Partial De-Annuitization of Public Pensions v.s. Retirement Age Differentiation. Which is Best to Account for Longevity Differences?

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Abstract

Extensive research by demographers and economists has shown that longevity differs across socioeconomic status (SES), with low-educated or low-income people living, on average, shorter lives than their better-endowed and wealthier peers. Therefore, a pension system with a unique retirement age is a priori problematic. The usual policy recommendation to address this problem is to differentiate the retirement age by SES. This paper explores the relative merits of `partial` de-annuitization of public pensions as a way of addressing the (imperfectly assessed) inequality of longevity.

Keywords: Pension Policy, Longevity Difference, Equity, Annuitization, Retirement Age Differentiation
JEL Codes: H55, J26, J14

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1 Introduction

The length of life of individuals (longevity hereafter) is correlated with socio-demographic characteristics: on average women outlive men, and low-income individuals live, on average, significantly shorter lives than their better-endowed and wealthier peers (Chetty et al., 2016; Olshansky et al., 2012). Therefore, a pension system with a unique retirement age (or uniform contribution or replacement rates) is a priori problematic. Unaccounted longevity differences in contributory pension systems amount to taxing short-lived people and subsidising their long-lived peers (Ayuso et al., 2016), potentially distorting labour supply. Also, the social gradient in life expectancy reduces the progressivity of public pensions in those countries (e.g. the US) where the replacement rate is a negative function of earned income (Bosworth et al., 2016; Bommier et al., 2011). Some would even argue that longevity difference makes public pensions regressive (Piketty and Goldhammer, 2015).

We show in this paper that unaccounted longevity differences violate the most basic definition of equity under both a Bismarckian (i.e. fully contributory) or a Beveridgian pension system.[1] One of the usual policy recommendations to address these problems is to differentiate the retirement age by socio-economic status (SES hereafter) (Ayuso et al., 2016; Leroux et al., 2015; Vandenberghe, 2021). Related proposals recommend differentiating contributions or replacement rates (Bismarck) or the amount of the instalment (Beveridge) based on expected longevity differences. We explain later in the paper that these are functionally equivalent to retirement age differentiation. That is because, at its core, the problem of pension differentiation is an `imperfect information` problem. What is at stake is the difficulty/impossibility to fully individualise treatment (i.e. generate an almost infinite number of pension regimes) using a pension parameter (or a series of parameters) whose variation is intrinsically more limited than the realised longevity it is supposed to match. The degree of differentiation of pensions that can be achieved via pension parameters corresponds to what statisticians call the between-SES expected longevity differences. It leaves aside the (potentially very important) within-SES longevity heterogeneity. Given this imperfect information conundrum, we argue in this paper there might be another, possibly more effective, option to improve lifetime pension equity. We call it `partial` de-annuitization of public pensions.

[1] It is common to distinguish Bismarckian and Beveridgean pension regimes. Bismarckian ones are contributory and, in that sense, work-related. Benefits are paid *prorata* the duration and level of contributions. This is a basic feature of the first fully-fledged public pension scheme introduced by German Chancellor Bismarck in 1889. By contrast, Beveridgean pensions (in reference to the British economist W. Beveridge who presided over the design the British system) are non-contributory and distribute basic universal benefits and so provide a (generally small minimal) pension to all, in particular those who do not qualify for a contributory pension (e.g. because they never worked...).

Annuitization is a common (implicit) feature of most, if not all, public pension systems organised on a pay-as-you-go basis (PAYG).[2] But in principle, if we leave aside liquidity and transition issues,[3] nothing prevents imagining a public PAYG pension scheme where `part` of the sums earmarked for someone are paid upfront (i.e. at the beginning of the retirement spell) as a lump sum.[4] In the universe of fully funded pension systems, including public or publicly sponsored ones, that option is available, and sometimes explicitly related to the perspective of a short life. Examples that we are aware of comprise

- the Netherlands. From 2023 onward, the Dutch Government will allow lump sum payments equal to a maximum of 10% of the accumulated capital when reaching the retirement date, under occupational pension plans (Dillingh and Zumbuehl, 2021).
- the US, State of New York where the public sector employees can upon retirement fill a form to receive a "Partial Lump Sum Payment" corresponding to 5 to (max) 25% of the accumulated capital, with "a reduced lifetime monthly benefit based on the remainder"(New York State Government, 2022).
- the UK, you may be able to take all the money in your occupational pension as a tax-free lump sum, if (. . .) you're expected to live less than a year because of serious illness, you're under 75, and you do not have more than the lifetime allowance of £1,073,100 in pension savings (UK Government, 2022).
- Canada, British Columbia, with the Public Service Pension Plan you may receive a lump-sum payment in lieu of a monthly pension if you have an illness or disability that has shortened your life expectancy (British Columbia Government, 2022).

In what follows, partial de-annuitization will not be an option but considered as automatic and universal (i.e. applicable to all pensioners). But the key intuition will remain the same as in the above UK or Canadian examples: if longevity varies and is a source of inequality, paying part of the accrued pension rights when (all) prospective pensioners are still alive is a way to minimize pension-related lifetime inequalities. The idea echoes the notion of `reverse retirement` introduced by Ponthiere (2020) who considers a model where individuals start their life in retirement (and thus "all" receive their pension) before moving to work. What

[2] A system in which pensions are explicitly financed by contributions levied on current workers.

[3] We will comment briefly on transition issues in Section 5: Context and Policy Feasibility.

[4] Until 2016 in the UK, the (small) contributory segment of the PAYG public pension system (the Additional State Pension) offered a one-off lump-sum payment option.

follows should be seen as a milder, but more realistic, version of that thought-provoking idea. Ours is more related to the notion of `front-loaded` annuities or benefits in pension economics (Brown, 2002; Palmer, 2000).[5]

Our realism also stems from the fact that we are not so much interested in the absolute level of lifetime equity gains that can be achieved via de-annuitization. It is almost tautological that full de-annuitization is very effective in dealing with longevity differences. But also that it would annihilate pensions' capacity to cover the longevity risk.[6] Our perspective in this paper is that of the `relative` performance of de-annuitization vs retirement age differentiation (and by analogy, vs differentiation policies targeting either the contributory phase or the payout phase of pensions (Sanchez-Romero et al., 2020)). The key question of this paper is how much de-annuitization is needed to match the equity gains delivered by retirement age differentiation?

Of course, for obvious budgetary reasons, introducing de-annuitization (and the upfront payment of a lump sum to all) implies a reduction of the value of the pension annuity. Answering the question of `how much` de-annuitization is needed is thus also a way to quantify the propensity of partial de-annuitization to come at the expense of one of the key objectives of annuitization i.e. insuring individuals against the risk of longevity. That risk – and the underlying shortsightedness of individuals – is regularly mentioned in the literature as a justification for the State to impose a minimal degree of annuitization of the pension capital (Barr and Diamond, 2006).

The results presented in this paper show that the reduction of the monthly instalment needed to match the equity gains achieved via extensive retirement age differentiation (up to 200 different ages) is quite small. As far as we know such a result is a novelty and constitutes the key contribution of this paper to the economic literature on pensions and annuities.

Using US data assembled by Chetty et al. (2016), we estimate it to be about 4%, and the corresponding payment of an upfront lump sum corresponds to a bit less than a full year of benefits. The modest reduction of the annuity suggests a limited risk of significantly eroding pensions' monthly payment adequacy[7] and their capacity to insure the risk of longevity. Also, partial de-annuitization is administratively less costly to implement than retirement age differentiation. It does not require an upstream statistical analysis of the determinants

[5]In Sweden, for instance, the individual replacement rate from the contributory public pension is higher at the beginning of the retirement spell.

[6]The risk that individuals outlive their money, dying in poverty or burdening relatives.

[7]Their ability to support a basic acceptable standard of living.

of longevity. And it does not force civil servants to systematically verify pensioners' SES category (which, given the stakes for the retirees, might be prone to misreporting). We also anticipate fewer legal challenges. De-annuitization amounts to treating everyone equally. By definition, this is not the case of retirement age differentiation. Just consider the idea of differentiating the age of retirement by gender to account for the well-documented gender longevity gap ?[8] Would that be considered as legally acceptable? For instance, a look at the jurisprudence of the European Court of Justice suggests that the answer is simply no.

Note that throughout this paper, we will consider that retirement age(s) or the degree of de-annuitization are decided paternalistically by the State. Such a perspective partially reflects the European context underpinning this paper, where retirement is still largely driven by State-edicted rules. This said, we also consider the `political economy` of the proposal, i.e. that of the number of people who could support it.

From a normative point of view, we will consider throughout the paper that `all realised` longevity differences matter. This means that we subscribe to `ex-post` egalitarianism when it comes to dealing with longevity inequalities (Fleurbaey et al., 2016).

The rest of the paper is organized as follows. Section 2 exposes a simple framework to assess the gains from retirement age differentiation. Section 3 does the same thing for the idea of de-annuitization and exposes how the two approaches are logically related. Section 4 exposes the longevity data we use. Section 5 presents the key numerical results of the paper. Section 6 examines the political economy of partial de-annuitization considering the number of winners vs losers. Section 7 concludes with a discussion of the policy context and feasibility of partial de-annuitization.

2 A simple framework to assess the equity gains of retirement age differentiation

We consider a world where longevity varies significantly across individuals forming a cohort ($l_i, i = 1 \dots N$) and in a way that is related with observable SES category j, with $j = 1 \dots k$ and n_j the number of individuals forming the category j. Logically, we have that $\sum_{j=1}^{k} n_j = N$. The full distribution of longevity is unknown to the planner/pension minister. Her knowledge is limited to the `correlation` between SES and longevity. Equivalently, the

[8] Even after conditioning on the SES status.

planner can only differentiate treatment (retirement age) based on the $j = 1 \dots k$ SES group to which individuals belong.

Throughout the paper, we consider the two canonical versions of pay-as-you-go (PAYG) public pensions: the fully contributory Bismarckian version where benefits are indexed on contributions, and the Beveridgian one where every individual receives the same pension.

2.1 Bismarckian contributory pension scheme

The problem of policymakers under such a regime is to equalise the *actuarial fairness ratio* (afr) of lifetime pensions benefits to lifetime pension contributions.[9] Abstracting from education length differences, career breaks, wage growth, demographic changes or discount and indexation rates, and considering that retirement age is uniform, that actuarial fairness ratio writes,

$$\begin{gathered} afr_{i,j}(\boldsymbol{ra}) = S_{i,j}(\boldsymbol{ra}) \frac{(l_{i,j} - \boldsymbol{ra})\, \delta\, w_{i,j}}{\boldsymbol{ra}\, \eta\, w_{i,j}} = S_{i,j}(\boldsymbol{ra}) \frac{(l_{i,j} - \boldsymbol{ra})}{\boldsymbol{ra}} \theta \\ \text{where } \theta \equiv \frac{\delta}{\eta} \end{gathered} \tag{1}$$

Centrally defined reference retirement age is $\boldsymbol{ra}$ and lifetime benefits are equal to the time spent in retirement times the annuity $\delta w_{i,j}$ where δ is the replacement rate and $w_{i,j}$ is the individual level of earnings. Note that people can die before reaching retirement age. So we have the dummy variable $S_{i,j}(\boldsymbol{ra}) = 0$ if $l_{i,j} \leq \boldsymbol{ra}$ and $S_{i,j}(\boldsymbol{ra}) = 1$ otherwise. By definition of a Bismarckian system, lifetime contribution corresponds to the duration of the career (here, the retirement age) that multiplies the annual contributions at a rate η. We define $\theta \equiv \frac{\delta}{\eta}$ as the (uniform) rate of replacement for each euro of contribution.[10]

An alternative way of expressing reference retirement age is $\boldsymbol{ra} \equiv \alpha \boldsymbol{l}$ where $\boldsymbol{l}$ is the unique reference longevity and $0 \leq \alpha \leq 1$ is the share of life the reference person is supposed to spend working. Reference retirement age and reference longevity are thus isomorphic formulations in our setting.

[9] Here actuarial fairness is considered in its simplistic setting, as no actual discounting rate or other rates are considered.

[10] In reality, with PAYG, θ is also driven by the evolution of the relative size of the generations of (old) pensioners vs (younger) contributors, and by the wage/productivity gains that have occurred between the contributory and the payout years (Aaron, 1966).

$$afr_{i,j}(\boldsymbol{ra}) = S_{i,j}(\boldsymbol{ra})\frac{(l_{i,j} - \alpha\boldsymbol{l})}{\alpha\boldsymbol{l}}\theta \tag{2}$$

The equalisation of lifetime ratios across individuals ($\forall i, j : afr_{i,j} = afr$) can be achieved via the full individualisation of the retirement age, or, equivalently, via the use of each individual's longevity $l_{i,j}$ when defining the retirement age.[11] For any value of α, if the retirement age is fully individualised (i.e. $ra_{i,j} \equiv \alpha l_{i,j}, \forall i, j$), we verify

$$afr = \frac{(1-\alpha)}{\alpha}\theta \tag{3}$$

Note that, by definition, if the retirement age is fully individualized, and if $\alpha < 1$,[12] $S_{i,j}()$ is always equal to 1. In other words, nobody dies before reaching his fully individualised retirement age.

Using a uniform reference retirement age (i.e. $\boldsymbol{ra} \equiv \alpha\boldsymbol{l}$) introduces a `gap` between the fair ratio and the actual one

$$gap_{i,j}(\boldsymbol{ra}) \equiv afr - afr_{i,j}(\boldsymbol{ra}) = \theta\Big[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\,\frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}}\Big] \tag{4}$$

or equivalently when the reference retirement age is differentiated by SES category (i.e. $\boldsymbol{ra}_j \equiv \alpha\boldsymbol{l}_j$), the (presumably smaller) gap is

$$gap_{i,j}(\boldsymbol{ra}_j) \equiv afr - afr_{i,j}(\boldsymbol{ra}_j) = \theta\Big[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}_j)\,\frac{l_{i,j} - \boldsymbol{ra}_j}{\boldsymbol{ra}_j}\Big] \tag{5}$$

A graphical representation of what happens under uniform vs differentiated retirement age appears in Figure 1. The lower part of the graph represents the distribution of realised longevity, while the upper part depicts the (lifetime) actuarial fairness ratio (afr_{ij}). The first-best situation amounts to ensuring that every person gets the same ratio corresponding to the horizontal dashed line. The achieved degree of actuarial fairness corresponds to the solid and doted rising lines. Under uniform retirement age (solid rising line), only those whose longevity coincides with reference longevity ($\boldsymbol{l}$) get the first best. All the others get

[11] The other one, that we will not discuss systematically is to differentiate θ by SES and make it inversely proportional to expected longevity.

[12] Some part of life goes to retirement.

less or more than what lifetime equality commands. Those who reach that retirement age but die before the reference longevity $\boldsymbol{l}$ get less than what they should. And those whose longevity exceeds $\boldsymbol{l}$ get too much. Under differentiated retirement age (2 different retirement ages are depicted in Figure 1 and correspond to the dotted rising lines), the number of people who get exactly what equity commands (i.e. those whose longevity corresponds to the intersection between the dashed and dotted lines) rises. Also, the integral of the distances between the dotted lines and the dashed horizontal line becomes lower, reflecting a reduction of the inequity gaps in eq(3). However, situations synonymous with $afr_{i,j}$ "undershooting" or "overshooting" still abound.

Note that Figure 1 also reveals what would happen if, instead of differentiating the age or retirement, policy markers were to differentiate by SES either the contribution rate η or the replacement rate δ. That would amount to differentiating $\theta \equiv \delta/\eta$, thus the slope of the solid line, while keeping the retirement age unchanged (i.e. uniform). Instead of relying on $\boldsymbol{ra}_j$ to (try) to reflect realised longevity differences (i.e. any value on the horizontal axis), the planner would use θ_j. More specifically, while here we explore what happens when $\boldsymbol{ra}_j$ becomes a `positive` function of expected longevity, in an alternative (but fairly equivalent approach) one could examine what happens were θ_j to become a `negative` function of expected longevity. And echoing the point we raised in the introduction, it is likely that the outcome of that type of differentiation would not fundamentally change. That is because, at its core, the problem of pension differentiation is an `imperfect information` problem. How to fully individualise (and thus generate an almost infinite number of pension regimes), using only a limited number of values for the pension parameters $(ra, \theta, \delta, \eta)$ mirroring the equally limited number of expected longevities (one for each SES $j = 1 \ldots k$) the policymaker knows? And combining several parameters of differentiation (i.e. simultaneously differentiating ra and θ for instance) would not bring much change either, because the information used to implement that "hybrid" approach of differentiation would still boil down to exploiting a mere correlation between SES category k and realised longevity.

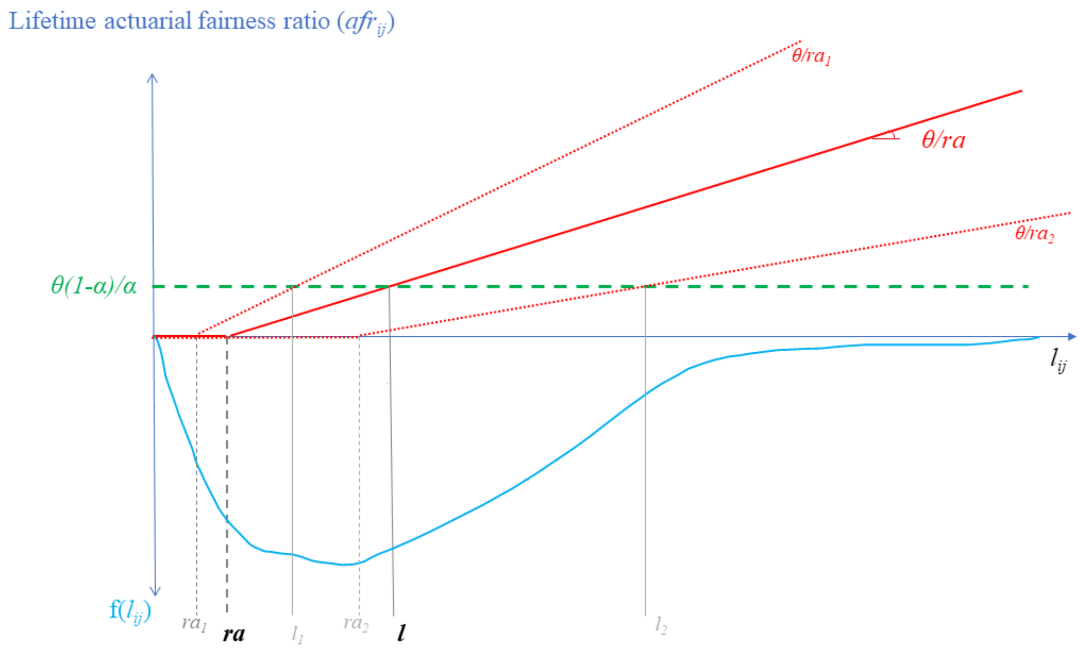

Figure 1: Bismarckian pension under uniform vs diff. ret age

Following Baurin (2021), the overall reduction in the propensity of retirement policy to deviate from the fully individualised first best can be expressed as a pension inequity index $I^{Bismarck} \in [0,1]$ where the numerator aggregates the (absolute) values of the individual equity gaps under retirement age differentiation by SES $j = 1 \dots k$, and the denominator does the same when there is no differentiation.[13] [14] Note that the index being a ratio, constant θ cancels out. In the numerical simulations underpinning the results of Section 4.2 the differentiated reference retirement ages/longevities correspond to averages by SES category j.

$$I^{Bismarck} \in [0,1] = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} \theta \,|\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}_j)\, \frac{l_{i,j}-\boldsymbol{ra}_j}{\boldsymbol{ra}_j}|}{\sum_{j=1}^{k}\sum_{i=1}^{n_j} \theta \,|\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\, \frac{l_{i,j}-\boldsymbol{ra}}{\boldsymbol{ra}}|} \tag{6}$$
$$\text{with } \boldsymbol{ra} \equiv \alpha \boldsymbol{l}\ ;\ \boldsymbol{ra}_j \equiv \alpha \boldsymbol{l}_j$$

[13]To be precise, our index deviates from that of Baurin (2021) in the sense that its building blocks are `monetised`. Gaps are expressed in monetary terms and not just in terms of years.

[14]Also, in the context of Bismarckian pensions, the index reads as an `inefficiency` index that quantifies the propensity to deviate from the actuarially fair first-best, synonymous with no labour supply or savings distortion.

Note finally that retirement age differentiation can generate budgetary savings (thus a surplus) in comparison with uniform retirement. This is because the overall (or average) time spent in retirement may go down. What is more, with a Bismarckian regime, the money not spent on those who live longer is likely to outweigh the cost of financing early retirement to the benefit of the short-lived individuals. In what follows, we will assume that the planner absorbs the net budgetary surplus generated by the move from uniform to differentiated retirement age. An alternative is to enforce cohortal budget equivalence, i.e. to use the surplus to increase the replacement rate θ in the numerator of our (in)equity index eq.(6). We explore that alternative in detail in Appendix 6.2. However, the key result is that the equity gains achievable via retirement age differentiation are likely to be lower. This is because the *time* component of the equity gaps [eq.(5)] — that inevitably still exists under retirement age differentiation — is *priced* at a higher rate.[15] Say differently, in the equity index,[16] the time component of the gaps in the numerator is now multiplied by a higher θ than in the denominator.[17] Thus the equity gains achieved in terms of years spent in retirement is eroded by the (remaining) year gaps weighing more in monetary terms.

2.2 Beveridgian pension scheme

By definition, a Beveridgian pension system would rather aim at equalising lifetime pension benefits ($B_{i,j}$).

$$B_{i,j}(\boldsymbol{ra}) \equiv b\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra}) \tag{7}$$

In the above expression, b is the standard uniform annual/monthly pension (which is independent of earning $w_{i,j}$ and contributions) that multiplies the time spent in retirement. Again, people can die before reaching retirement age. So we have the dummy variable $S_{i,j}(\boldsymbol{ra}) = 0$ if $l_{i,j} \leq \boldsymbol{ra}$ and $S_{i,j}(\boldsymbol{ra}) = 1$ otherwise. The equalisation of lifetime benefits ($\forall i, j : B_{i,j} = B$) can only be achieved via full-individualisation of retirement age, or the corresponding reference longevity ($ra_{ij} \equiv l_{i,j} - \kappa$), where κ is the reference number of years spent in retirement.

[15] In eq.(5), the equity gap $\theta\left[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\,\frac{l_{i,j}-\boldsymbol{ra}}{\boldsymbol{ra}}\right]$ consists of a price θ that multiplies a year- or time equity gap $\left[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra})\,\frac{l_{i,j}-\boldsymbol{ra}}{\boldsymbol{ra}}\right]$.

[16] eq.(6).

[17] We no longer have that the θ's cancel out.

$$B \equiv b(l_{i,j} - ra_{i,j}) = b\,(l_{i,j} - l_{i,j} + \kappa) = b\,\kappa \tag{8}$$

Note again that $S_{i,j}() = 1$ if there is perfect individualisation and if $\kappa > 0$. We logically assume κ is the time spent in retirement by the person whose longevity is equal to the reference longevity ($\boldsymbol{l}$) under a uniform retirement policy ($\kappa = \boldsymbol{l} - \boldsymbol{ra} = (1-\alpha)\boldsymbol{l}$).[18]

Key is that the use of a uniform retirement age/longevity reference ($\boldsymbol{ra}$) leads to lifetime benefits gaps

$$gap_{i,j}(\boldsymbol{ra}) \equiv B - B_{i,j}(\boldsymbol{ra}) = b\Big[\kappa - S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\Big] \tag{9}$$

or equivalently when the reference retirement age is differentiated by SES category ($\boldsymbol{ra}_j$), the (presumably smaller) gap is

$$gap_{i,j}(\boldsymbol{ra}_j) \equiv B - B_{i,j}(\boldsymbol{ra}_j) = b\Big[\kappa - S_{i,j}(\boldsymbol{ra}_j)(l_{i,j} - \boldsymbol{ra}_j)\Big] \tag{10}$$

Again, we can produce a graphical representation of what is at stake under uniform vs differentiated retirement. It is to be found in Appendix 6.1. In more analytical terms, the reduction in the overall propensity of retirement policy to deviate from the fully individualised first best can be expressed as the following index, where the constants b cancel out

$$\begin{aligned} I^{Beveridge} \in [0,1] = &\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} b\,|\kappa - S_{i,j}(\boldsymbol{ra}_j)\,(l_{i,j} - \boldsymbol{ra}_j)|}{\sum_{j=1}^{k}\sum_{i=1}^{n_j} b\,|\kappa - S_{i,j}(\boldsymbol{ra})\,(l_{i,j} - \boldsymbol{ra})|} \\ \text{with } &\boldsymbol{ra} \equiv \boldsymbol{l} - \kappa = \alpha \boldsymbol{l} \\ &\boldsymbol{ra}_j \equiv \boldsymbol{l}_j - \kappa \end{aligned} \tag{11}$$

Note again that, so far, we have assumed that the planner absorbs any net surplus/deficit generated by the move from uniform to differentiated retirement. The alternative policy would be to raise(reduce) b prorata in the numerator of the (in)equity index eq.(11), and enforce cohortal budget equivalence. Under a Beverigian regime, retirement age differentiation is likely to generate a surplus(deficit) if the overall time spent in retirement goes down(up). Not absorbing that surplus(deficit) means that policymakers raise(reduce) b. This again

[18] Thus the equalising retirement age can also be written as $ra_{i,j} = l_{i,j} - \boldsymbol{l} + \boldsymbol{ra}$.

means that year equity gaps — that still exist under retirement age differentiation — are "priced" differently. Thus, the equity gains achieved in terms of years spent in retirement are eroded(amplified) by the (remaining) year gaps weighted differently. This translates into different equity gains than those reported in result Section 4.2. We analyse this cohortal-budget-equivalence alternative in detail in Appendix 6.2. Results, in short, suggest that under a Beveridgian regime retirement age differentiation could generate a (small) surplus, allowing for a (slightly) higher b.[19] Thus, again, our main results in Section 4.2 (derived without imposing cohort budgetary equivalence) should be seen as an upper bound of the achievable equity gains.

To sum up, minimising both the Bismarckian and Beveridgian inequity indices [equ. (6),(11)] depends on the social planner being able to match the full distribution of longevity across individuals $l_{i,j}$ i.e., the different values of the horizontal axis forming the longevity distribution on the lower part of Figures 1,7. If she can only go for tagging (Akerlof, 1978) i.e. use $j = 1 \ldots k < N$ proxies $\boldsymbol{l}_j$ that are simply correlated to realised longevity $l_{i,j}$ to differentiate treatment, and if unaccounted longevity differences are important and matter, then both policies should translate into values of our indices that are relatively close to 1. In Section 4.2, we will show simulation results illustrating this, using US data on longevity heterogeneity.

3 De-annuitization

We now consider Bismarckian and Beveridgian pension schemes with some de-annuitization: i.e. with an upfront lump-sum payment LS.[20] With the Bismarckian pension, the actuarially fair ratio becomes

$$afr_{i,j} = \frac{LS_{i,j} + S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\,\delta'\,w_{i,j}}{\boldsymbol{ra}\,\eta\,w_{i,j}} = \mu + S_{i,j}(\boldsymbol{ra})\frac{(l_{i,j} - \boldsymbol{ra})}{\boldsymbol{ra}}\theta' \tag{12}$$

with the (logically lower) annuity corresponding here to a lower replacement rate i.e. $\delta' < \delta$, $\theta' \equiv \frac{\delta'}{\eta} < \theta$, and $\mu \equiv \frac{LS_{i,j}}{\boldsymbol{ra}\,\eta\,w_{i,j}}$, $\forall\, i, j$. Note that the lump sum paid varies for each

[19] It is likely this result is purely driven by the US data used to do the simulation. In principle, under a Beveridgian regime, the only factor playing a role is the relative importance of the groups spending more(less) time in retirement.

[20] We leave aside complications stemming from those who did not survive until prime age i.e. the moment from which longevity heterogeneity is considered conceptually or empirically (i.e. 40 hereafter).

individual, but it is strictly proportional to lifetime contributions, guaranteeing that each individual achieves some positive uniform (up-front) ratio.[21] And it is that uniform μ that contributes to reducing equity gaps in eq.(14) below. In Appendix 6.3 we show that the value of μ that is compatible with the budget-equivalence constraint (i.e. same sums spent on a cohort with and without de-annuitization) is

$$\mu = (\theta - \theta') \, \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra}) \, w_{i,j}}{N \boldsymbol{ra} \overline{w}} \tag{13}$$

where the term that post-multiplies $(\theta - \theta')$ is the ratio of the (wage-weighted) number of years spent in retirement to the (wage-weighted) number of years spent contributing. As to the denominator remember that N is the size of the cohort of pensioners considered and $\overline{w}$ the average wage earned by the members of that cohort. Note finally that $\mu > 0$ if $\theta' < \theta$.

The interesting point is what happens with the inequity gap indices when θ is reduced to θ'. The building blocks of the Bismarckian version of that index consist (to the numerator) of the gaps between the actuarially fair and the one achieved via the policy envisaged. With de-annuitization (assuming a unique retirement age), the gaps become

$$\begin{aligned} gap_{i,j}(\boldsymbol{ra}, \mu, \theta') &= \Big[(\mu + \theta' \frac{1-\alpha}{\alpha}) - (\mu + \theta' S_{i,j}(\boldsymbol{ra}) \, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}}) \Big] \\ &= \theta' \Big[\frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}) \, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}} \Big] \end{aligned} \tag{14}$$

The index capturing the gains achieved via de-annuitization (the reference policy being one with no de-annuitization and uniform retirement age) now writes:

$$\begin{aligned} I^{Bismarck} \in [0,1] &= \frac{\theta' \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left| \frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}) \, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}} \right|}{\theta \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left| \frac{1-\alpha}{\alpha} - S_{i,j}(\boldsymbol{ra}) \, \frac{l_{i,j} - \boldsymbol{ra}}{\boldsymbol{ra}} \right|} \\ &= \frac{\theta'}{\theta} \\ \text{with } \boldsymbol{ra} &\equiv \alpha \boldsymbol{l} \end{aligned} \tag{15}$$

Crucial is that the gain achieved via de-annuitization is strictly proportional to the re-

[21] Note that, unlike lifetime benefits, lifetime contributions $(\boldsymbol{ra} \, \eta \, w_{i,j})$ are potentially fully known by policy makers or their pension administration. So there is no problem fully individualising LS to achieve uniformity across individuals in terms of guaranteed ratio μ.

duction of the annuity ($\frac{\theta'}{\theta} < 1$).[22] And an interesting numerical exercise, based on actual longevity data, is to compute the gains that can be achieved via retirement age differentiation. This will provide a certain value of the index $I^{Bismark} < 1$, from which we can infer the corresponding value of $\frac{\theta'}{\theta}$ (and thus also of μ) ensuring the `same` fairness improvement. Thus, quantifying the gains that can be achieved via retirement age differentiation – as we do in Section 4.2 – amounts to computing the degree de-annuitization that will provide exactly the same pension fairness gains.

A similar equivalence can be established between retirement age differentiation and partial de-annuitization of Beveridgian pensions. This time, the lump sum LS paid upfront is the same for every individual (hence the absence of subscripts i, j) and writes:

$$B_{i,j} \equiv LS + b'\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra}) = LS + b'\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{l} + \kappa) \tag{16}$$

with a logically lower annuity $b' < b$. In appendix 6.3 we show that the value of LS that is compatible with the budget-equivalence constraint (i.e. same sums spent on a given cohort with and without de-annuitization) is

$$LS = (b - b')\, \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})}{N} \tag{17}$$

where the term that post-multiplies $(b - b')$ is just the average number of years spent in retirement. Note that $LS > 0$ if $b' < b$.

The key point is again to consider what happens with the indices exposed above when b is reduced to b'. The building blocks of the Beveridgian version of that index consist (to the numerator) of the gaps between the fair annuity and the one actually achieved via the policy envisaged. With de-annuitization (assuming again a unique retirement age), the gaps become

$$\begin{aligned} gap_{i,j}(\boldsymbol{ra}, LS, b') &= (LS + b'\kappa) - (LS + b'\, S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})) \\ &= b' \left[\kappa - S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\right] \end{aligned} \tag{18}$$

[22] Strictly speaking, the (reduced) annuity is $\delta' w_{i,j}$. But δ' is directly related to θ' as $\theta' = \frac{\delta'}{\eta}$. Thus, the new annuity becomes $\theta' \eta w_{i,j}$.

Hence, the index capturing the gains achieved via de-annuitization (the reference policy still being one with no de-annuitization and uniform retirement age) writes

$$\begin{aligned} I^{Beveridge} \in [0,1] &= \frac{b' \sum_{j=1}^{k} \sum_{i=1}^{n_j} |\kappa - S_{i,j}(\boldsymbol{ra})\,(l_{i,j} - \boldsymbol{ra}_j)|}{b \sum_{j=1}^{k} \sum_{i=1}^{n_j} |\kappa - S_{i,j}(\boldsymbol{ra})\,(l_{i,j} - \boldsymbol{ra})|} \\ &= \frac{b'}{b} \\ \text{with } \boldsymbol{ra} &\equiv \alpha \boldsymbol{l} \end{aligned} \tag{19}$$

So, paralleling the result for Bismarckian pensions in eq.(15), we see that the gains achieved here via de-annuitization are strictly proportional to the reduction of the annuity ($\frac{b'}{b} < 1$).

4 Data & Results

4.1 Data construction

The data used to analyse partial de-annuitization vs retirement age differentiation are from the US. They consist of a `simulation` of the full distribution of longevity across a cohort of N individuals with different socio-demographic backgrounds ($l_{i,j}; i = 1 \dots n_j; j = 1 \dots k$) who have survived until prime age. At its core, the simulation rests on the (unavailable to us) mortality rates assembled by Chetty et al. (2016).

The underlying microdata comprises a sample of 1.4 billion observations from anonymised tax records, covering the years 1999 to 2014. Mortality data start at age 40 and are available either by gender, US state of residence and income quartile; or by gender and income percentile. We retain the gender $\times$ income version of the Chetty data. More precisely, we use the (publicly available) parameters of Gompertz functions they provide for each gender $\times$ income cell j, alongside the number of people in the US population belonging to these cells ($n_j, j = i \dots k$).[23] The parameters of the Gompertz function capture the expected

[23] A Gompertz function is sigmoid which describes growth (here mortality) as being slowest at the start and end of a given period (respectively age 40 and age 120 with the Chetty data). The right-hand or future value asymptote of the function is approached much more gradually by the curve than the left-hand or lower valued asymptote. This is in contrast to the simple logistic function in which both asymptotes are approached by the curve symmetrically. The Gompertz is a special case of the generalised logistic function

differences of mortality between categories j. Whereas the predicted values delivered by each Gompertz function j provides the "within" category distribution of mortality rates for each (potential) age of death.[24] These mortality rates by age can then be multiplied by the number of individuals forming each cell j to know the number of individuals whose longevity is equal to $40, 41, \ldots, 120$.

In Figure 2 we display the Gompertz-generated distribution of longevity for men belonging to the lowest income percentile of the US male population vs the equivalent distribution for women forming the highest income percentile of the female population. Expected/average longevity (corresponding to the dashed vertical lines) vary between the two groups: the expected longevity gap is larger than 16 years (88.7 v.s. 71.9 years). Still, the solid curves capturing the full distribution of realised longevities within each group, reveal that quite many women forming the upper-income percentile die before the average age of 88.7 (and would deserve to retire early), while some low-income men live beyond that age (and to not deserve to early retire).[25]

that has proved adequate to describe human mortality as an (accelerating) function of age.

[24] The latter is known by demographers as a life table (Chiang, 1984).

[25] Remember that reference retirement ages are defined as $\boldsymbol{ra}_j \equiv \alpha\, \boldsymbol{l}_j$ with $\alpha < 1$ the share of life spent working.

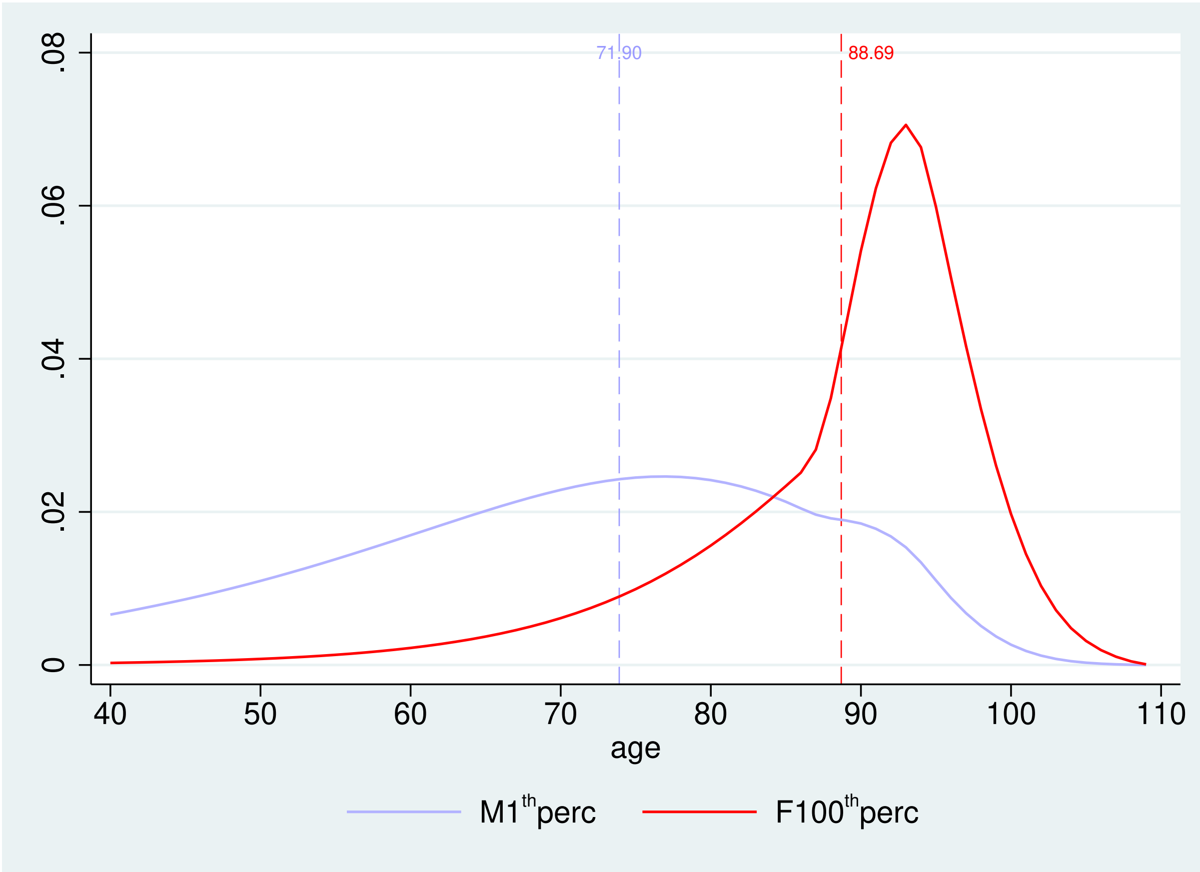

Figure 2: USA- [M]ale lowest income percentile and [F]emale highest percentile; expected/average longevity difference between the two groups vs full distribution of realised longevities within these groups. Source: Chetty et al. (2016), based on mortality data beyond the age of 40.

4.2 Numerical simulation results

We start by assuming that our reference longevity (i.e. $\boldsymbol{l}$) underpinning the uniform retirement policy is the average longevity in the Chetty data. Without loss of generality, and for the sake of clarity, the value of the share of life spent in employment α is chosen so that the corresponding uniform retirement age is equal to 65. Hereafter, the results for the retirement age policy are centred on that age of 65 ($\boldsymbol{ra} = 65$).

Given the Chetty data documenting longevity differences only past the age of 40, the minimum retirement age under retirement age differentiation is 40. And with de-annuitization, the lump-sum payment is also assumed to intervene at 40. By definition, it consists of paying a lump-sum to all whose longevity is considered, and that is only feasible before the first individual dies, thus here at 40.[26] Also, given the data used, the two policies examined here (equally) ignore the problem of the very short-lived, i.e. those who don't survive up to the age of 40. Note that the hypothetical use of a data set documenting longevity only from the age of 50 or 55 would simply inflate the number of individuals who are de facto not compensated, but without affecting our key analytical results markedly. Remember that our prime interest is to assess de-annuitization needed to match whatever can be achieved via retirement age differentiation, and by that we also mean the extent to which that policy ignores some short-lived. Remember also that the major factor driving our results is the (in)ability of the planner to match the full distribution of longevity using a few proxies. Whether it is the post-40, 50 or 55 distribution does not matter much.

Our results consist of the simulated values of the gains generated by retirement age differentiation in terms of inequity gap indexes exposed in eq.(15) and eq.(19), one for the Biskmarckian system and one for the Beveridgean one. In both cases we estimate numerically the gains achieved by resorting to 200 different reference retirement ages/longevities (i.e $j = 1 \dots 200$, corresponding to 2 genders $\times$ 100 income percentiles). The differentiated retirement ages we use are visible in Figures 3, 4. They correspond to each of our SES category j's average longevity (multiplied by α with Bismarck, or minus κ with Beveridge).

The key result is the one about the equity gains achieved via differentiation. It is reported on top of Table 1. We see values of .963 and .964 for (respectively) the Biskmarckian and Beveridgian schemes. These are our best estimates of how much inequity indices can be brought down via retirement age differentiation. Remember that these indices are equal to $\theta'/\theta; b'/b$ as stated in eq.(15),(19). Thus $1 - .963$ or $1 - .964$ tell us about the % reduction of pension instalment[27] required to generate an equivalent gain in terms of pension fairness. We see it is relatively limited: less than 4 % points of reduction of θ (Bismarck) or of the basic pension b (Beveridge) would be enough to generate the same equity gains as extensive retirement age differentiation across 200 gender$\times$ income categories; with retirement ages ranging from 56.5 to 69.7 (Bismarckian) or 54.2 to 71 (Beveridge) displayed in the lower part of Table 1. Also, one can get an idea of what the above results imply for the importance of the lump-sum paid upfront. Assuming that the (by 4%) reduced pension instalments are paid from the age of 65 to retirees with a 20-year life expectancy, following eq. (17), in a Beveridgian system, the lump-sum would represent approximately $.004 \times 20 \times 12 = 9.6$

[26]The results that follow can be considered as fully representative of what would happen if the lump-sum is paid at reference retirement age (65) only if mortality between 40 and 65 is negligible.

[27]Remember that for Bismark we have that the pension instalment is strictly proportional to θ as it is equal to $\theta' \eta w_{i,j}$.

month-equivalents of the current pension instalment. The calculus is less straightforward for the Bismarckian system but the results are similar.

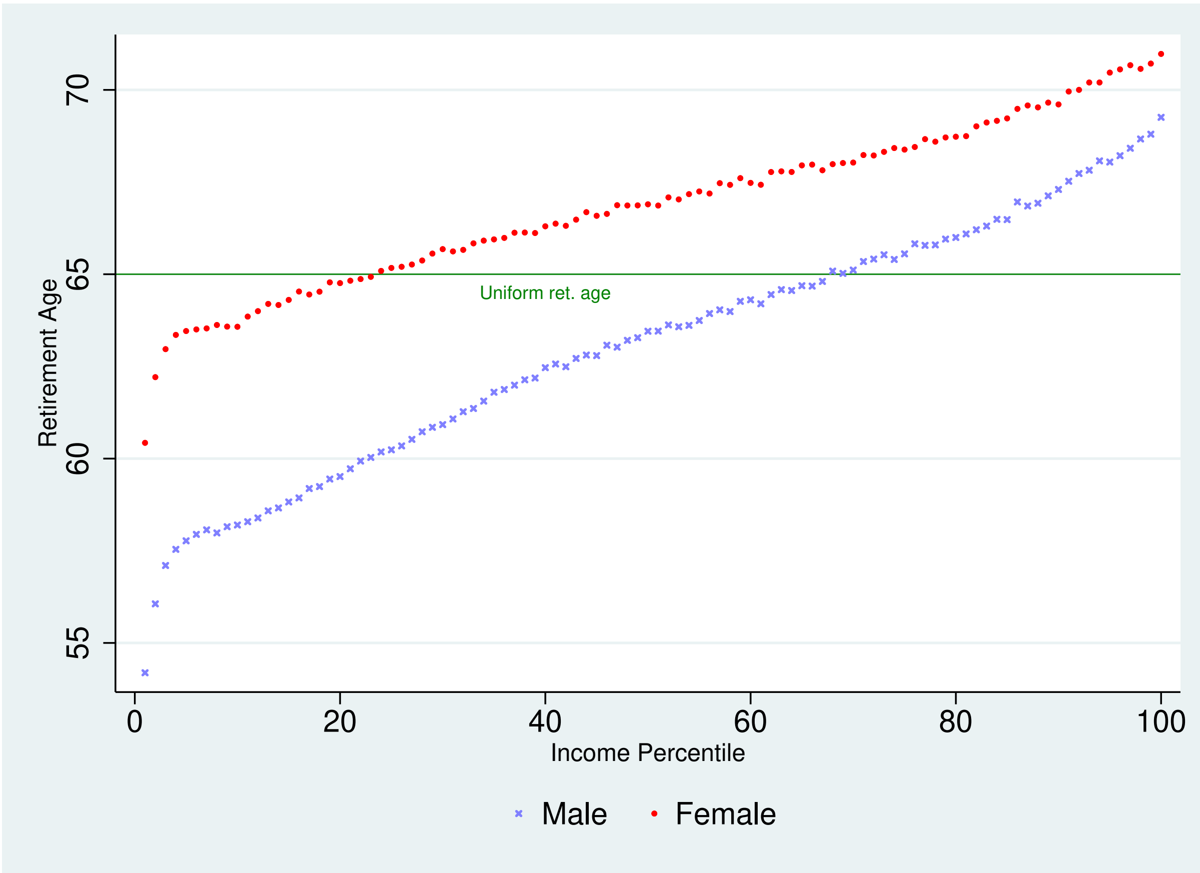

Figure 3: Bismarckian differentiated pension ages

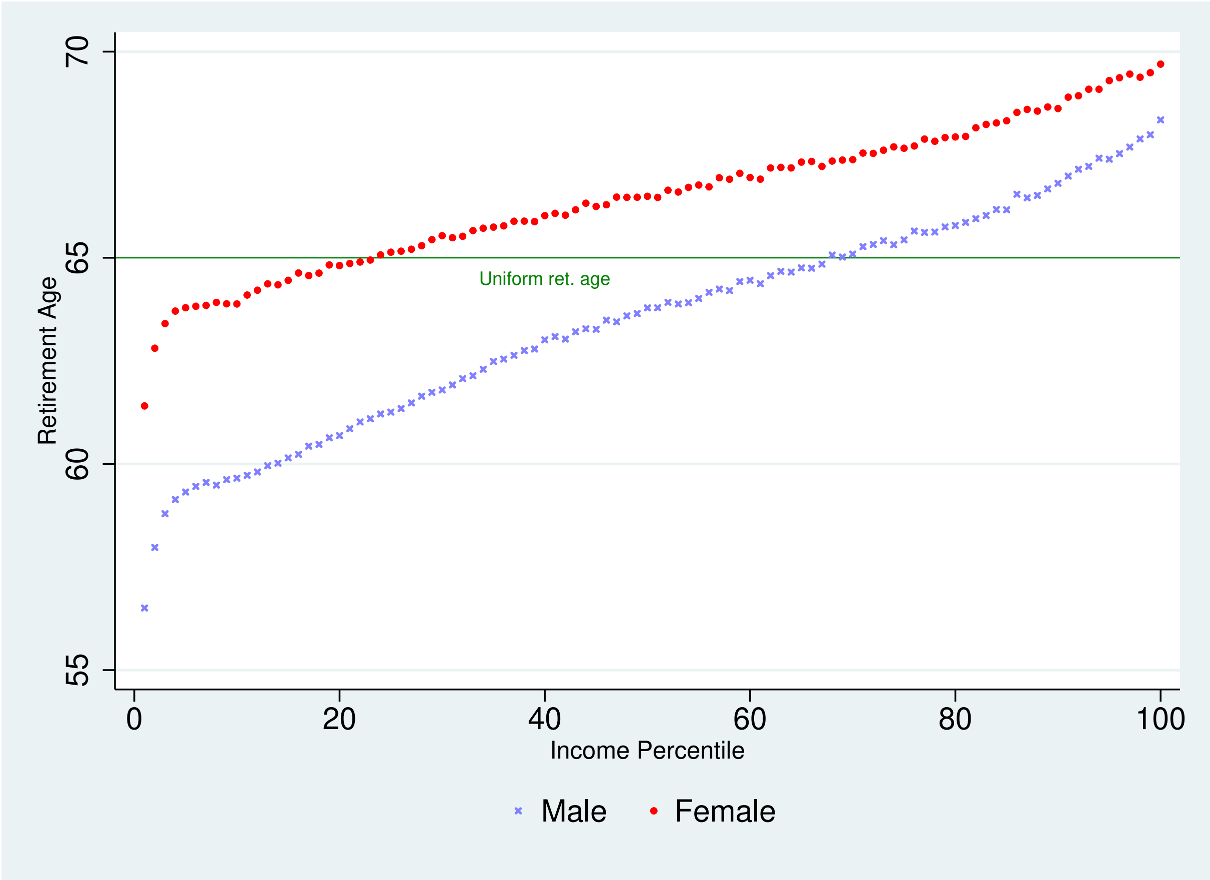

Figure 4: Beveridgian differentiated pension ages

Table 1: Numeric results: values of the Equity Gap Index[a] achieved via retirement age differentiation and the degree of de-annuitization needed to match these gains [without cohortal budget equivalence[b]]

	Pension regime			
	Bismark		Beveridge	
	F	M	F	M
Equity Gap Index[a]	0.963[c]		0.964[d]	
Inc. perc.				
1	61.407	56.507	60.428	54.192
10	63.881	59.656	63.576	58.200
20	64.812	60.688	64.760	59.513
30	65.535	61.795	65.681	60.922
40	66.022	63.010	66.301	62.467
50	66.491	63.785	66.898	63.455
60	66.948	64.456	67.479	64.307
70	67.381	65.090	68.029	65.114
80	67.931	65.784	68.730	65.997
90	68.620	66.807	69.607	67.300
100	69.697	68.345	70.976	69.256
Ref. ret. age	65	65	65	65

[a]: The simulated values of equity gap indexes exposed in eq.(15) and eq.(19). The propensity of these indices to be below 1 tells us `simultaneously` about *i)* the equity gains that can be achieved via retirement age differentiation and *ii)* `the degree of de-annuitization required to generate an equivalent gain in terms of pension fairness`. We report here the gains achieved by resorting to 200 different reference retirement ages/longevities (i.e $j = 1 \ldots 200$, corresponding to 2 genders $\times$ 100 income percentiles). The differentiated retirement ages we use are those visible in Figures 3, 4. In the lower part of this table, we report only a sample of them.
[b]: Meaning that we assume that the planner "absorbs" the net budgetary surplus(deficit) generated by the move from uniform to differentiated retirement age. Thus the move does not translates into a higher(lower) value of θ and b to guarantee that the same total budget is spent on the cohort of pensioners.
[c]: $I^{Bismarck} \in [0,1]$ as defined by eq.(15)
[d]: $I^{Beveridge} \in [0,1]$ as defined by eq.(19)

4.3 Winners, losers

In this section, we explore the question of the support/opposition that partial de-annuitization might encounter. Our approach is quite simple. We consider that the support for de-annuitization depends on the share of short-lived individuals who get a higher lifetime actuarial fairness ratio (or benefits with a Beveridgian system). In Figure 5, it corresponds to all the individuals whose longevity is inferior to l^*.

Algebraically and numerically, one can show that the "indifferent" pensioner is not influenced by the degree of de-annuitization. The point of indifference in Figure 5 and the corresponding longevity l^* are `fixed`. Whatever the intensity of de-annuitization, there is always the same share of pensioners who gain from de-annuitization vs uniform retirement age. Proof of this is in Appendix 6.4.[28]

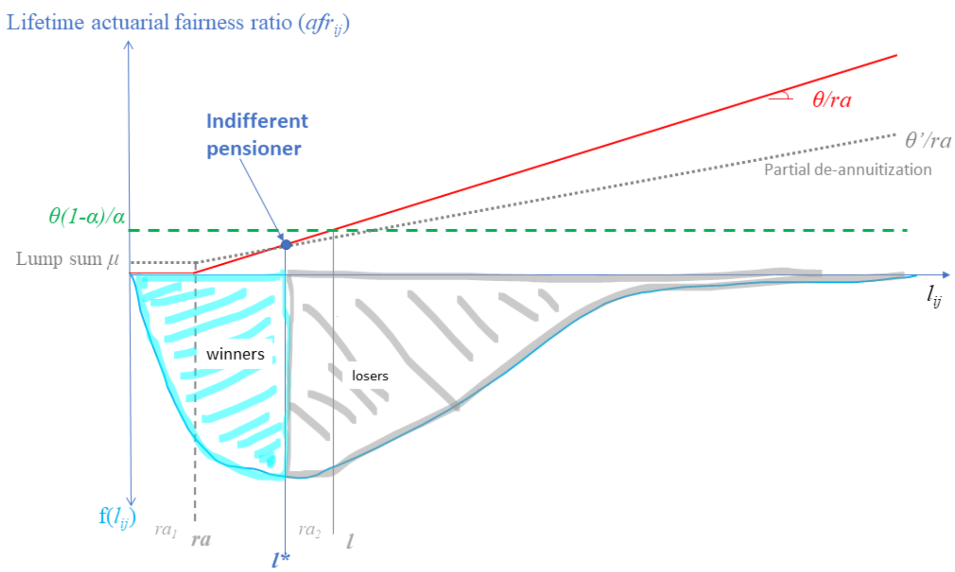

Figure 5: Partial de-annuitisation: winners vs losers

This said, it is important to stress that the value $\boldsymbol{l^*}$ varies with the reference retirement age ($\boldsymbol{ra}$). In Figure 6 we report simulation results on the share of (relatively short-lived) pensioners who would gain from de-annuitization. It is clearly a rising function of the reference retirement age. In policy terms this means that the support for de-annuitization is likely to rise the more policy-markers increase the (unique) retirement age.

[28]The same cannot be said about how much, in monetary terms, is gained and lost because of various degrees of de-annuitization. More de-annuitization means more monetary gains(losses) for those who gain(lose). The headcount approach and the monetary one are not equivalent.

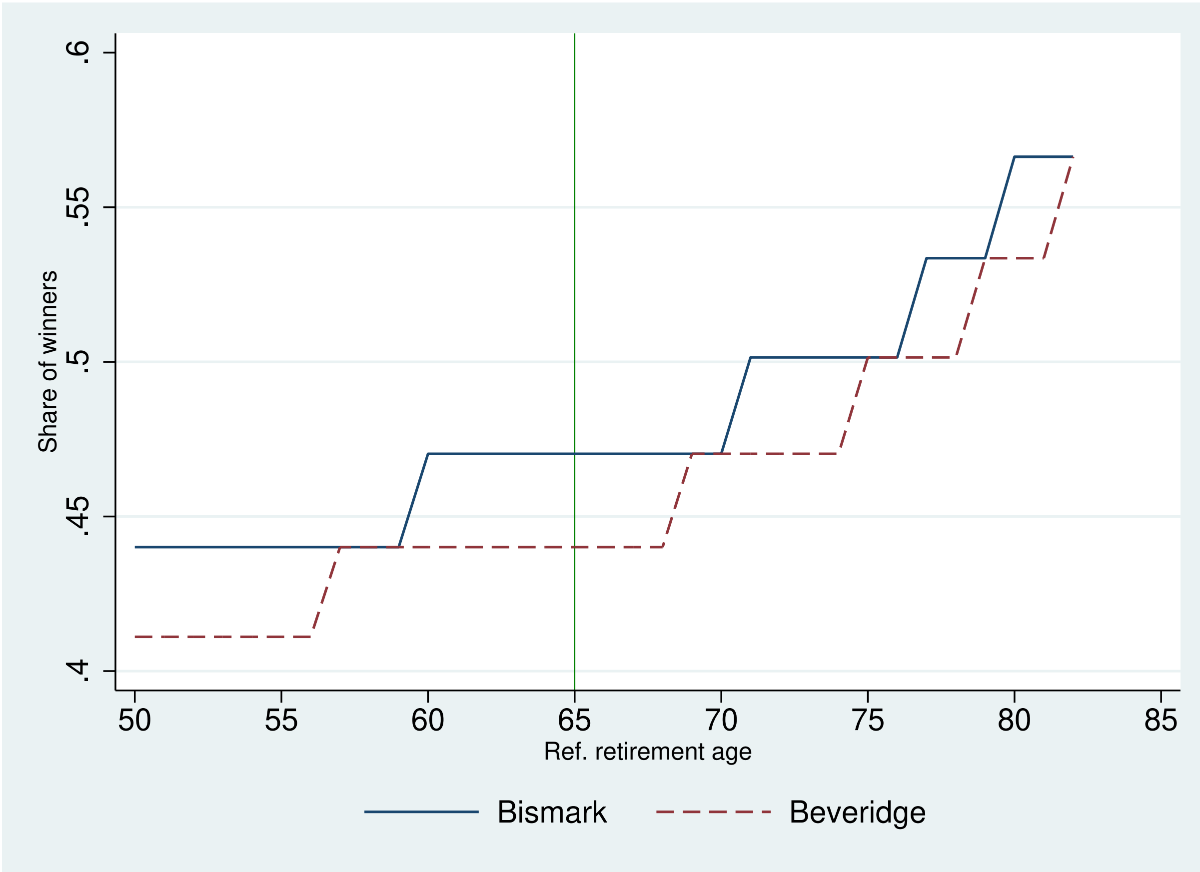

Figure 6: Partial de-annuitisation: share of winners and ref. retirement age

5 Context and Policy Feasibility

In the US context, and among economists who work on private/funded pensions, the dominant view is that people are "under-annuitized" for their privately provided and funded pension. By that, our colleagues mean they are underinsured against the risk of outliving their pension capital (Yaari, 1965; Diamond and Mirrlees, 1985; Diamond and Sheshinski, 1995, Brown, 2007). And one challenge is to understand the so-called `annuity puzzle` i.e. the fact that annuities are rarely purchased (voluntarily) despite the longevity insurance they provide. One of the problems seems to be that people are too pessimistic about their longevity (Benartzi et al., 2021; O'Dea and Sturrock, 2020). A lot of recent effort has been made to understand the public's rejection of full-annuitization (Vidal-Meliá and Lejárraga-García, 2006; Brown et al., 2021; Clark et al., 2019) and its preference for partial de-annuitization (Beshears et al., 2014; Dillingh and Zumbuehl, 2021). These authors, for instance, find that allowing individuals to annuitize a fraction of their wealth increases their support for annuitization relative to a situation where annuitization is an "all or nothing" decision.

The take-home message of this paper is also that pensions should be partially de-annuitized, even PAYG public ones. But the underlying context is quite different. In Europe and for public pensions organised on a PAYG basis, full and mandatory annuitization is the (unquestioned) rule. Also our key argument in favour of partial de-annutization is not so much that the public prefers it, but that it is a simple and effective way of addressing the problem of the short-lived individuals. In contrast with the US/private and funded pension debate our starting point is not the risk of poverty at (very) old age but the risk of inequality inherent to full annuitization when the length of life varies a lot across individuals; what a burgeoning literature calls the risk of early death (Fleurbaey et al., 2016; Leroux and Ponthiere, 2018; Ponthiere, 2020; Fleurbaey et al., 2022).

The latter problem, and also evidence of a resurgent gap in life expectancy (Chetty et al., 2016; Auerbach et al., 2017), are getting more and more attention among pension economists (Bommier et al., 2011; Gustman and Steinmeier, 2001; Haan et al., 2019), but the focus is only on differentiation of treatment based on expected longevity differences across socio-demographic groups (Ayuso et al., 2016; Holzmann et al., 2017; Vandenberghe, 2021) or occupations (Vermeer et al., 2016). The parameters of differentiation investigated in that literature comprise the retirement age, and also the replacement rate or the contribution rate during the pension build-up phase (Biskmarckian pensions), or simply the amount of the basic pension (Beveridgian pensions).

What we show in this paper is that partial de-annuitization of PAYG pensions would be as effective at addressing the inequalities and inefficiencies generated by longevity differences. And as far as we know this result is a novelty in the literature on pensions. If all (or most) longevity differences matter from a normative point of view,[29] for both the Bismarckian and Beverigdian versions of public PAGY pensions, we show that a modest de-annuitization – 4 % point reduction of the monthly pension and the corresponding payment of an upfront lump sum – would be enough to match the equality gains recorded via extensive retirement differentiaion.

Finally, the partial de-annuitization we propose here appears feasible from both a technical/financial and political point of view. First, being limited in magnitude, it is unlikely to

[29]Because they are unrelated to risky lifestyles.

generate insurmountable liquidity and transition difficulties. We show that the lump sums that would be paid correspond (on average) to less than one year of annuity payments (less than 12 monthly instalments). These sums could be borrowed from the market and financed by savings corresponding to the stream of reduced monthly instalments. Also, we do not a priori detect "cohabitation" problems: older generations could stick to full annuitization, while the next ones would get partial de-annuitization. Second, partial de-annuitization as we model it in this paper is unlikely to significantly compromise the longevity insurance role of public pensions; the very one that pushes our US colleagues to recommend more annuitization, and corresponds to the historical role of pensions that the general public probably still support to a large extent. Third, administratively, it is much less costly and easier to implement than retirement age differentiation.[30] It does not require upstream analysis of the determinants of longevity or the verification of pensioner's category as would, by definition, be the case with retirement age differentiation.[31] Fourth, unlike retirement age differentiation, de-annuitization is not prone to legal challenges. It amounts to treating everyone equally. By definition, this is not the case of differentiation. Would differentiating retirement age by gender be legal? Probably not in the US and in Europe. The European Court of justice bans any form of difference in treatment between women and men as to the legal age of retirement. Finally, it is also exempt from the risk of misreporting or moral hazard. De-annuitization amounts to paying a lump sum at a certain age to everyone. As shown repeatedly in public economics, these types of payments are exempt from the risk of misreporting or disincentive to perform (here earn a lower wage conducive to a lower income percentile to get classified as a short-lived person entitled to early retirement).

Acknowledgement

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[30] Or any policy that consists of differentiating a parameter of the pension system based on expected longevity differences.

[31] The difficulty to differentiate a policy is known in economics as the "tagging" problem (Akerlof, 1978).

6 Appendix

6.1 Graphical representation of Beveridge pensions

The first-best situation amounts to ensuring that every person receives the horizontal dashed line in terms of (lifetime) benefits ($b\kappa$) where κ is the number of years spent in retirement by the reference pensioner with longevity $\boldsymbol{l}$. Actual/realised lifetime benefits correspond to the solid/doted rising lines. People who die before retirement age receive no benefits. Beyond that point, lifetime benefits rise at a rate b.[32] Under uniform retirement age (solid rising line), only individuals whose longevity coincides with reference $\boldsymbol{l}$ get the first best. Under differentiated retirement age (doted rising lines, only 2 different retirement ages are depicted in Figure 7), the number of people who get on the green line a priori rises. But note again that situations synonymous with "undershooting" or "overshooting" are still very frequent.

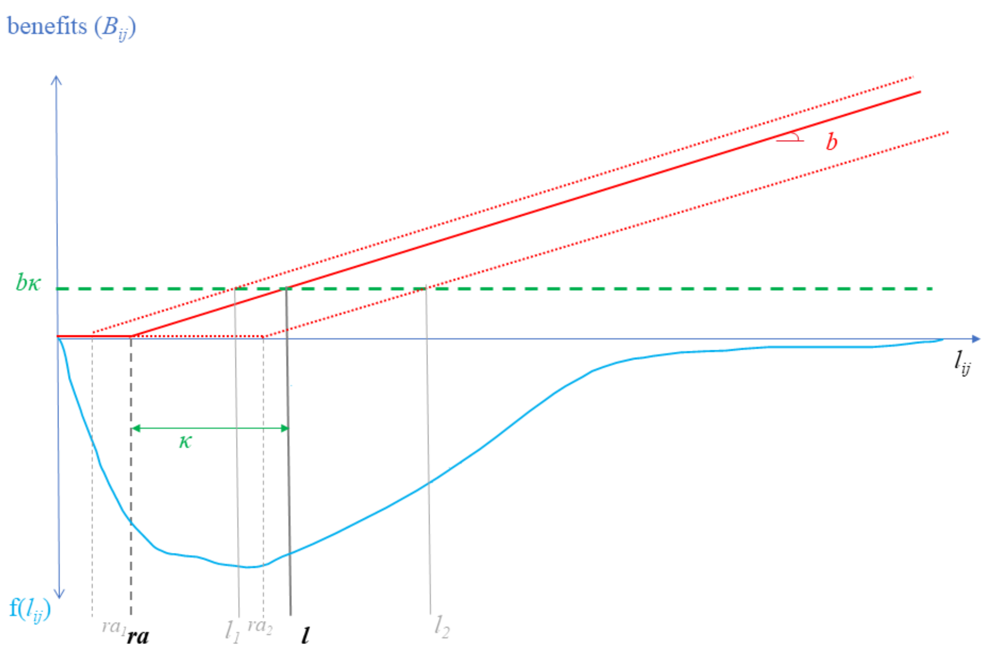

Figure 7: Beveridgian pension under uniform vs diff. ret age

[32]Here again we get a sense of what would happen if, instead of differentiating the age or retirement, policy markers were to differentiate pension instalment b by SES. That would amount to differentiating the slope of the red curve to increase the chance of crossing the green line.

6.2 Cohortal budget equivalence under retirement age differentiation

Retirement age differentiation can generate budgetary savings in comparison with uniform retirement. This is because the overall (or average) time spent in retirement may go down. What is more, with a Bismarckian regime, the money not spent on those who live longer is likely to outweigh the cost of financing early retirement to the benefit of the short-lived individuals. If these budgetary savings are used to increase b (Beveridge) or θ (Bismarck) and so achieve cohortal budget equivalence, then the equity gaps [eq.(5)] — that inevitably still exist under retirement age differentiation — will be "priced" at a higher rate. Say differently, in our (in)equity index, the time component of the gaps will be multiplied by a higher "price" b (Beveridge) or θ (Bismarck). Thus, the equity gains achieved in terms of years spent in retirement will be eroded by the (remaining) year gaps weighing more in monetary terms.

The algebra below identifies b (Beveridge) or θ (Bismarck) ensuring cohortal budget equivalence and discusses their determinants. But the key take-home result is visible in Table 2. The second line of the table reports the equity gains achieved by retirement age differentiation when cohortal budget equivalence is imposed. We see that, in particular for Bismarckian pensions, equity gains are lower and potentially negative, suggesting that the negative effect of higher "prices" applicable to persistent gaps dominates the gains achieved by reducing the (time) magnitude of these gaps.

Table 2: Numeric results: values of the Equity Gap Index[a] achieved via retirement age differentiation [without and with cohortal budget equivalence]

	Pension regime			
	Bismark		Beveridge	
	F	M	F	M
Equity Gap Index[a],[b] without cohortal budget equivalence	0.963[c]		0.964[d]	
Equity Gap Index[a] with cohortal budget equivalence	1.015[c]		0.969[d]	
Inc. perc.				
1	61.407	56.507	60.428	54.192
10	63.881	59.656	63.576	58.200
20	64.812	60.688	64.760	59.513
30	65.535	61.795	65.681	60.922
40	66.022	63.010	66.301	62.467
50	66.491	63.785	66.898	63.455
60	66.948	64.456	67.479	64.307
70	67.381	65.090	68.029	65.114
80	67.931	65.784	68.730	65.997
90	68.620	66.807	69.607	67.300
100	69.697	68.345	70.976	69.256
Ref. ret. age	65	65	65	65

[a]: The simulated values of equity gap indexes exposed in eq.(15) and eq.(19). The propensity of these indices to be below 1 tells us about the equity gains that can be achieved via retirement age differentiation. We report here the gains achieved by resorting to 200 different reference retirement ages/longevities (i.e $j = 1 \dots 200$, corresponding to 2 genders $\times$ 100 income percentiles). The differentiated retirement ages we use are those visible in Figures 3, 4. In the lower part of this table we report only a sample of them.
[b]: Meaning that we assume that the planner does not "absorb" the net budgetary surplus(deficit) generated by the move from uniform to differentiated retirement age, and thus translates these into higher(lower) value of θ and b to guarantee that the same total budget is spent on pensions.
[c]: $I^{Bismarck} \in [0, 1]$ as defined by eq.(15)
[d]: $I^{Beveridge} \in [0, 1]$ as defined by eq.(19)

Now we turn the algebra. To ensure strict budgetary equivalence, the Beveridgian planner should use an annuity b^{rad} such that

$$
\begin{aligned}
b^{rad}\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{ij}-\boldsymbol{ra}_j) &= b\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j}-\boldsymbol{ra}) \\
\frac{b^{rad}}{b} &= \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j}-\boldsymbol{ra})}{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{i,j}-\boldsymbol{ra}_j)}
\end{aligned} \tag{20}
$$

The budget-balancing b^{rad} is thus inversely proportional to the change (possibly reduction) of aggregate time spent in retirement due to introducing different retirement ages.

The Bismarkian planner's equivalent problem is a bit more complex. She needs the ratio of benefits to contribution to be equivalent to what it is under uniform retirement. Formally we need

$$
\frac{BEN^{rad}}{CONT^{rad}}\Big/\frac{BEN}{CONT} = 1
$$

where

$$
\begin{aligned}
BEN^{rad} &\equiv \delta^{rad}\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{i,j}-\boldsymbol{ra}_j)\, w_{i,j} \\
BEN &\equiv \delta\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j}-\boldsymbol{ra})\, w_{i,j} \\
CONT^{rad} &\equiv \eta^{rad}\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left[S_{i,j}(\boldsymbol{ra}_j)\boldsymbol{ra}_j \ + (1-S_{i,j}(\boldsymbol{ra}_j))\,\boldsymbol{l_{i,j}}\right] w_{i,j} \\
CONT &\equiv \eta\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left[S_{i,j}(\boldsymbol{ra})\boldsymbol{ra} + (1-S_{i,j}(\boldsymbol{ra}))\,\boldsymbol{l_{i,j}}\right] w_{i,j}
\end{aligned} \tag{21}
$$

or equivalently

$$\frac{\theta^{rad}}{\theta} = \frac{WYIR(\boldsymbol{ra})}{WYIR(\boldsymbol{ra}_j)} \frac{WYIE(\boldsymbol{ra}_j)}{WYIE(\boldsymbol{ra})}$$

where

$$\begin{aligned}
\theta^{rad} &= \frac{\delta^{rad}}{\eta^{rad}}; \theta = \frac{\delta}{\eta} \\
WYIR(\boldsymbol{ra}_j) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra}_j)(l_{i,j} - \boldsymbol{ra}_j)\, w_{i,j} \\
WYIR(\boldsymbol{ra}) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{i,j} - \boldsymbol{ra})\, w_{i,j} \\
WYIE(\boldsymbol{ra}_j) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[S_{i,j}(\boldsymbol{ra}_j)\boldsymbol{ra}_j \; + (1 - S_{i,j}(\boldsymbol{ra}_j))\boldsymbol{l}_{i,j} \right] w_{i,j} \\
WYIE(\boldsymbol{ra}) &\equiv \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left[S_{i,j}(\boldsymbol{ra})\boldsymbol{ra} + (1 - S_{i,j}(\boldsymbol{ra}))\boldsymbol{l}_{i,j} \right] w_{i,j}
\end{aligned} \tag{22}$$

So θ^{rad} should be inversely proportional to the change of the (wage-weighted)[33] years spent in retirement ($WYIR(\boldsymbol{ra}_j)$) and proportional to the change in the (wage-weighted) years spent in employment ($WYIE(\boldsymbol{ra}_j)$). If retirement age differentiation leads to fewer wage-weigted years spent in retirement[34] (fraction 1 to the rhs of equ. (22) is > 1) and simultaneously more (wage-weighted) years spent working (fraction 2 >1) than the planner can finance $\theta^{rad} > \theta$. This means that each years-in-retirement gap in equ.(6) are "priced" at a higher rate, contributing to the erosion of the equity gains generated by retirement age differentiation.

6.3 Cohortal budget equivalence under de-annuitization

Here, we identify the conditions for de-annuitization to generate, for a cohort, overall benefits matching what is spent under uniform retirement age.

[33]In our simulations, we have assumed (average) by SES pension-relevant wages with a gradient of 1 (lowest income percentile) to 4 (highest income percentile), and a .2 gender wage gap.

[34]The `overall` number of years that in principle should not change much (as some individuals spend more time and others less time in retirement). By contrast, the wage-weighted version should fall as the people who spent less time earn less and those who spend more time

We start with partial de-annuitization of Bismarckian pensions, where a uniform fraction ($\mu \equiv \frac{LS_{i,j}}{\boldsymbol{ra}\,\eta\, w_{i,j}}$) of lifetime contributions (i.e. the "guaranteed" part of the ratio) is handed over to every retiree. For a cohort, budget equivalence is achieved if the lump-sum payment $LS_{i,j}$ and the reduced annuity, calculated with a lower replacement rate δ', verify

$$\begin{aligned}
\sum_{j=1}^{k}\sum_{i=1}^{n_j} LS_{i,j} + \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,\delta' w_{i,j} &= \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,\delta w_i \\
\sum_{j=1}^{k}\sum_{i=1}^{n_j} \mu\,\boldsymbol{ra}\,\eta w_{i,j} + \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,\delta' w_{i,j} &= \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,\delta w_{i,j} \\
\mu\,\boldsymbol{ra}\,\eta N\overline{w} &= (\delta - \delta')\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,w_{i,j} \\
\mu &= (\theta - \theta')\,\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})\,w_{i,j}}{N\boldsymbol{ra}\overline{w}} \\
\mu &= (\theta - \theta')\,\frac{WYIR(\boldsymbol{ra})}{WYIE(\boldsymbol{ra})}
\end{aligned} \tag{23}$$

To move from line 2 to line 3, we regroup expressions with δ to the right-hand side. Considering that $\mu, \boldsymbol{ra}, \eta$ are constant, we are left with $\mu\boldsymbol{ra}\eta \sum_{j=1}^{k}\sum_{i=1}^{n_j} w_{i,j}$ or equivalently $\mu\,\boldsymbol{ra}\,\eta\,N\overline{w}$ where $N$is the size of the cohort and $\overline{w}$ its average wage. On line 5 we have that $WYIR(\boldsymbol{ra})$ is the overall (wage-weighted) number of years spent in retirement and $WYIR(\boldsymbol{ra})$ is the overall (wage-weighted) number of years spent in employment.[35]

In a Beveridgian scheme, the uniform lump-sum payment LS and the reduced annuity b' must verify

[35]In our simulations, we have assumed (average) by SES (pension-relevant) wages with a gradient of 1 (lowest income percentile) to 4 (highest income percentile), and a .2 gender wage gap.

$$
\begin{aligned}
\sum_{j=1}^{k}\sum_{i=1}^{n_j} LS + \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\, b' &= \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\, b \\
LS\ N &= (b-b')\ \sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra}) \\
LS &= (b-b')\ \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})}{N} \\
LS &= (b-b')\ AYIR(\boldsymbol{ra})
\end{aligned}
\tag{24}
$$

where $AYIR(\boldsymbol{ra})$ is the average number of years spent in retirement

6.4 De-annuitization and the indifferent pensioner

Here we characterize algebraically the `indifferent` retiree (Figure 5). This is the person who is (or should be) indifferent[36] between what he gets under partial de-annuitization and under uniform retirement. With a Bismarckian system, that person has longevity l^* such that

$$
\begin{aligned}
&\frac{\theta(l^*-\boldsymbol{ra})}{\boldsymbol{ra}} = \mu + \theta' \frac{(l^*-\boldsymbol{ra})}{\boldsymbol{ra}} \\
&\qquad \text{with, given the budget equivalence condition} \\
&\qquad \mu = (\theta-\theta')\ \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\, w_{i,j}}{N\boldsymbol{ra}\overline{w}}
\end{aligned}
\tag{25}
$$

After some simple algebraic transformations we get

$$
\begin{aligned}
&l^* = \frac{\mu\,\boldsymbol{ra}}{(\theta-\theta')} + \boldsymbol{ra} \\
&\qquad \text{with from the budget equivalence condition} \\
&\qquad \frac{\mu\,\boldsymbol{ra}}{(\theta-\theta')} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij}-\boldsymbol{ra})\, w_{i,j}}{N\overline{w}} = \text{constant}|\boldsymbol{ra}
\end{aligned}
\tag{26}
$$

[36] Assuming perfect foresightedness.

In other words, l^* is independent of de-annuitization parameters μ and $\theta' < \theta$. So the intensity of de-annuitization has no impact on the longevity identifying the indifferent retiree. Note, however, that l^* is a function of (uniform) retirement age ($\boldsymbol{ra}$).

Similarly, with Beveridgian pension system, the indifferent pensioner has longevity l^* that verifies

$$b(l^* - \boldsymbol{ra}) = LS + b'(l^* - \boldsymbol{ra})$$
$$\text{with, given the budget equivalence condition} \tag{27}$$
$$LS = (b - b')\,\frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})}{N}$$

After some simple algebraic transformations we get

$$l^* = \frac{LS}{(b - b')} + \boldsymbol{ra}$$
$$\text{with, given the budget equivalence condition} \tag{28}$$
$$\frac{LS}{(b - b')} = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_{i,j}(\boldsymbol{ra})(l_{ij} - \boldsymbol{ra})}{N} = \text{constant}|\boldsymbol{ra}$$

where, again, l^* turns out to be independent of de-annuitization parameters LF and $b' < b$. So the intensity of de-annuitization has no impact on the longevity identifying the indifferent retiree. But note again that l^* is a function of (uniform) retirement age ($\boldsymbol{ra}$).

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