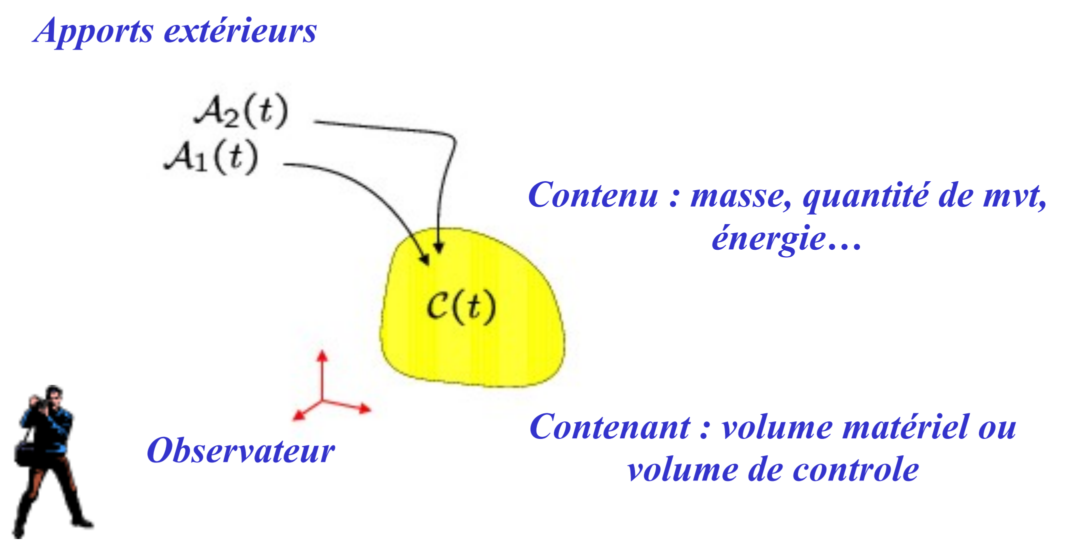
Lois de conservation

$$\frac{d\mathcal{C}}{dt}(t) = \mathcal{A}_1(t) + \mathcal{A}_2(t) + \ldots$$



1 DERIVEE MATERIELLE

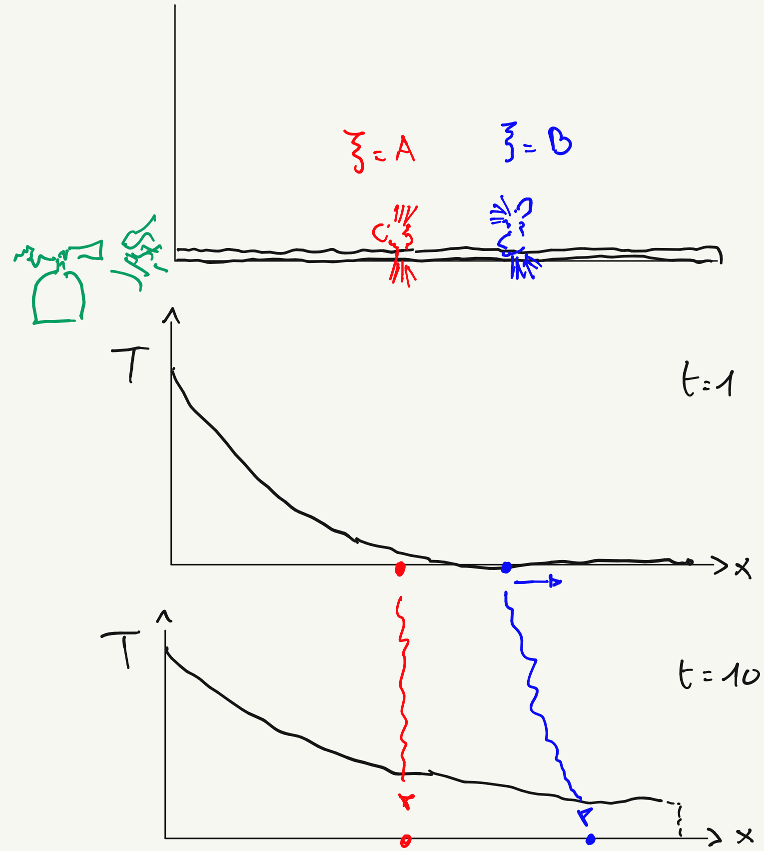
$$f = f_L(\xi, t) = f_E(x(\xi,t), t)$$

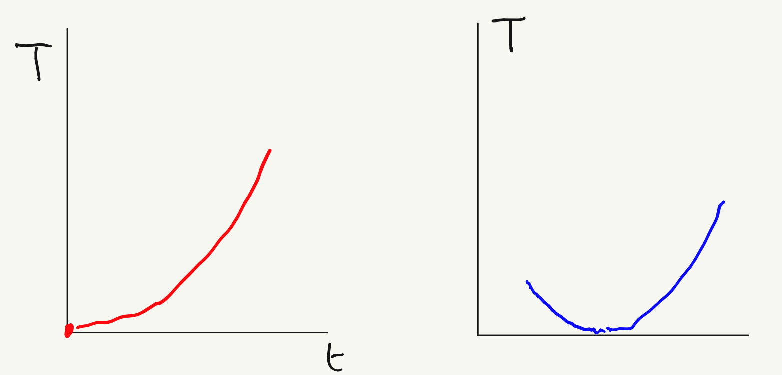
$$\frac{df}{dt} = \frac{\partial f_L}{\partial t} = \frac{\partial f_E}{\partial t} + \frac{\partial f_E}{\partial x} \underbrace{\frac{dx}{dt}}_{v_E = v_L}$$

$$\frac{df}{dt} = \boxed{\frac{\partial f_E}{\partial t}} + \boxed{v_E \frac{\partial f_E}{\partial x}}$$

CHANGEMENT DE f A UNE POSITION DONNEE

CHANGEMENT DE f OBSERVÉ PAR UN POINT MATERIEL QUI SE DEPLACE DANS UN CHAMPS NON CONSTANT !





$\boxed{2}$ UN EXEMPLE POUR VERIFIER

$$\boxed{v_E(x,t) = \frac{2xt}{(1+t^2)} + (1+t^2)}$$

? $\underbrace{x(\xi,t)}_{\text{TRAJECTOIRE}}$

$$\frac{dx}{dt}(t) = \frac{2x(t)\,t}{(1+t^2)} + (1+t^2)$$

$(ab)' = a'b + ab'$

$$x' - \frac{2xt}{(1+t^2)} = (1+t^2)$$

$$\underbrace{\frac{x'}{(1+t^2)} - \frac{2xt}{(1+t^2)^2}} = 1$$

$$\left(\frac{x}{(1+t^2)}\right)' = 1$$

$$\downarrow$$

$$\frac{x}{1+t^2} = t + \xi$$

$x(\xi,0) = \xi$:-)

$$\boxed{x(\xi,t) = (1+t^2)(t+\xi)}$$

2 UN EXEMPLE POUR VERIFIER

$$\boxed{v_E(x,t) = \frac{2xt}{(1+t^2)} + (1+t^2)}$$

$$a_E = \frac{Dv_E}{Dt} = \frac{\partial v_E}{\partial t} + v_E \frac{\partial v_E}{\partial x}$$

$$a_E = \frac{2x}{(1+t^2)} - \cancel{\frac{4xt^2}{(1+t^2)^2}} + 2t + \left[\cancel{\frac{2xt}{(1+t^2)}} + (1+t^2)\right]\frac{2t}{(1+t^2)}$$

$$2t$$

$$= \frac{2x}{(1+t^2)} + 4t$$

$$a_E = 6t + 2\left[\frac{x}{(1+t^2)} - t\right]$$

$$= \frac{2x}{(1+t^2)} + 4t$$

$$x(\xi,t) = t + t^3 + \xi + \xi t^2$$

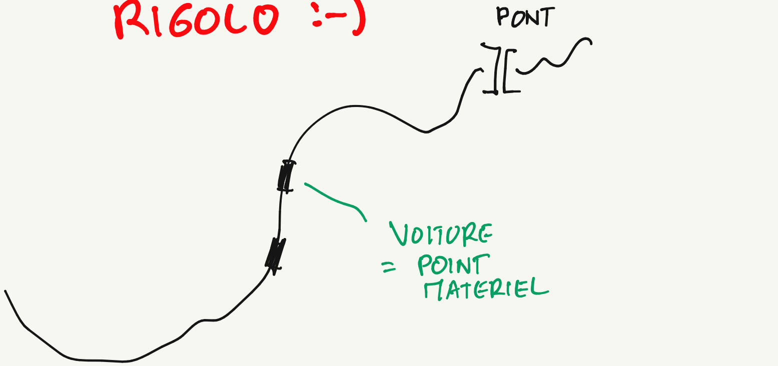
$$v_L(\xi,t) = 1 + 3t^2 + 2\xi t$$

$$a_L(\xi,t) = 6t + 2\xi$$

$$\xi(x,t) = \frac{x}{(1+t^2)} - t$$

$$\boxed{x(\xi,t) = (1+t^2)(t+\xi)}$$

3 UN PETIT MODÈLE RIGOLO :-)



$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0}$$

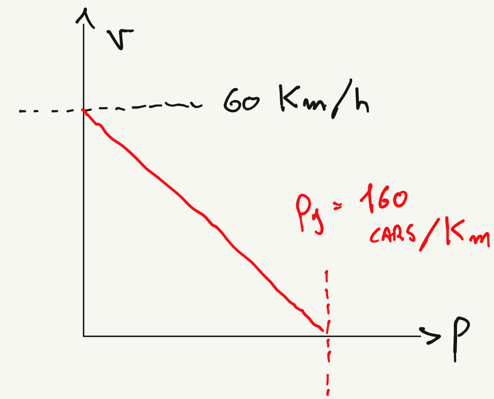
CONSERVATION MASSE

$\rho(x,t)$ DENSITE DE VOITURES [cars / km]

$v(x,t)$ VITESSE

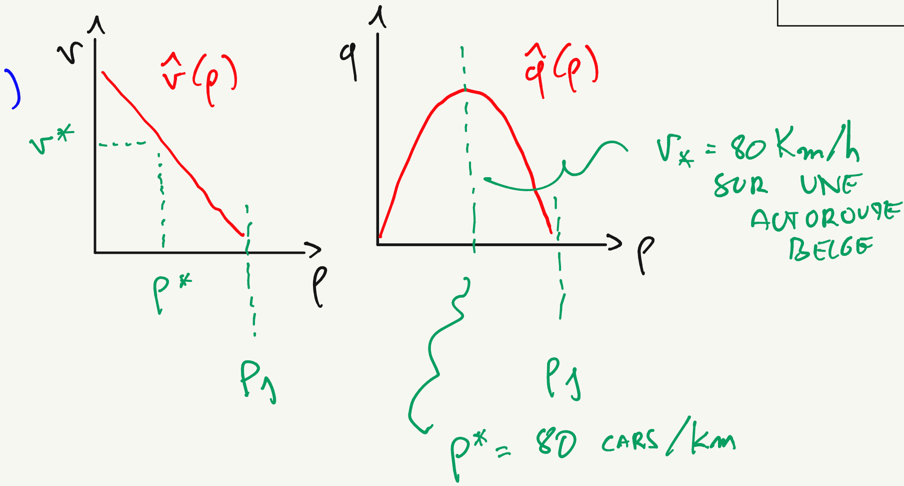
$$\boxed{v = \hat{v}(\rho)}$$

EQUATION DE COMPORTEMENT



FLUX $q = \rho v$

$$q = \hat{q}(\rho) = \rho\, \hat{v}(\rho)$$



MAXIMISER LE FLUX SUR UNE AUTOROUTE !

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\hat{q}(\rho)\right) = 0$$

$$\frac{\partial \rho}{\partial t} + \underbrace{\hat{q}'(\rho)}_{\hat{c}(\rho)} \frac{\partial \rho}{\partial x} = 0$$

EQUATION AUX DERIVEES PARTIELLES
NON-LINEAIRE
HYPERBOLIQUE

PLUS SIMPLE
LE CAS LINEAIRE

$$\frac{\partial \rho}{\partial t} + c \frac{\partial \rho}{\partial x} = 0$$

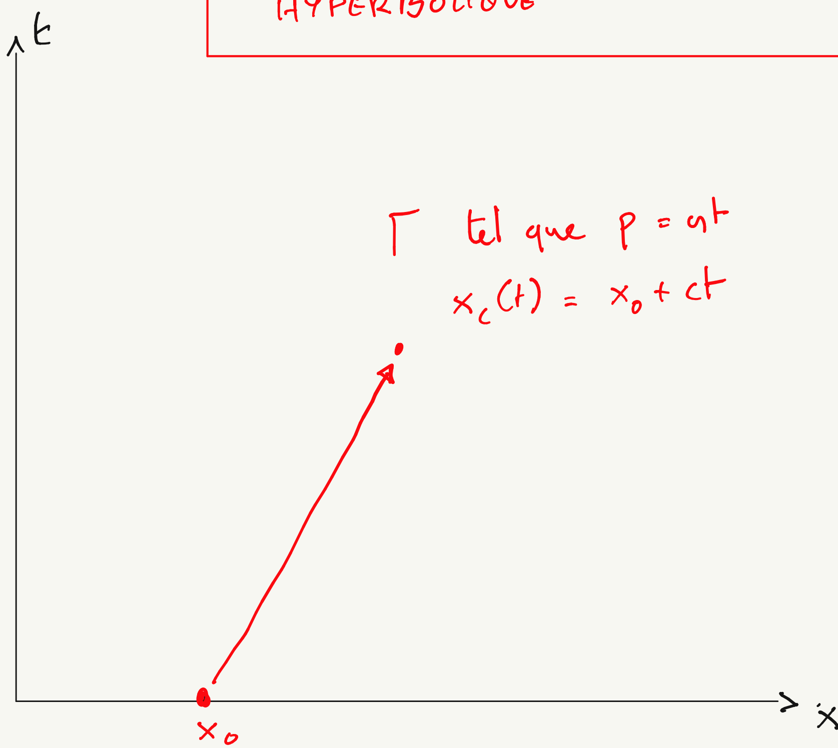
EQUATION DE TRANSPORT

$$\rho(x,t) = \rho_0(x - ct)$$

SOLUTION D'ALEMBERT

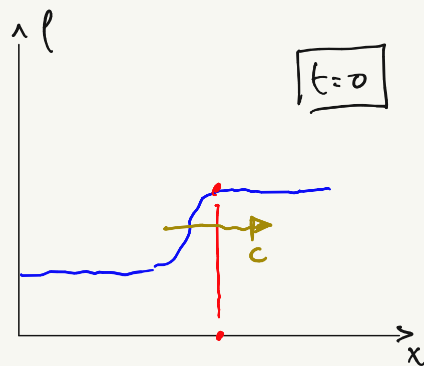
$$\frac{\partial \rho}{\partial t} = -c\,\rho_0'$$

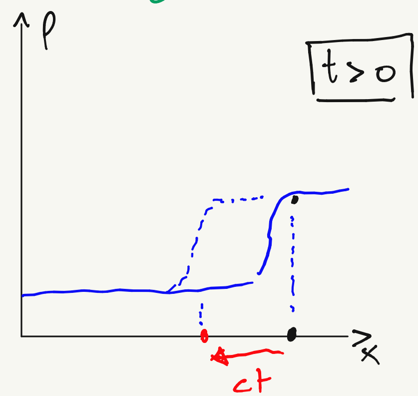
$$\frac{\partial \rho}{\partial x} = \rho_0'$$

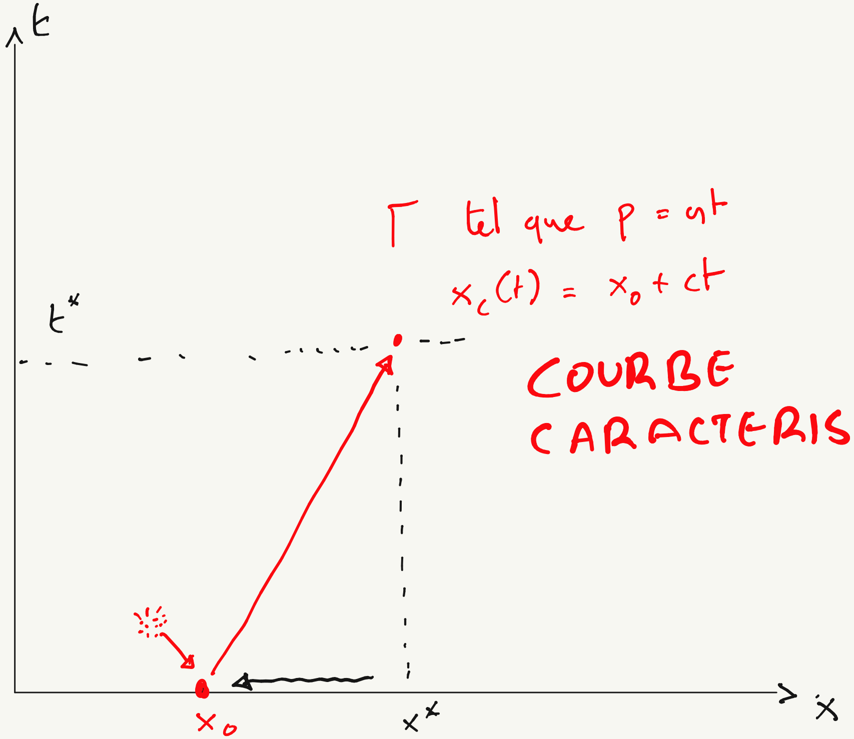


Γ tel que $\rho = \text{cst}$

$x_c(t) = x_0 + ct$







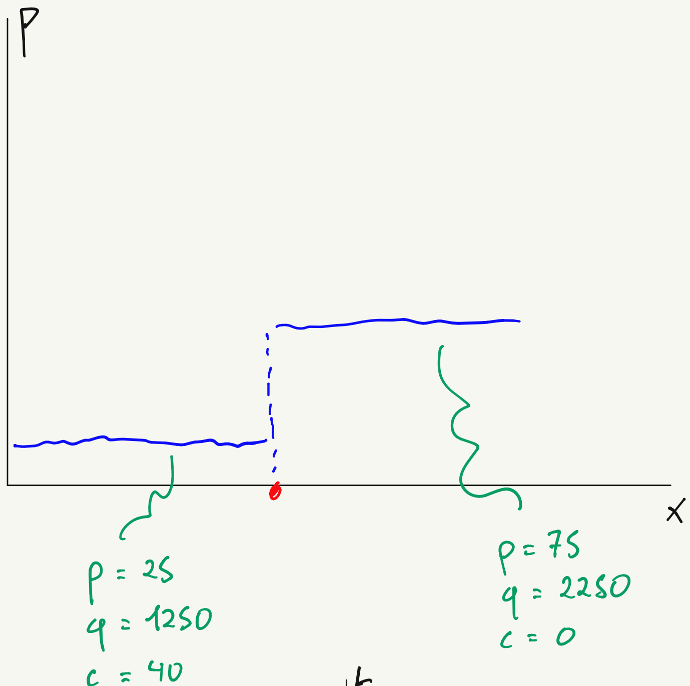
Γ tel que $\rho = c^{t}$

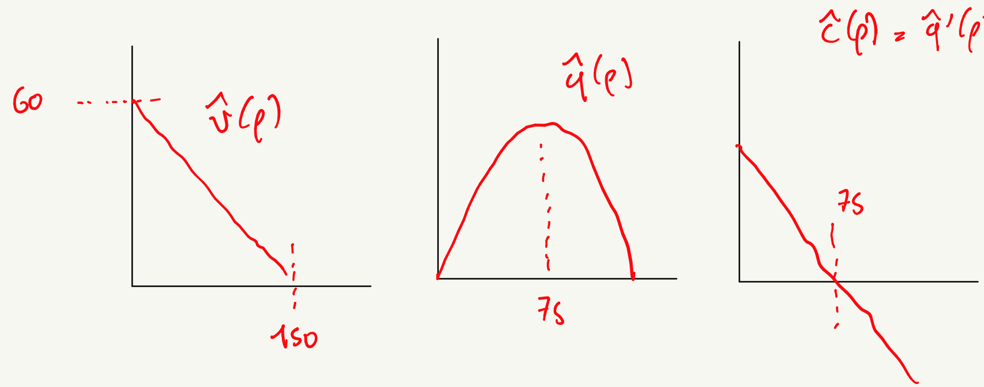
$x_c(t) = x_0 + ct$

COURBE CARACTERISTIQUE :-)

Sur Γ

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\underbrace{\frac{dx}{dt}}_{c} = \underbrace{\frac{\partial \rho}{\partial t} + c\,\frac{\partial \rho}{\partial x}}_{=0} = 0$$

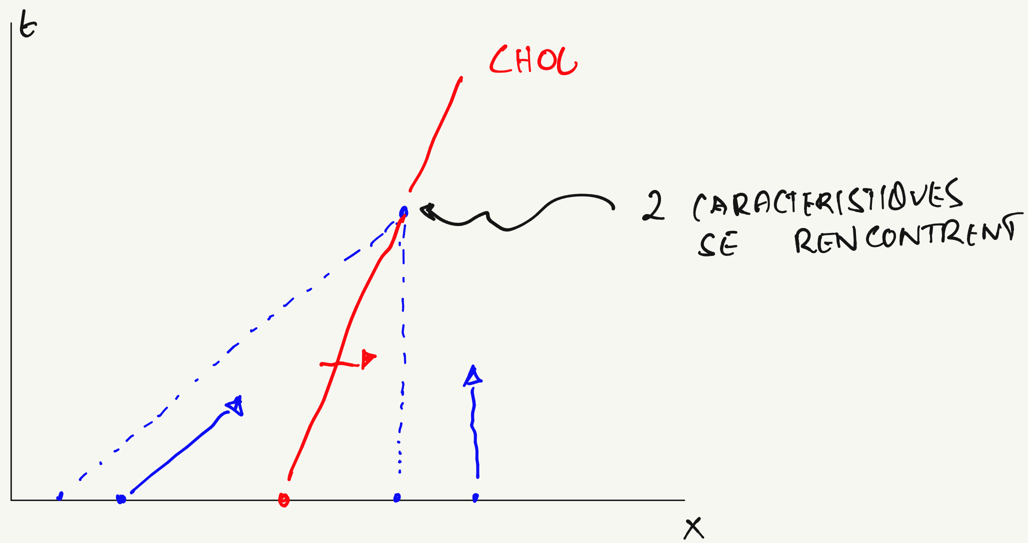




$$v(\rho) = 60\left(1 - \frac{\rho}{150}\right)$$

$$q(\rho) = 60\rho\left(1 - \frac{\rho}{150}\right)$$

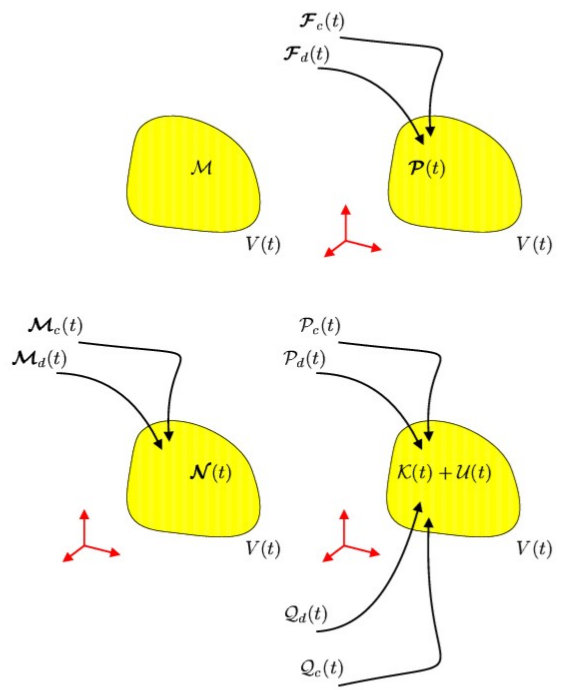
$$c(\rho) = 60\left(1 - \frac{2\rho}{150}\right)$$



Lois de conservation, lois de comportement, conditions aux limites.

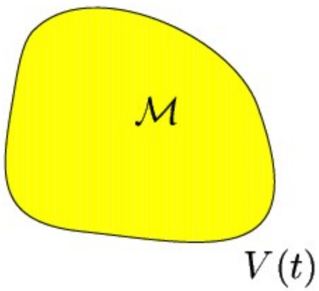
Lois de conservation | Lois de comportement

Conditions aux limites



Consevation de la masse,
de la quantité de mouvement,
du moment de la quantité de mouvement
et de l'énergie.

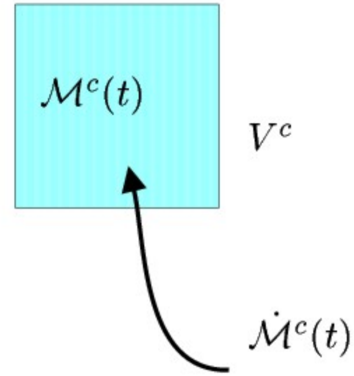
Formes globales de la conservation de la masse



$$\frac{d\mathcal{M}}{dt} = 0, \qquad \forall \; V(t),$$

Volume matériel

***Ensemble de points matériels en mouvement se déplaçant à une vitesse macroscopique* v(x,t)**



$$\frac{d\mathcal{M}^c}{dt}(t) = \dot{\mathcal{M}}^c(t), \qquad \forall \; V^c,$$

Volume de controle

Ensemble de points eulériens

$$\mathbf{v}(\mathbf{x}, t) = v_i(x_j, t)\mathbf{e}_i$$

Conservation de la masse

Forme globale

$$\frac{d\mathcal{M}}{dt} = 0, \qquad \forall \; V(t),$$

satisfaite pour une certaine classe de systèmes, à tout instant

$$\mathcal{M} = \int_{V(t)} \rho \, dV$$

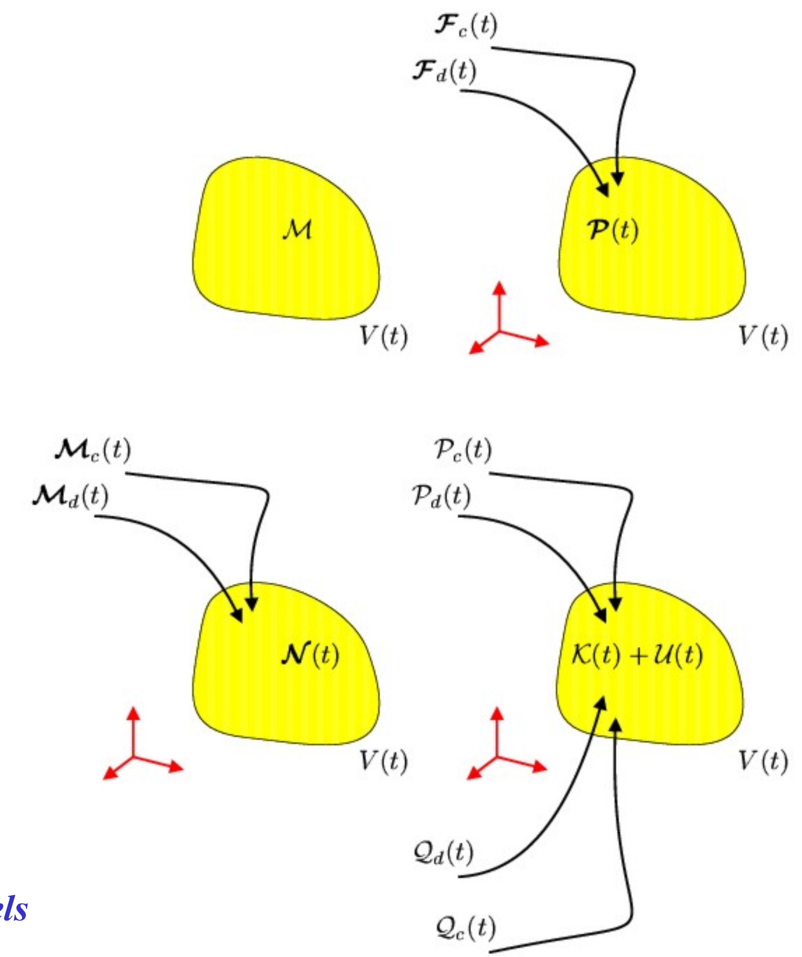
sous certaines conditions de continuité..

Forme locale

$$\frac{D\rho}{Dt} + \rho \boldsymbol{\nabla} \cdot \mathbf{v} = 0,$$

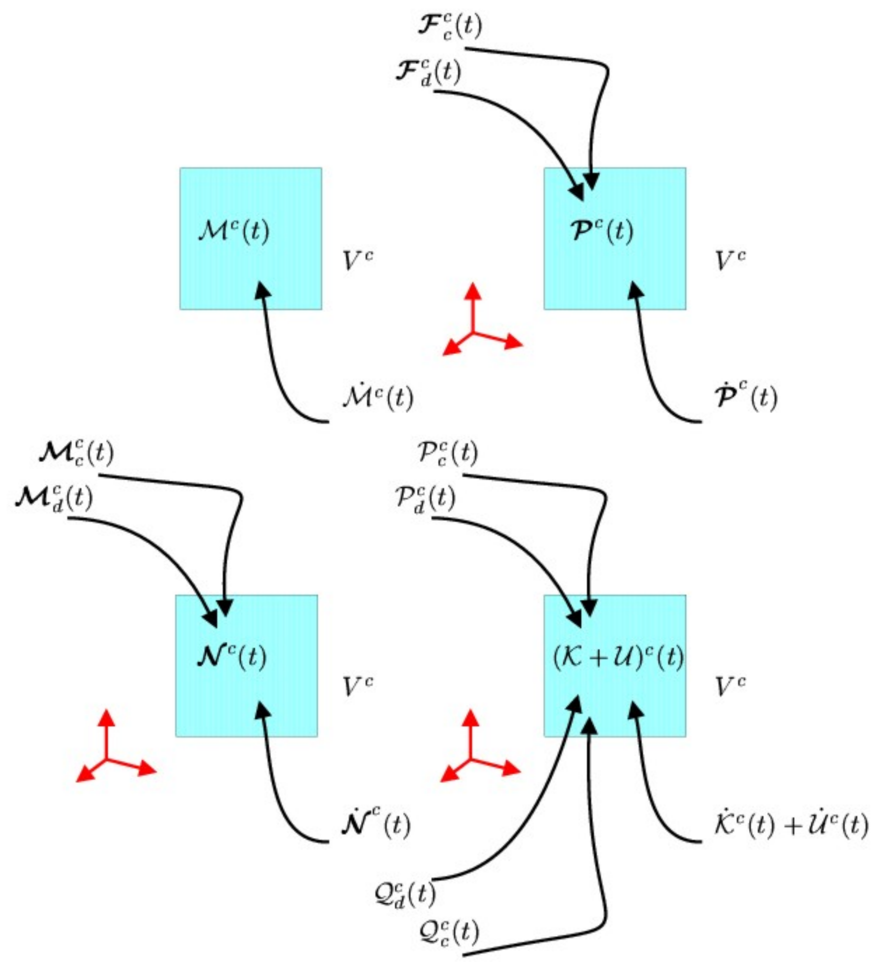
satisfaite en tout point et à tout instant

Toutes les lois de conservation, en un clin d' oeil…



Forme globale
pour des volumes matériels

Sous un autre angle, ces lois de conservation…



Forme globale
pour des volumes de controle

…dont on peut déduire des formes locales

$$
\begin{array}{rcl}
\mathbf{t}(\mathbf{n}) & = & \boldsymbol{\sigma}^T \cdot \mathbf{n} \\
\boldsymbol{\sigma} & = & \boldsymbol{\sigma}^T \\
q(\mathbf{n}) & = & -\mathbf{q} \cdot \mathbf{n}
\end{array}
\qquad
\begin{array}{rcl}
\dfrac{D\rho}{Dt} + \rho \boldsymbol{\nabla} \cdot \mathbf{v} & = & 0, \\
\rho \dfrac{D\mathbf{v}}{Dt} & = & \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \rho \mathbf{g} \\
\rho \dfrac{DU}{Dt} & = & \boldsymbol{\sigma} : \mathbf{d} + r - \boldsymbol{\nabla} \cdot \mathbf{q}
\end{array}
$$

Forme locale dite non-conservative

q

FLUX DE MASSE

$$
\begin{array}{rcl}
\mathbf{t}(\mathbf{n}) & = & \boldsymbol{\sigma}^T \cdot \mathbf{n} \\
\boldsymbol{\sigma} & = & \boldsymbol{\sigma}^T \\
q(\mathbf{n}) & = & -\mathbf{q} \cdot \mathbf{n}
\end{array}
\qquad
\begin{array}{rcl}
\dfrac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) & = & 0 \\
\dfrac{\partial (\rho \mathbf{v})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}\mathbf{v}) & = & \boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \rho \mathbf{g} \\
\dfrac{\partial (\rho U)}{\partial T} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v} U) & = & \boldsymbol{\sigma} : \mathbf{d} + r - \boldsymbol{\nabla} \cdot \mathbf{q}
\end{array}
$$

Forme locale dite conservative

Viscosité de volume

Viscosité de cisaillement

$$
\begin{aligned}
\boldsymbol{\sigma} &= -p\boldsymbol{\delta} + 3\boxed{\hat{\kappa}(p,T)}\mathbf{d}^s + 2\boxed{\hat{\mu}(p,T)}\mathbf{d}^d, \\
\mathbf{q} &= -\boxed{\hat{k}(p,T)}\boldsymbol{\nabla} T, \\
\rho &= \hat{\rho}(p,T), \\
H &= \hat{H}(p,T), \\
S &= \hat{S}(p,T).
\end{aligned}
$$

Conductibilité thermique

L'équation de comportement pour l'entropie n'est utile que pour vérifier que le second principe est bien satisfait !

$$
T\,dS = dH - \frac{dp}{\rho} = dU - \frac{p\,d\rho}{\rho^2},
$$

$$
\begin{aligned}
k &\geq 0, \\
\kappa &\geq 0, \\
\mu &\geq 0.
\end{aligned}
$$

Contraintes à respecter pour satisfaire Clausius-Duhem

Modèle du fluide visqueux newtonien

Le compte est bon !

$$
\begin{aligned}
\boldsymbol{\sigma} &= -p\boldsymbol{\delta} + 3\widehat{\kappa}(p,T)\mathbf{d}^s + 2\widehat{\mu}(p,T)\mathbf{d}^d, \\
\mathbf{q} &= -\widehat{k}(p,T)\boldsymbol{\nabla}T, \\
\rho &= \widehat{\rho}(p,T), \\
H &= \widehat{H}(p,T), \\
S &= \widehat{S}(p,T).
\end{aligned}
$$

conservation locale de la masse	ρ	1
conservation locale de la quantité de mouvement	$\mathbf{v}$	3
conservation locale de l'énergie	T	1
constitution pour les contraintes	$\boldsymbol{\sigma}$	6
constitution pour le flux calorifique	$\mathbf{q}$	3
constitution pour la masse volumique	p	1
constitution pour l'enthalpie	H	1
constitution pour l'entropie	S	1

Remarque : si une équation de comportement pour l'enthalpie est donnée… on en déduit automatique l'énergie interne et vice-versa.

$$U = -\frac{p}{\rho} + H$$