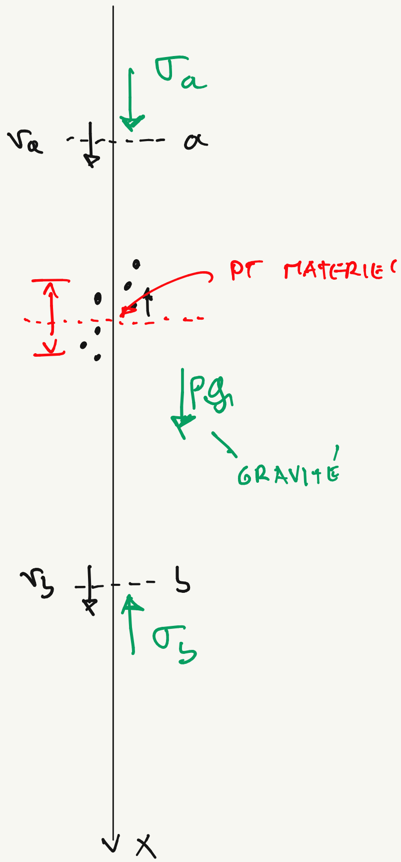
$$\rho = \frac{M}{\int_V dV}$$

$$\mathfrak{p} = \frac{\underset{\sim}{P}}{\int_V dV}$$



TRACTION

$$\underset{\sim}{v}(x,t) \triangleq \frac{\mathfrak{p}(\underset{\sim}{x},t)}{\rho(\underset{\sim}{x},t)}$$



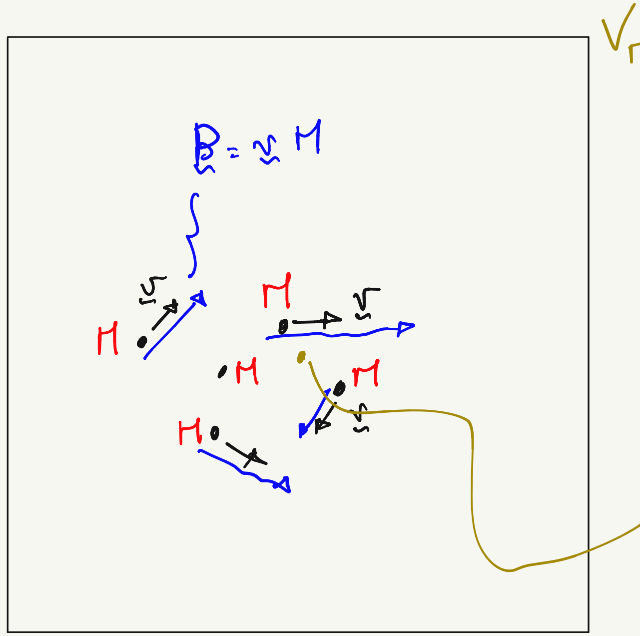
$\forall a,b$

$$\frac{d}{dt}\int_a^b \rho v\, dx = \underbrace{\rho_a v_a^2 - \rho_b v_b^2}_{-[\rho v^2]_a^b} + \overbrace{(-\sigma_a) - (-\sigma_b)}^{[\sigma]_a^b} + \int_a^b \rho g\, dx$$

σ COMPRESSION < 0

TRACTION > 0

$$\boxed{\int_a^b \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2)\, dx = \int_a^b \frac{\partial \sigma}{\partial x} + \rho g \; dx}$$



MOYENNE DE TOUTES LES MASSES

$$\rho(x,t) \triangleq \frac{M}{V_r}$$

$$\underset{\sim}{p}(\rho,t) \triangleq \frac{\underset{\sim}{p}}{V_r}$$

MOYENNE DE LA QUANTITÉ DE MVT

$$v(x,t) \triangleq \frac{\underset{\sim}{p}(x,t)}{\rho(x,t)}$$

ET POURQUOI ON FAIT PAS

$$\underset{\sim}{v}(x,t) = \frac{\underset{\sim}{v}}{V_r}$$

FORME CONSERVATIVE

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\underbrace{\frac{\partial}{\partial t}\rho v + \frac{\partial}{\partial x}(\rho v^2)}_{(*)} = \frac{\partial \sigma}{\partial x} + \rho g$$

2 EQUATIONS

3 INCONNUES

$\rho \quad v \quad \sigma$

$$\underbrace{\frac{\partial}{\partial t}\rho v + \frac{\partial}{\partial x}(\rho v^2)}_{(*)} = \rho\Big(\underbrace{\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}}_{\frac{Dv}{Dt}}\Big) + v\Big(\underbrace{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v)}_{=0}\Big)$$

$$\underbrace{\frac{\partial}{\partial t}\rho + v\frac{\partial \rho}{\partial x}}_{\frac{D\rho}{Dt}} + \rho\frac{\partial v}{\partial x} = 0$$

FORME NON-CONSERVATIVE

$$\rho\frac{Dv}{Dt} = \frac{\partial \sigma}{\partial x} + \rho g$$

$$\frac{D\rho}{Dt} + \rho\frac{\partial v}{\partial x} = 0$$

$$\rho a = \frac{\partial \sigma}{\partial x} + \rho g$$

$$m\vec{a} = \sum \vec{F}$$

$$x + 4 = 0$$

$$x = -4$$

IL FAUT
UNE EQUATION
DE COMPORTEMENT
POUR CLORE
LE MODÈLE !

$$\boxed{pV = n R_* T}$$

LOI DES
GAZ IDEAUX

$$\rho = \frac{n}{V}$$

$$\sigma = -p + \underbrace{\cancel{\text{FRICTION TERMS}}}_{\text{NEGLIGEABLE}}$$

$$T = K \rho^{\gamma - 1} \qquad \boxed{\gamma = 7/5 \text{ AIR}}$$

CONSTANTE
DU GAZ

$$\overbrace{-\sigma}^{p} = \rho R_* T$$

$$= R_* K \rho^{\gamma}$$

$$= \left(\frac{k^2}{\gamma}\right) \rho^{\gamma} \qquad k = \sqrt{\gamma R_* K}$$

$$-\sigma \quad :-)$$

$$p = A \rho^{\gamma}$$

EQUATION
COMPORTEMENT

WIKIPEDIA EST MON AMI :-)

WIKIPÉDIA

L'Encyclopédie libre

$\boxed{2}$ LA VITESSE DU SON

$$= \sqrt{\frac{c_p}{c_v} R_* T}$$

$$\mathrm{M_{ACH}} = \frac{U}{\sqrt{\frac{c_p}{c_v} R_* T}}$$

Modèle de gaz idéal

$$\boxed{\widehat{\rho}(p, T) \;=\; \frac{p}{R_* T}}$$

Constante du gaz

Un exemple d'équation d'état pour la masse volumique

Ecoulements compressibles

Propagation des sons au sein de l'air : c'est un effet de la compressibilité de l'écoulement.
Caractérisation par le nombre de Mach

Presque comme en thermo…

Concentration molaire [mole/m³]

$$\boxed{\rho} = \boxed{c}\boxed{M}$$

Masse volumique [kg/m³]

Masse molaire [kg/mole]

$$pV = nRT$$

$$c = \frac{n}{V}$$

$$c = \frac{p}{RT}$$

Constante des gaz

$$\rho = \frac{p}{R_* T}$$

Constante du gaz

$$R = 8.314\ [J/moleK]$$

$$R_{*,air} = \frac{R}{M_{air}} = 287\ [m^2/s^2K]$$

Nombre de Mach

$$Ma = \frac{U}{\sqrt{\frac{c_p}{c_v} R_* T}}$$

Born: 18 Feb 1838, Turas, Moravia

Died: 19 Feb 1916, Munchen, Germany

caractérise un écoulement d'un fluide !

$$\frac{\textbf{Vitesse caractéristique du fluide}}{\textbf{Vitesse caractéristique de propagation du son}}$$

3

MODELE DE NEWTON

$T = \text{cst}$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2) = -\frac{\partial p}{\partial x}$$

$$p = R_* T \rho$$

$$\rho(x,t) = \rho_0 + \rho'(x,t)$$

$$p(x,t) = p_0 + p'(x,t)$$

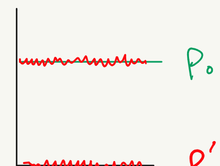
$$v(x,t) = \underbrace{v'(x,t)}_{\substack{\text{PETITES PERTURBATIONS} \\ \mathcal{O}(\varepsilon)}}$$

$$\underbrace{\frac{\partial \rho'}{\partial t}}_{\mathcal{O}(\varepsilon)} + (\underbrace{\rho_0}_{\mathcal{O}(1)} + \cancel{\underbrace{\rho'}_{\mathcal{O}(\varepsilon)}}) \underbrace{\frac{\partial v'}{\partial x}}_{\mathcal{O}(\varepsilon)} + \cancel{\underbrace{v'}_{\mathcal{O}(\varepsilon)} \underbrace{\frac{\partial \rho'}{\partial x}}_{\mathcal{O}(\varepsilon)}} = 0$$

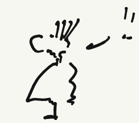
$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v'}{\partial x} = 0$$

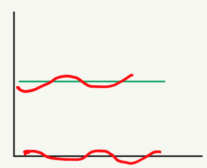
$$\rho_0 \frac{\partial v'}{\partial t} = -R_* T \frac{\partial \rho'}{\partial x}$$

MODELE DES PETITES PERTURBATIONS



$\rho' \ll 1 \quad \frac{\partial \rho'}{\partial x} \gg 1 \quad$ KO :-)





$\rho' \ll 1 \quad \frac{\partial \rho'}{\partial x} \ll 1$

CELA DEPEND DE LA FREQUENCE !!!

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v'}{\partial x} = 0$$

$$\rho_0 \frac{\partial v'}{\partial t} = -R_* T \frac{\partial \rho'}{\partial x}$$

MODELE
DES PETITES
PERTURBATIONS

$$\frac{\partial^2 \rho}{\partial t^2} = -\rho_0 \frac{\partial^2 v}{\partial x \partial t}$$

$$-\rho_0 \frac{\partial^2 v}{\partial x \partial t} = R_* T \frac{\partial^2 \rho}{\partial x^2}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \underbrace{R_* T}_{c^2} \frac{\partial^2 \rho}{\partial x^2}$$

VITESSE
DU SON

$$c = \sqrt{R_* T}$$

KO !

L'ERREUR
DE NEWTON :-)

Calcul de la vitesse du son : l'erreur de Newton !

$$\begin{aligned}
\rho(x,t) &= \rho_0 + \rho'(x,t) \\
v(x,t) &= \cancel{v_0} + v'(x,t) \\
p(x,t) &= p_0 + \underbrace{p'(x,t)}_{\mathcal{O}(\epsilon)}
\end{aligned}$$

Petites pertubations de vitesse, pression et de densité.
Les effets visqueux sont négligeables.
L'air est un gaz idéal.

Que devient la conservation de la masse ?

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho v\right) = 0$$

$$\downarrow$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial v'}{\partial x} + \underbrace{\cancel{\rho' \frac{\partial v'}{\partial x}} + \cancel{v' \frac{\partial \rho'}{\partial x}}}_{\mathcal{O}(\epsilon^2)} = 0$$

Equation linéarisée en termes de petites perturbations

Par paresse de notations, nous noterons désormais les perturbations sans apostrophe :-)

Modèle 1 :
Ecoulement isotherme :-(

$$\left\{\begin{array}{rcl} \dfrac{\partial \rho}{\partial t} + \rho_0 \dfrac{\partial v}{\partial x} & = & 0 \\ \rho_0 \dfrac{\partial v}{\partial t} & = & -\dfrac{\partial p}{\partial x} \end{array}\right.$$

$$\boxed{p = \rho R_* T} \qquad \boxed{T = cst}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} \qquad\qquad \frac{\partial^2 p}{\partial x^2} = R_* T \frac{\partial^2 \rho}{\partial x^2}$$

Modèle 1 :
Ecoulement isotherme :-(

$q = -k \frac{\partial T}{\partial x}$

FLUX CHALEUR

CONDUCTIBILITE THERMIQUE

$$\boxed{p = \rho R_* T} \qquad \boxed{T = cst}$$

$$\frac{\partial^2 \rho}{\partial t^2} = R_* T \frac{\partial^2 \rho}{\partial x^2}$$

La vitesse du son ainsi prédite ne correspond pas aux valeurs mesurées expérimentalement ...

$$c = \sqrt{R_* T}$$

$\boxed{4}$ MODÈLE ADIABATIQUE

$\partial T / \partial x = 0$

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v}{\partial x} = 0$$

$$\rho_0 \frac{\partial v}{\partial t} = - \frac{\partial p}{\partial x}$$

$$p = R_* T \rho$$

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt}$$

$$\frac{p}{RT} c_p \frac{dT}{dt} = \frac{dp}{dt}$$

$$\frac{c_p}{R} \underbrace{\frac{1}{T} \frac{dT}{dt}}_{\frac{d}{dt}(\ln T(t))} = \underbrace{\frac{1}{p} \frac{dp}{dt}}_{\frac{d}{dt}(\ln p(t))}$$

Modèle 2 :
Ecoulement adiabatique :-)

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}$$

Petites pertubations de vitesse, pression et de densité.
Les effets visqueux sont négligeables : pas de dissipation.
L'air est un gaz idéal.

Il s'agit donc d'un écoulement adiabatique réversible ou encore d'un <u>écoulement isentropique</u>

$$\left\{\begin{array}{rcl} \displaystyle\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v}{\partial x} & = & 0 \\ \displaystyle\rho_0 \frac{\partial v}{\partial t} & = & \displaystyle -\frac{\partial p}{\partial x} \\ \displaystyle\rho c_p \frac{dT}{dt} & = & \displaystyle\frac{dp}{dt} \end{array}\right.$$

$$\boxed{p = \rho R_* T} \quad \boxed{\frac{\partial T}{\partial x} = 0}$$

L'air est un mauvais conducteur !
C'est même un bon isolant !

Un peu d'algèbre fastidieuse

$$\boxed{p = \rho R_* T} \qquad \boxed{\frac{\partial T}{\partial x} = 0}$$

$$\rho c_p \frac{dT}{dt} = \frac{dp}{dt}$$

$$\downarrow$$

$$\frac{p}{R_* T} c_p \frac{dT}{dt} = \frac{dp}{dt}$$

$$\frac{d}{dt}\left(\ln\left(T^{c_p}\right)\right) = \frac{c_p}{T}\frac{dT}{dt} = \frac{c_p - c_v}{p}\frac{dp}{dt} = \frac{d}{dt}\left(\ln\left(p^{c_p - c_v}\right)\right)$$

$R = c_p - c_v$

$\gamma = \frac{c_p}{c_v}$

$$\frac{d}{dt}\left(\ln\left(\frac{p^{c_p - c_v}}{T^{c_p}}\right)\right) = 0$$

$$\frac{p^{c_p - c_v}}{T^{c_p}} = C_1$$

$$\boxed{\frac{\rho^\gamma}{p} = C_2}$$

Il faut bien imposer que l' écoulement est parfaitement isentropique !

On retrouve une relation bien connue !

Modèle 2 :
Ecoulement adiabatique :-)

$$\left\{\begin{array}{rcl} \frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v}{\partial x} & = & 0 \\ \rho_0 \frac{\partial v}{\partial t} & = & -\frac{\partial p}{\partial x} \end{array}\right.$$

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 p}{\partial x^2}$$

$$p = A\rho^\gamma$$

$$p = \rho R_* T$$

$$\frac{\partial T}{\partial x} = 0$$

$$\frac{\partial p}{\partial x} = A\gamma\rho^{\gamma-1}\frac{\partial \rho}{\partial x} = \gamma R_* T \frac{\partial \rho}{\partial x}$$

$$\frac{\partial^2 p}{\partial x^2} = \gamma R_* T \frac{\partial^2 \rho}{\partial x^2} + \gamma R_* \cancel{\frac{\partial T}{\partial x}} \frac{\partial \rho}{\partial x}$$

Modèle 2 :
Ecoulement adiabatique :-)

$$\boxed{p = \rho R_* T} \quad \boxed{\frac{\partial T}{\partial x} = 0}$$

$$\frac{\partial^2 \rho}{\partial t^2} = \gamma R_* T \frac{\partial^2 \rho}{\partial x^2}$$

$\Downarrow$

La vitesse du son ainsi prédite <u>correspond</u> bien aux valeurs mesurées expérimentalement ...

$$c = \sqrt{\gamma R_* T} = 342[m/s]$$

$\boxed{5}$

RESOLUTION DU PROBLEME NON-LINEAIRE

$$p = A \rho^{\gamma}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial x} = -\gamma A \rho^{\gamma - 1} \frac{\partial \rho}{\partial x}$$

$\gamma = 1$
SANS PERTE DE GENERALITÉ :-)

$\boxed{\text{SUPPOSONS QUE } \rho = R(v)}$

$$R' \frac{\partial v}{\partial t} + R' \frac{\partial v}{\partial x} v + R \frac{\partial v}{\partial x} = 0$$

$$R \frac{\partial v}{\partial t} + R \frac{\partial v}{\partial x} v + A R' \frac{\partial v}{\partial x} = 0$$

$$\frac{R}{R'} = \frac{A R'}{R}$$

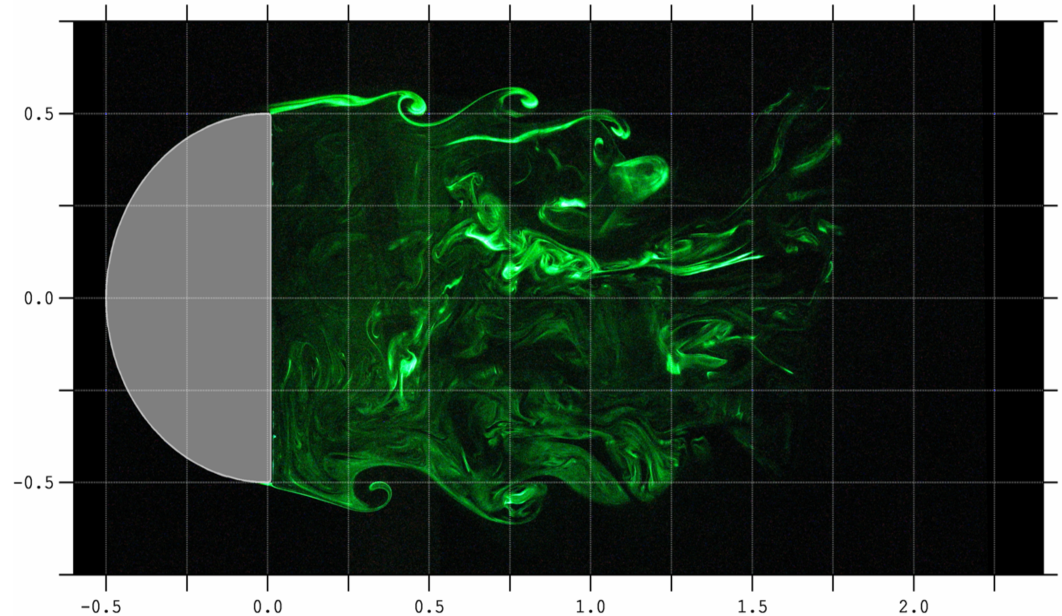
$$(R)^2 = A (R')^2$$

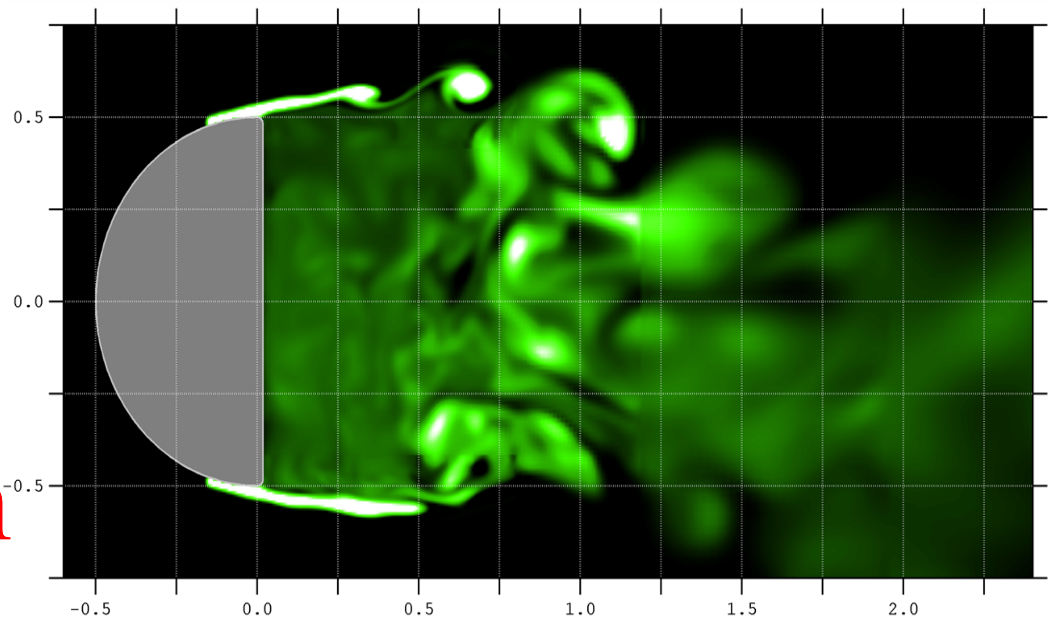
$$\boxed{R(v) = \exp\left(\pm \frac{v}{\sqrt{A}}\right) \rho_*}$$

$$\frac{\partial v}{\partial t} + \left(\pm \sqrt{A} + v\right) \frac{\partial v}{\partial x} = 0$$

Expérience…

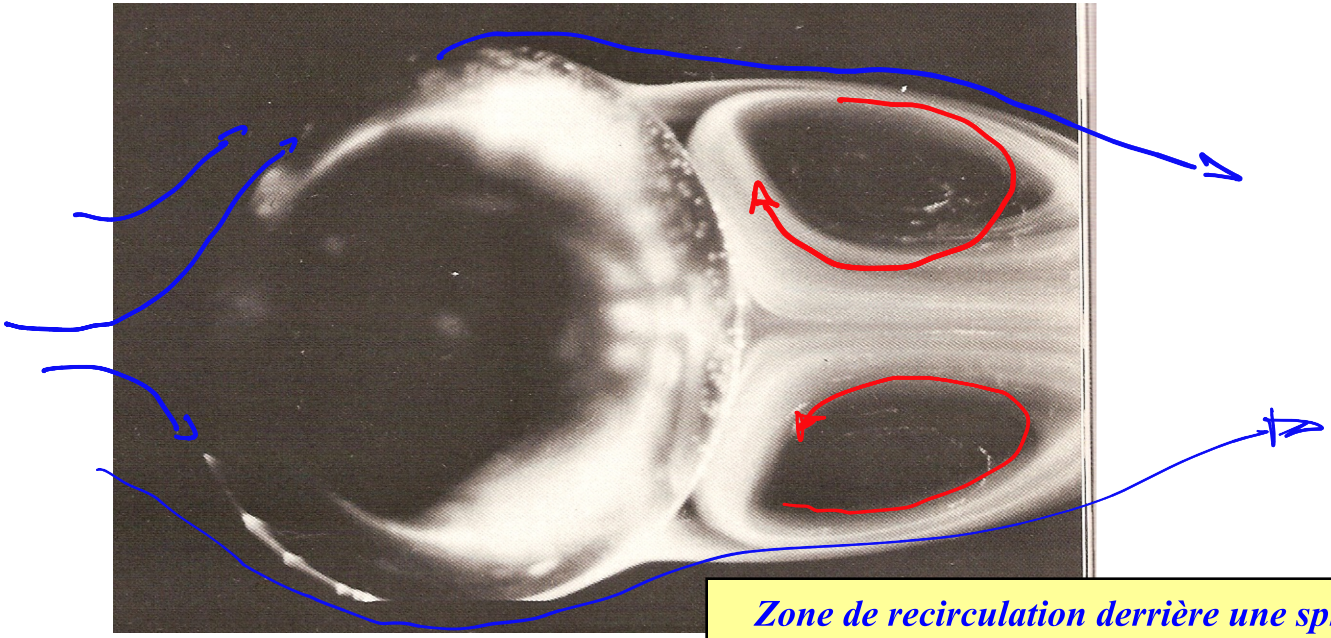
Experimental visualization of the flow past the hemisphere at Re=3,000 in a towing tank. Fluorescein is injected in the boundary layer at the front of the hemisphere.





…et simulation

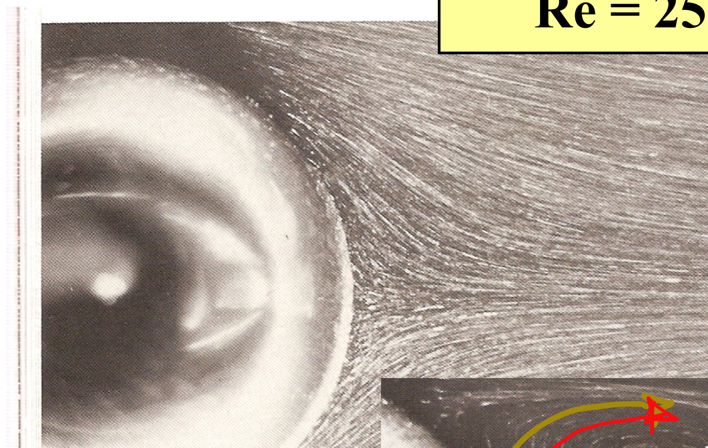
Ecoulement laminaire : Re = 104



Zone de recirculation derrière une sphère Re = 104

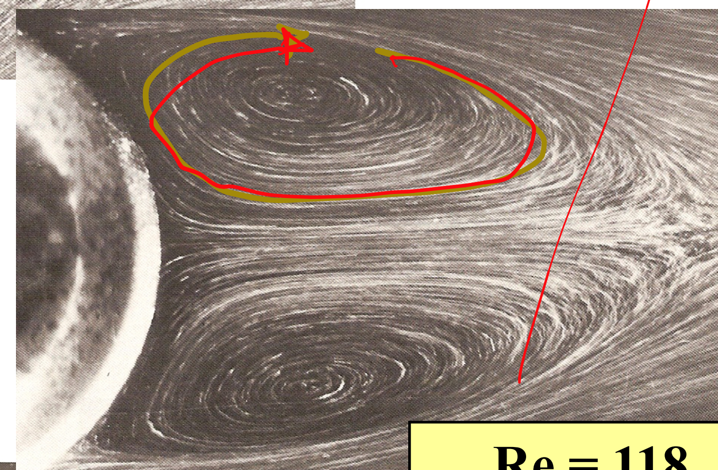
Taneda 1956
(from An Album of Fluid Motion, Van Dyke)

Ecoulement laminaire ?

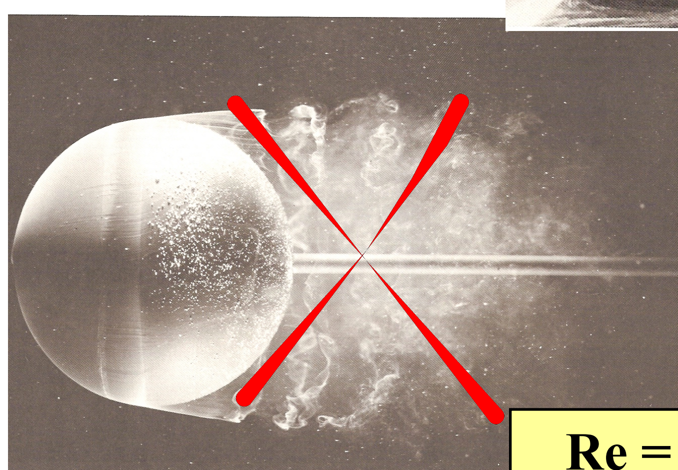


Re = 25

ECOULEMENTS LAMINAIRES



Re = 118



Re = 15000

ECOULEMENT TURBULENT

(Van Dyke, 1982)

Nombre de Reynolds

caractérise un écoulement d'un fluide !

$$Re = \frac{\rho_0 u_0 L}{\mu}$$

VISCOSITE

Born: 23 Aug 1842 in Belfast, Ireland

Died: 21 Feb 1912 in Watchet, Somerset, England

$\boxed{1}$ **EN 3D !**

NOTATIONS TENSORIELLES

$\perp$ SYST COORDONNEES

CART / CYL / SPH / CURVILIGNES

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = \nabla \cdot \underline{\underline{\sigma}} + \rho \mathbf{g}$$

FORME CONSERV.

ECOULEMENT INCOMPRESSIBLE

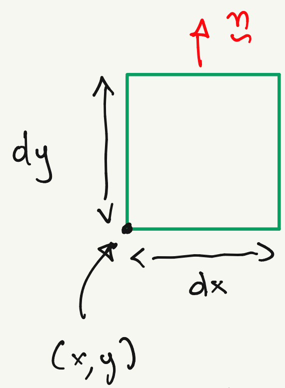
$$\rho = cst$$

$$\nabla \cdot \mathbf{v} = 0$$

FORME NON CONS.

$$\frac{D}{Dt}\rho + \rho \nabla \cdot \mathbf{v} = 0$$

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \underline{\underline{\sigma}} + \rho \mathbf{g}$$

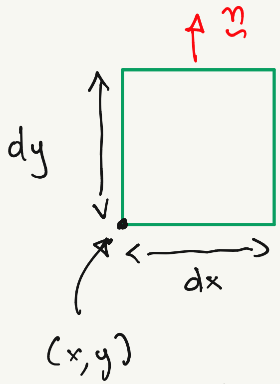


$$\int_V \nabla \cdot (\rho \mathbf{v})$$

THEOREME DE OSTROGRADSKY

$$\frac{d}{dt}\int_\Omega \rho = -\overbrace{\int_{\partial V} \rho \mathbf{v} \cdot \mathbf{n}}$$

NORMALE SORTANTE



$$\int_V \nabla\cdot(\rho\,\underline{v})$$

THEOREME DE OSTROGRADSKY

$$\frac{d}{dt}\int_\Omega \rho = -\overbrace{\int_{\partial V} \rho\,\underline{v}\cdot\underline{n}}$$

NORMALE SORTANTE

$$dx\,dy\,\frac{\partial \rho}{\partial t} = \Big[\overbrace{\rho u(x,y)}^{(x*)} - \overbrace{\rho u(x+dx,y)}^{*}\Big]\,dy + \Big[\rho v(x,y) - \rho v(x,y+dy)\Big]\,dx$$

$dx, dy \to 0$

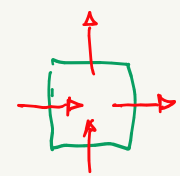
$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho u) - \frac{\partial}{\partial y}(\rho v)$$
$$= -\nabla\cdot(\rho\,\underline{v})$$

CECI EST UN ABUS DE NOTATION "CONVENIENT"

REPRESENTATION DE $\underline{v}$ DANS UN SYST CARTESIEN

$$\underline{v} = \overbrace{(v_x, v_y)} = (u, v)$$

$$\underline{v} = v_x\,\hat{e}_x + v_y\,\hat{e}_y$$
$$= v_r\,\hat{e}_r + e_\theta\,\hat{e}_\theta + e_z\,\hat{e}_z$$



$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\,\underline{v}) = 0$$

$$\begin{bmatrix} \partial/\partial x \\ \partial/\partial y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

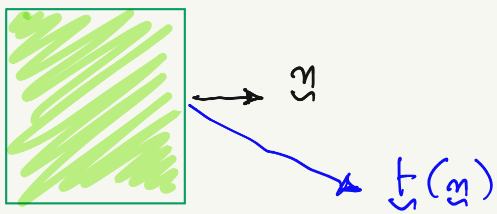
SIMPLE CONTRACTION

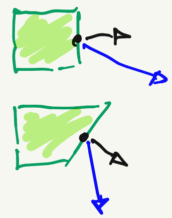
$\neq$ ALGEBRE MATRICIELLE

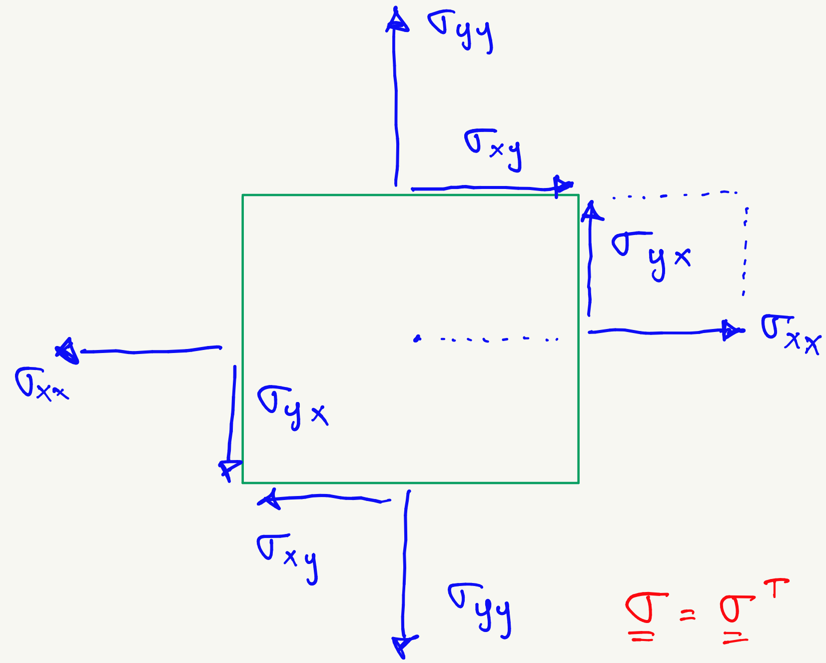
$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0$$

$\boxed{2}$

$\underline{\underline{\sigma}}$ TENSEUR DES CONTRAINTES







$$\underline{t}(\underline{n}) = \underline{\underline{\sigma}}^T \cdot \underline{n}$$

$$\underbrace{\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}^T \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\sigma_{ji}\, n_j} = \begin{bmatrix} \sigma_{xx} \\ \sigma_{yx} \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T$$

PAS DE SOURCE PONCTUELLE DE COUPLE PUR

$$\boxed{\begin{aligned} &\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho u v) \\ &\quad = \frac{\partial}{\partial x}(\sigma_{xx}) + \frac{\partial}{\partial y}(\sigma_{xy}) + \rho g_x \end{aligned}}$$

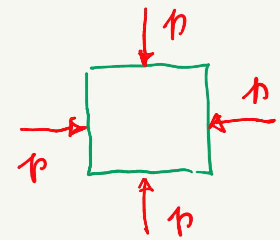
$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot (\rho \underline{v}\, \underline{v}) = \underline{\nabla} \cdot \underline{\underline{\sigma}} + \rho \underline{g}$$

FLUIDE NEWTONIEN

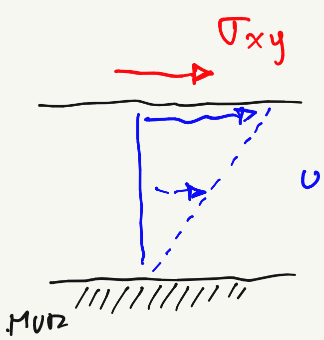
$$\underline{\underline{\sigma}} = -p\,\underline{\underline{\delta}} + 2\mu\,\underline{\underline{d}}$$

$$\begin{bmatrix}\sigma_{xx} & \sigma_{xy}\\ \sigma_{xy} & \sigma_{yy}\end{bmatrix} \qquad \begin{bmatrix}-p & 0\\ 0 & -p\end{bmatrix} + 2\underbrace{\mu}_{\text{VISCOSITÉ}} \underbrace{\begin{bmatrix}d_{xx} & d_{xy}\\ d_{xy} & d_{yy}\end{bmatrix}}_{\text{TENSEUR DES TAUX DE DÉFORMATIONS}}$$

$$\begin{bmatrix}\frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\\ \cdots & \frac{\partial v}{\partial y}\end{bmatrix}$$



$$\underline{\underline{d}} = \frac{1}{2}\left[\underline{\nabla}\,\underline{v} + \underline{\nabla}\,\underline{v}^{T}\right]$$



$$u(y) = \dot{\gamma}\, y$$

$$\frac{\partial u}{\partial y} = \dot{\gamma}$$

$$\sigma_{yx} = \mu \frac{\partial u}{\partial y}$$

$$q_x = -k \frac{\partial T}{\partial x}$$