Estimating Productivity-Related Labour Adjustment Costs.

How do new hires and different categories of leavers, including retirees, compare?

Preliminary Version. Do not quote

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Abstract

In this paper, we exploit unique firm-level private economy Belgian data on labour turnover containing annual detailed information on shares of stayers, new hires, and also different categories of leavers: those who were fired, those who left voluntarily, and those who retired as they become eligible for old-age (early) pension benefits. We use the Hellerstein-Neumark framework to estimate the relative labour productivity of these different labour categories and posit that these are good estimates of the so-called internal/productivity-related Labour Adjustment Costs (LACs) that have received limited attention in the existing eponymous literature. Results point to gains for new hires and relatively important losses for leavers, driven by sizeable productivity handicaps during the final year of work.

Keywords: Labour productivity, labour cost, labour turnover, prospective retirees

JEL Codes: H55, J26, J14

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1 Introduction

The concept of labour adjustment costs (LACs) can be seen as an extension of the seminal work of Oi (1962) on labour being a quasi-fixed factor of production. The key idea is that firms incur additional costs when changing the composition and the overall level of their labour force. These costs are considered economically significant and have important implications for the dynamic of labour demand. They comprise costs of search (e.g., recruiter agency fees), selection and hiring (e.g., interviews), and training, as well as costs related to firing (e.g., lawsuits, severance packages). But also productivity losses (e.g. lack of firm-related human capital for new hires, peer disruption or demotivation for leavers...). Labour economists regularly mobilise the assumption of the existence of significant LACs. For example, in the presence of LACs, they predict that a firm cannot adjust its labour demand costlessly and has the incentive to minimize labour turnover (Dixit, 1997), something called the labour demand "stickyness" or labour "hoarding".

The LACs literature (Hamermesh, 1995) distinguishes *i)* gross LACs incurred when a worker leaves or joins the firm, and are independent of the change in the overall level of employment: these can be linked to the profile or status of the individual joining or about to leave a job, and *ii)* net LACs strictly related to changing the overall number of workers/hours. The literature (Hamermesh, 1993) also distinguishes internal/productive vs. external adjustment costs. Internal or productivity-related LACs point to the loss of output caused by employment changes strictly inherent to the production process (e.g. disruption to the accustomed flow of work among experienced employees, demotivation and divestment when people[1] know the work horizon is short...). External LACs refer to any additional cost related to employment changes occurring off the production/shop floor (e.g. recruitment/training costs, firing costs).

While the labour adjustment cost hypothesis has been mobilized in many important works on labour demand, conducting a formal test of this hypothesis has been challenging due to the inherent difficulty of identifying accurate proxies or directly measuring LACs, and even more so their different components (Golden et al., 2020). The existing literature has mostly relied on proxies of LACs and has generally focused on one aspect of LACs. For example, Banker et al. (2013) consider country-level employment protection legislation (EPL) as a proxy for LACs associated with the legal impediments to firing workers in an international context. Nguyen (2022) use level of labour skills as a proxy for LACs costs

[1] Both employees and employers.

incurred to search, hire, train, retain, and fire employees. Another example is Dierynck et al. (2012) considering the Belgian legal requirements that result in ex-ante higher costs related to adjusting the number of white-collar employees compared to the costs of adjusting the number of blue-collar employees. But, in these three studies, the proxies only capture "external" LACs and ignore "internal/productivity-related" ones.

More direct and more encompassing measures of LACs come from surveys and/or accounting data and have been reviewed by Hamermesh (1995). These are also generally focused on external i.e. hiring/separation costs. They are supportive of the idea that LACs can be significant[2] but deliver divergent results as to the magnitude of costs. However, none of them attempts to `estimate the internal/productivity cost of labour adjustment`. One of the main contributions of this paper to the LACs literature is to deliver reliable estimates of these internal/productivity-related costs of labour turnover. More in detail, its contribution to the literature on LACs is fourfold:

- First, show that internal/productivity-related LACs can be `directly retrieved from firm-level data` containing information about labour flows. By "directly retrieved" we mean as econometric estimates of productivity variations caused by labour turnover – thus saving the time devoted to detailed cost accounting analysis as is traditionally done in the LACs literature. We suggest that this can be done by implementing methods commonly used by the production function literature (Hellerstein et al., 1999; Vandenberghe et al., 2013; Ackerberg et al., 2015) to assess the relative productivity of different types of labour. Using firm-level panel information on shares of stayers, new hires and leavers (and using stayers as a reference), we propose *i)* estimating the productivity handicap (or advantage) of the two latter groups and *ii)* considering these as reasonable estimates of LACs.

- Second, show that internal/productivity-related LACS can be retrieved for `the different categories of workers` contributing to labour turnover. What we show in this paper is that with the Hellerstein-Neumark framework (HN hereafter) and firm-level semi-aggregated data on labour level and turnover, one can estimate LACs of new hires and leavers. Also, we can compare LACs for different categories of leavers: people who leave because they have been dismissed vs. people who leave for other reasons (presumably more voluntarily).

[2]Results by Blatter et al. (2012) analysing Swiss accounting data show that average hiring costs range, depending on firm size, from 10 to 17 weeks of wage payments.

- Third, say something about the LACs stemming from `people who retire`. As far as we know, this is a group that has never been examined by the LACs literature, although it fully qualifies as a potential source of turnover cost. We show evidence supporting the idea that retirement (both early and mandatory age retirement), comes with important productivity declines during the final year of work.

- Fourth, assess the relative importance of so-called gross vs. net LACs. We do that by re-estimating productivity-related LACs for firms that have not experienced a significant change in their overall labour force over the years present in our panel and comparing these to the estimates delivered by the entire sample of firms.

2 The Hellerstein-Neumark (HN) Framework

As said above, we propose using the framework developed by Hellerstein et al. (1999) to quantify internal/productivity-related LACs for different categories of workers generating labour turnover. The strategy these authors propose is to estimate (relative) marginal labour productivity for different categories of workers using between- and within-firm variations of their share in total employment. That model is traditionally used to assess gender/age/education-related productivity differences (Ours and Stoeldraijer, 2011;Dostie, 2011;Vandenberghe, 2011;Vandenberghe et al., 2013; Vandenberghe, 2013;Lebedinski and Vandenberghe, 2014). We will show below that it can be applied to situations where labour heterogeneity consists of stayers vs. new hires vs. (different categories of) leavers.

HN consider an augmented Cobb-Douglas production function

$$Y_i = AQL_i^{\alpha} K_i^{\beta} \tag{1}$$

where Y_i is output in firm i, K_i is capital, and labour heterogeneity $\rightarrow$ [quality of] labour index QL_i.

We have that L_{ij} is the number of type j workers (e.g. young/prime-age/old; men/women; stayers/new recruits/leavers) in firm i. Types are perfectly substitutable with different marginal products μ_j (assumed identical across firms, meaning that i can be dropped from μ). Each type j is an input of the quality of labour aggregate:

$$QL_i = \sum_j \mu_j L_{ij} = \mu_0 L_{i0} + \mu_j L_{i1} + ...\mu_n L_{in} \tag{2}$$

with $j = 0, \ldots, n$ and μ_0 the marginal productivity of (reference) type 0 workers.

$$\begin{aligned} Y_i = A\left(\mu_0 L_{i0} + \mu_1 L_{i1} + \ldots \mu_n L_{in}\right)^{\alpha} K_i^{\beta} \\ Y_i = A(\sum_j \mu_j L_i)^{\alpha} K_i^{\beta} \end{aligned} \tag{3}$$

Note that the marginal labour productivity (MLP) of labour types is

$$\begin{aligned} MLP_0 \equiv \frac{\partial Y}{\partial L_0} = A\alpha[\mu_0 L_0 + \mu_1 L_1 + \mu_n L_n]^{\alpha-1}\mu_0 K^{\beta} \\ MLP_1 \equiv \frac{\partial Y}{\partial L_1} = A\alpha[\mu_0 L_0 + \mu_1 L_1 + \mu_n L_n]^{\alpha-1}\mu_1 K^{\beta} \end{aligned} \tag{4}$$

thus the relative $MLP_{1;0} = \mu_1/\mu_0$.

From there, it is easy to show (see Appendix 1.1) that the production function becomes:

$$lnY_i = lnA + \alpha\left(ln\mu_0 + lnL_i + \sum_{j>0}(\lambda_j - 1)S_{ij}\right) + \beta lnK_i \tag{5}$$

or, equivalently, as a fully linearized expression

$$y_i = B + \alpha l_i + \sum_{j>0} \eta_j S_{ij} + \beta k_i \tag{6}$$

where $y_i \equiv lnY_i, l_i \equiv lnL_i, k_i \equiv lnK_i; B \equiv lnA + \alpha ln\mu_0$ and in particular $\eta_j = \alpha(\lambda_j - 1); j > 0$ with the estimated MLP of type j equal to $\widehat{\lambda}_j = \widehat{\eta}_j/\widehat{\alpha} + 1$

3 Data

Our data come from the Bel-first data basis published by the bureau Van Dijk. It consists of a firm-level panel (2014-2023) containing, for each firm i, the number of (full-time-equivalent) employees at the end of the year ($L_{i,t}^{eoy}$), but also the number of new hires ($L_{i,t}^{hires}$) and the number of people who left the company (for a variety of motives k) **during** the year

($L_{i,t}^{leavers^k}$). Assuming the actual moment of departure is uniformly distributed over the year, the total number of workers that actually worked during that year can be decomposed as

$$\begin{aligned}
\tilde{L}_{i,t} &= \tilde{L}_{i,t}^{stayers} + \tilde{L}_{i,t}^{hires} + \sum_j \tilde{L}_{i,j,t}^{leavers} \\
\text{with } \tilde{L}_{i,t}^{stayers} &\equiv L_{i,t}^{eof} - L_{i,t}^{hires} - \sum_j L_{i,j,t}^{leavers} \\
\tilde{L}_{i,t}^{hires} &\equiv .5\ L_{i,t}^{hires} \\
\tilde{L}_{i,t}^{leavers^k} &\equiv .5\ L_{i,t}^{leavers^k} \\
k &= \text{retired, early-retired, dismissed, other}
\end{aligned} \tag{7}$$

The key descriptive statistics for the data used in the paper are reported in Table 1.

Table 1: Descriptive statistics

	mean	sd	min	max
Net value added th. EUR (log)	8.917	1.317	0.000	15.572
Capital th. EUR (log)	10.194	1.407	4.419	18.966
Labour (log)	4.117	1.396	0.000	10.412
Purch. int. goods & serv. th. EUR (log)	10.249	1.221	1.386	18.029
Share new hires	0.137	0.126	0.000	1.000
Share leavers (overall)	0.123	0.112	0.000	1.000
Share retirees	0.006	0.018	0.000	1.000
Share early retirees	0.001	0.007	0.000	0.500
Share dismissed	0.022	0.043	0.000	1.000
Share oth. leavers	0.100	0.104	0.000	1.000
N		49,529		

Source: Bel-first
Reported total labour and labour shares correspond to $\tilde{L}$'s defined in eq. (7).

4 Coping with simultaneity/endogeneity

The estimation of a production function is complicated by the presence of endogeneity problems. To deal with the one that is the most likely to affect our estimation of labour productivity (i.e. the simultaneity bias[3]), in this paper, we resort to the control function method à

[3]For instance, the simultaneity of a negative productivity shock (due to the loss of a major contract) and a recruitment freeze or even downsizing (i.e. less or no new hires, more leaves), source of reverse causality.

la Ackerberg, Caves & Frazer (Ackerberg et al., 2015) (ACF hereafter). The idea consists of using intermediate inputs to control/proxy short-term unobserved productivity shocks that may bias our estimates of labour productivity. The control function approach proposed by Ackerberg et al. (2015), capitalises on (and improves) the methods developed by Olley and Pakes (1996) and Levinsohn and Petrin (2003). In practice, ACF uses the firm's demand for intermediate inputs (i.e. electricity consumption, purchase of services ...). The rationale behind this is that changes is the use of intermediate inputs is strongly related the firm's unobserved productivity shocks, typically caused by short-term variations of the demand for the goods or services they produce. The identifying assumption also rests on the assumption that firms can relatively easily (and quickly) adjust their use of intermediate inputs, in response to the productivity shocks.

The full exposure of the ACF estimator and its logic is to be found in Appendix 1.2.

5 Results

In Section 5.1, (Table 2) we first report the full OLS econometric results (col1) corresponding to teh estimation of eq. (6). We then (col2) present our preferred results, i.e. those delivered by the ACF method that addresses the risk of simultaneity/endogeneity bias. Further down, we report (and synthesise visually) the LACs implied by these econometric results (Section 5.2).

5.1 Econometric results

In the upper part of Table 2, we report the OLS and ACF estimates for parameters η in eq. (6). For new hires, the ACF-estimated η of .120 (statistically different from zero) suggests some productivity gains relative to the stayers. We see in the lower part of Table 2 that the corresponding implied marginal productivity (λ) is 1.257 (not statistically different from one). In other words, we find evidence that new hires are 25.7% more productive than stayers.

Things are quite different when we turn to the different categories of leavers. For instance, prospective retirees display an η equal to -.527 (statistically significantly different from zero). Their implied (relative) marginal productivity is estimated to be -.127 (and statistically

significantly different from one). In other words, their relative productivity is so low that it corresponds to "value destruction". we estimate the productivity handicap of early retirees to be even larger (λ =-1.277) than that of retirees. Finally, both dismissed and other leavers display statistically significant productivity handicaps (λ of respectively .643 and .632). Overall, the leavers tend to be synonymous with large productivity-related LACs.

Table 2: OLS estimates of relative labour productivity and labour cost (Standard errors)

	1.OLS	2.ACF
Capital	0.394***	0.468***
	(0.004)	(0.039)
Labour	-0.439***	-0.507***
	(0.004)	(0.015)
Share new hires	-0.027	0.096***
	(0.037)	(0.035)
Share retirees	-0.825***	-0.613***
	(0.228)	(0.078)
Share early retirees	-1.021***	-1.325***
	(0.311)	(0.114)
Share dismissed	-0.173*	-0.241***
	(0.091)	(0.030)
Share oth. leavers	-0.214***	-0.226***
	(0.042)	(0.030)
Controls	Year, Region, NACE2	
N	49,529	40,721
* p<0.10, ** p<0.05, *** p<0.01		
Standard errors in parentheses		
Implied marginal values	λ, (p-value H0: $\lambda = 1$)	
Hires	0.952(0.465)	1.194(0.012)
Retirees	-0.470(0.000)	-0.243(0.000)
E.retirees	-0.820(0.001)	-1.687(0.000)
Dismissed	0.691(0.057)	0.512(0.000)
Oth. leavers	0.618(0.000)	0.542(0.000)

Source: Bel-first

5.2 Implied Labour Adjustment Costs: a Synthetic View

In the figures below, we synthesise the key results that can be retrieved for the econometric results displayed in detail in the previous section. In Fig. **??** we display internal/productivity-related (λ's) implied by the econometric results. The reported distance to the magenta

vertical line expresses LACs as percentage-point gaps vis-à-vis the reference group (i.e. the stayers). We also translate these values in Euros (Fig. 2). For that, we multiply our estimates of (marginal) labour productivity losses (λ) by the year 2021 average labour productivity of stayers underpinning our Bel-first data that we estimate to be equal to 87,300 euros. In Fig. 1, referring to our preferred ACF results, we verify what was already visible in the previous tables. New hires represent a productivity gain of max. 22,000 euros. By contrast, all leavers are synonymous with (relative) productivity losses. They range from -31,000 (other leavers), -32,000 (dismissed leavers), -98,000 (retirees) to -198,000 euros (early retirees).

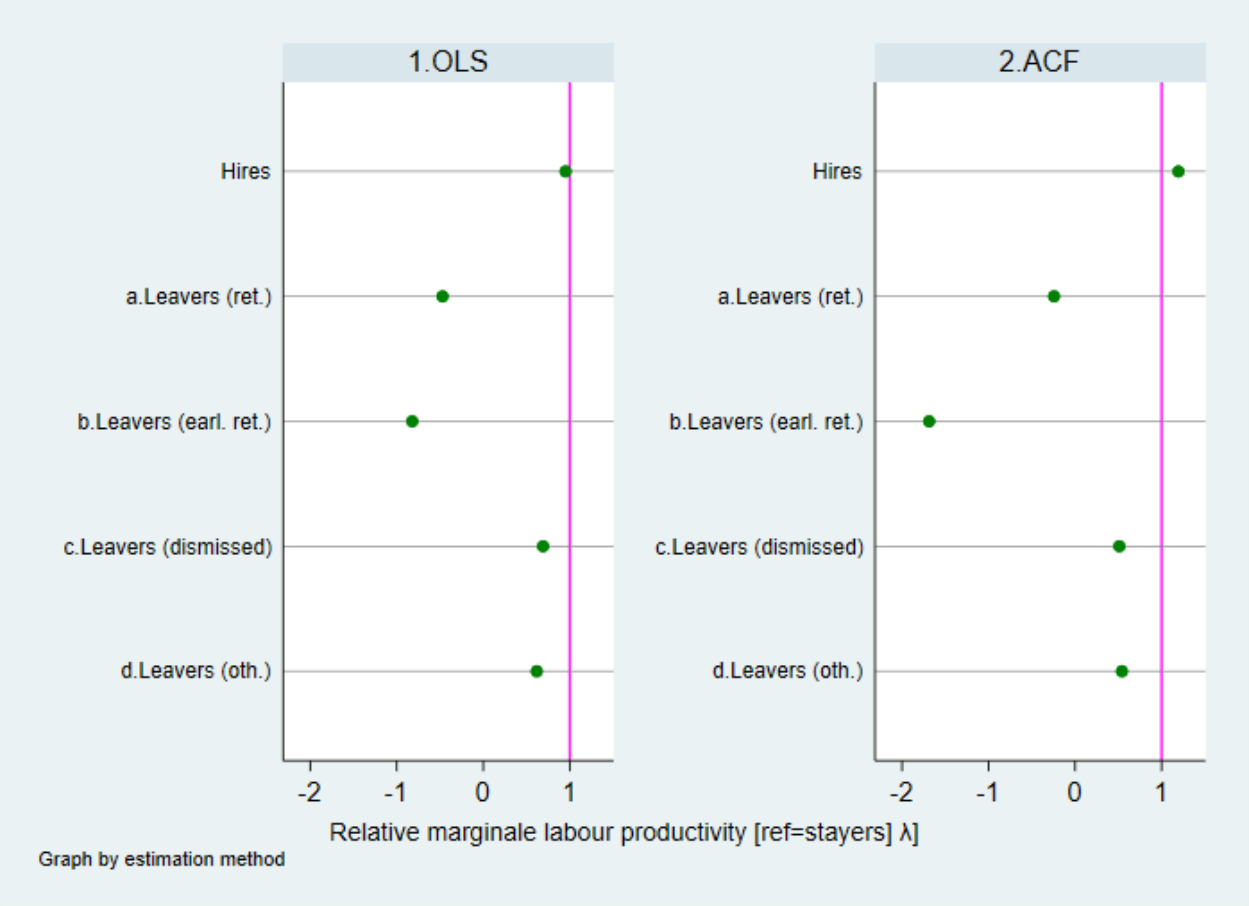

Figure 1: Productivity-related LACs (ref. stayers' productivity)

Reported values correspond to λ.

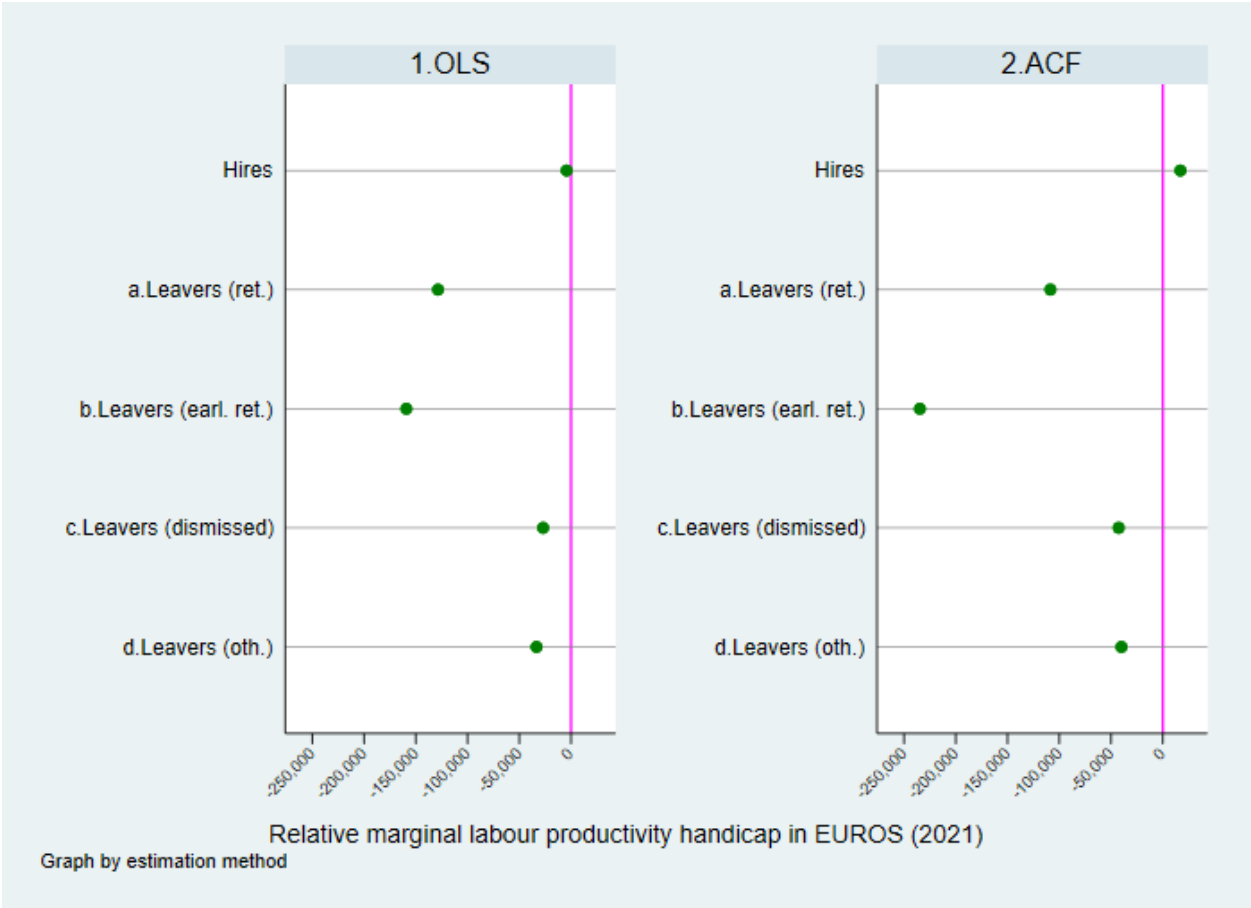

Figure 2: Productivity-related LACs in EUROs (ref. stayers' productivity)

Reported values are obtained by multiplying the estimates of (marginal) labour productivity losses (λ) by the year 2021 average labour productivity of stayers i.e. 87,300 euros.

5.3 Assessing the contaminating role of NET adjustment costs

So far, we have implicitly assumed that we were estimating what the literature calls "gross" LACs. Strictly speaking, however, it could be that our results also capture "net" adjustment costs, namely those stemming from changes in the overall level of employment.

To assess the potential role of those, we implement a simple robustness strategy: we characterize the firms forming our simple in terms of change of the overall level of employment and re-estimate all the above results using only firms with low changes. How do we define low vs. high change? For each firm, we compute the standard deviation of the level of employment over the panel years (2014-2023). To account for the inflationary

effect of being observed for more years, we regress the standard deviation over a constant and the number of spells characterizing the firm. We retain the residuals as an adjusted measure of change of employment.[4] The final step is to compute the deciles of that adjusted measure of change and to re-estimate our results for firms belonging to the 5 first deciles; the low-change ones.

Results are reported in Appendix 1.3. Due to the absence of significant deviation with the results reported in Table 2, we conclude that what we have estimated so far corresponds to gross LACs. And this is something that aligns with Hamermesh (1995): the largest part of LACs firms are confronted with correspond to gross LACs.

6 Concluding remarks

Most of the existing empirical literature has focused on estimating – and most of the time just proxying – the "external" Labour Adjustment Costs (LACs) i.e. those incurred to search, hire, train, retain, and fire employees. The purpose of this paper was to show that it is possible to estimate econometrically the importance of the so-called "internal" (ie. productivity-related) LACs; those that correspond to a lack of firm-related human capital for new hires or, when it comes to leavers, to team-work disruption or demotivation stemming from both the demand- or the supply-side of the employment relationship. To that end, we suggest implementing the Hellerstein-Neumark (HN) framework commonly used in the production function literature and applying it to firm-level data containing information about worker flows.

Applying that HN framework to unique Belgian firm-level data covering the 2014-2023 period, we retrieve estimates of productivity-related LACs separately for new hires and different categories of leavers: retirees, early retirees, dismissed leavers and other (presumably voluntary) leavers. First, we show that it is important to distinguish new hires from leavers. Whereas new hires compare favourably to stayers, leavers invariably represent a labour productivity loss, implying that internal/productivity-related LACs are primarily driven by these workers. Second, productivity-related LACs for leavers are sizeable, possibly as large as external/labour cost-related LACs traditionally emphasized by the LACs literature. Third, We show that the magnitude of productivity-related LACs varies significantly across

[4] A measure of employment change netted out of the impact of the flow of time.

categories of leavers. It is for prospective retirees and early retirees that productivity losses are the largest, as they exceed the annual output (i.e. value added) of the typical stayer. This latter result calls for further research. Discussions on the relationship between productivity and retirement have been rare in the pension or labour literature.[5] A starting point could be to investigate the role of short-horizon effects on labour productivity.

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[5] One possible exception is Ippolito (1998).

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Appendix

1.1 The HN model: development

Consider the following Cobb-Douglas-augmented HN production function in logs

$$lnY_i = lnA + \alpha lnQL_i + \beta lnK_i \tag{8}$$

and then transform the labour quality index as

$$\begin{aligned} QL_i &= \mu_0 L_{i0} + \mu_1 L_{i1} + \dots \mu_n L_{in} \\ &= \mu_0 L_{i0} + \mu_1 L_{i1} + \dots \mu_n L_{in} + \mu_0 L_i - \mu_0 L_i \\ &= \mu_0 L_i + \mu_0 L_{i0} + \mu_1 L_{i1} + \dots \mu_n L_{in} - \mu_0 L_{i0} - \mu_0 L_{i1} - \dots \mu_0 L_{in} \\ &= \mu_0 L_i + (\mu_1 - \mu_0) L_{i1} + \dots (\mu_n - \mu_0) L_{in} \\ &= \mu_0 L_i + \sum_{j>0} (\mu_j - \mu_0) L_{ij} \end{aligned} \tag{9}$$

Multiplying/dividing 2nd right-hand-side term by $\mu_0 L_i$

$$\begin{gathered} QL_i = \mu_0 L_i + \mu_0 L_i \sum_{j>0} (\frac{\mu_j}{\mu_0} - 1) \frac{L_{ij}}{L_i} \\ \mu_0 L_i (1 + \sum_{j>0} (\lambda_j - 1) S_{ij}) \end{gathered} \tag{10}$$

where $S_{ij} \equiv L_{ij}/L_i$ and $\lambda_j \equiv \mu_j/\mu_0$.

Taking the log

$$lnQL_{it} = ln\mu_0 + lnL_i + ln(1 + \sum_{j>0} (\lambda_j - 1) S_{ij}) \tag{11}$$

Since $ln(1+x) \approx x$ for small values of x[6], in eq. (12) $\sum_{j>0} (\lambda_j - 1) S_{ij}$ becomes

[6] Mathematical reminder:

- Linear approx. (Taylor expansion) rule for $f(x)$ in $x = a$
  $f(x) \approx f(a) + f'(a)[x - a]$ if $x - a$ is small

$$lnQL_{it} \approx ln\mu_0 + lnL_i + \sum_{j>0}(\lambda_j - 1)S_{ij} \tag{12}$$

The production function becomes:

$$lnY_i = lnA + \alpha\left(ln\mu_0 + lnL_i + \sum_{j>0}(\lambda_j - 1)S_{ij}\right) + \beta lnK_i \tag{13}$$

or, equivalently, as a fully linearized expression (that can be estimated with OLS)

$$y_i = B + \alpha l_i + \sum_{j>0}\eta_j S_{ij} + \beta k_i \tag{14}$$

where $y_i \equiv lnY_i, l_i \equiv lnL_i, k_i \equiv lnK_i$
$B \equiv lnA + \alpha ln\mu_0; \eta_1 = \alpha(\lambda_1 - 1)\ldots\eta_n = \alpha(\lambda_n - 1)$

1.2 The ACF estimation method

Turning to the `econometric estimation` of HN using firm-level panel data, the OLS sample-error term potentially could consist of:
- short-term TFP term χ_{it}, is observed by the firm (but not by the econometrician) and (partially) anticipated by the firm (hence it is correlated with labour inputs $l_{i,t}; S_{i,j,t}$)
- a purely random shock $\epsilon_{i,t}$

In other words, we would have

$$\begin{aligned} y_{it} = B + \alpha l_{it} + \sum_{j>0}\eta_j S_{ijt} + \beta k_{it} + \nu_{it} \\ \nu_{it} = \chi_{it} + \epsilon_{it} \end{aligned} \tag{15}$$

- Application to $f(z) = ln(z)$ in $z = a = 1$ with $z \equiv 1 + x$
  $ln(z) \approx ln(a) + ln(a)'[z - a]$
  $ln(1 + x) \approx ln(1) + 1/1[1 + x - 1]$
  $\rightarrow ln(1 + x) \approx x$ if x is small
  NB: $ln(z)' = 1/z = 1$ in $z = 1$

where $cov(\chi_{it}, l_{it}) \neq 0, cov(\chi_{it}, S_{ijt}) \neq 0, E(\epsilon_{it}) = 0$, t

How to cope with χ_{it} ? Our preferred approach consists of proxying it (Olley and Pakes, 1996; Levinsohn and Petrin, 2003) using demand for intermediate inputs (Ackerberg et al., 2015)

$$\begin{aligned} \nu_{it} &= \chi_{it} + \epsilon_{it} \\ int_{it} &= f(\chi_{it}, k_{it}, ql_{it}) \\ ql_{it} &\equiv \alpha l_{it} + \sum_{j>0} \eta_j S_{itj} \end{aligned} \tag{16}$$

Assuming $f(.)$ can be inverted

$$y_{it} = B + ql_{it} + \beta k_{it} + \overbrace{f^{-1}(int_{it}, k_{it}, ql_{it})}^{\chi_{it}} + \epsilon_{it} \tag{17}$$

The ACF algorithm consists of two stages.
Stage 1: regress y_{it} on a composite term Ψ_{it} that comprises a constant, a 3rd order polynomial expansion in $int_{it}, k_{it}, ql_{it}$.

$$y_{it} = \Psi(const, int_{it}, k_{it}, ql_{it}) + \epsilon_{it} \tag{18}$$

Note that Ψ encompasses $\chi_{it} = f^{-1}(int_{it}, k_{it}, ql_{it})$ and that β, η_j, α are not identified yet

Stage 2: generate implied values for χ_{it} using stage 1 estimates $\widehat{\Psi}$ & candidate values[7] for the coefficients β, η_j, α

$$\chi_{it} = \widehat{\Psi} - ql_{it}(\alpha^c, \eta_j^c) - \beta^c k_{it} \tag{19}$$

ACF assume further that the evolution of χ_{it} follows a first-order Markov process

$$\chi_{it} = E[\chi_{it} \mid \chi_{it-1}] + \xi_{it} \tag{20}$$

realization of χ_{it} depends on some function $g(.)$ (known by the firm) of $t-1$ realisation

[7] For example OLS estimates

and a (random) innovation term ξ_{it}.

$$\chi_{it} = g(\chi_{it-1}) + \xi_{it} \tag{21}$$

Assuming capital in period t was determined at period $t-1$ (or earlier) (i.e. may take a full period for new capital to be ordered and put to use) it must be uncorrelated with the implied innovation terms ξ_{it}, hence the first moment condition

$$E[\xi_{it} \mid k_{it}] = 0 \tag{22}$$

Labour inputs observed in t are probably chosen sometime before, although after capital – say in $t-b$, with $0 < b < 1$. As a consequence, l_{it}, ql_{it} will be correlated with at least part of the productivity innovation term ξ_{it}. But assuming labour inputs were chosen at time $t-b-1$ (or earlier), labour inputs in $t-1, t-2, \ldots$ should be uncorrelated with ξ_{it}. Moments conditions write

$$E[\xi_{it} \mid ql_{it-1}, ql_{it-2} \ldots] = 0 \tag{23}$$

or more explicitly, given the composite nature of ql_{it}, we have:

$$\begin{aligned} E[\xi_{it} \mid l_{it-1}, l_{it-2} \ldots] = 0 \\ E[\xi_{it} \mid S_{ijt-1}, S_{ijt-2} \ldots] = 0 \end{aligned} \tag{24}$$

1.3 Assessing the contaminating role of NET adjustment costs

The results reported in the two tables below are obtained with the firms displaying a lower level of change in their overall labour force; thus those where, presumably, so-called net LACs (strictly related to changes in the overall level of employment/hours) are minimal. Table 3 reports the econometric results whereas the visualisation of the key implied marginal productivities is to be found in Fig. 3, 4.

Table 3: OLS-ACF estimates of relative labour productivity and labour cost (Standard errors)- Assessing the contaminating role of NET adjustment costs

	1.OLS	2.ACF
Capital	0.423***	0.476***
	(0.006)	(0.050)
Labour	-0.470***	-0.532***
	(0.006)	(0.020)
Share new hires	-0.031	0.120***
	(0.056)	(0.027)
Share retirees	-0.624***	-0.527***
	(0.229)	(0.075)
Share early retirees	-0.423	-1.066***
	(0.417)	(0.158)
Share dismissed	-0.142	-0.167***
	(0.124)	(0.036)
Share oth. leavers	-0.136**	-0.173***
	(0.059)	(0.031)
Controls	Year, Region, NACE2	
N	24,751	21,181
* p<0.10, ** p<0.05, *** p<0.01		
Standard errors in parentheses		
Implied marginal values	λ, (p-value H0: $\lambda = 1$)	
Hires	0.941(0.576)	1.257(0.000)
Retirees	-0.178(0.006)	-0.127(0.000)
E.retirees	0.201(0.310)	-1.277(0.000)
Dismissed	0.732(0.253)	0.643(0.000)
Oth. leavers	0.744(0.021)	0.631(0.000)

Source: Bel-first. Reported results are obtained with the subsample of firms displaying a lower level of change in their overall labour force; thus those where, presumably, so-called net LACs (strictly related to changes in the overall level of employment/hours) are minimal.

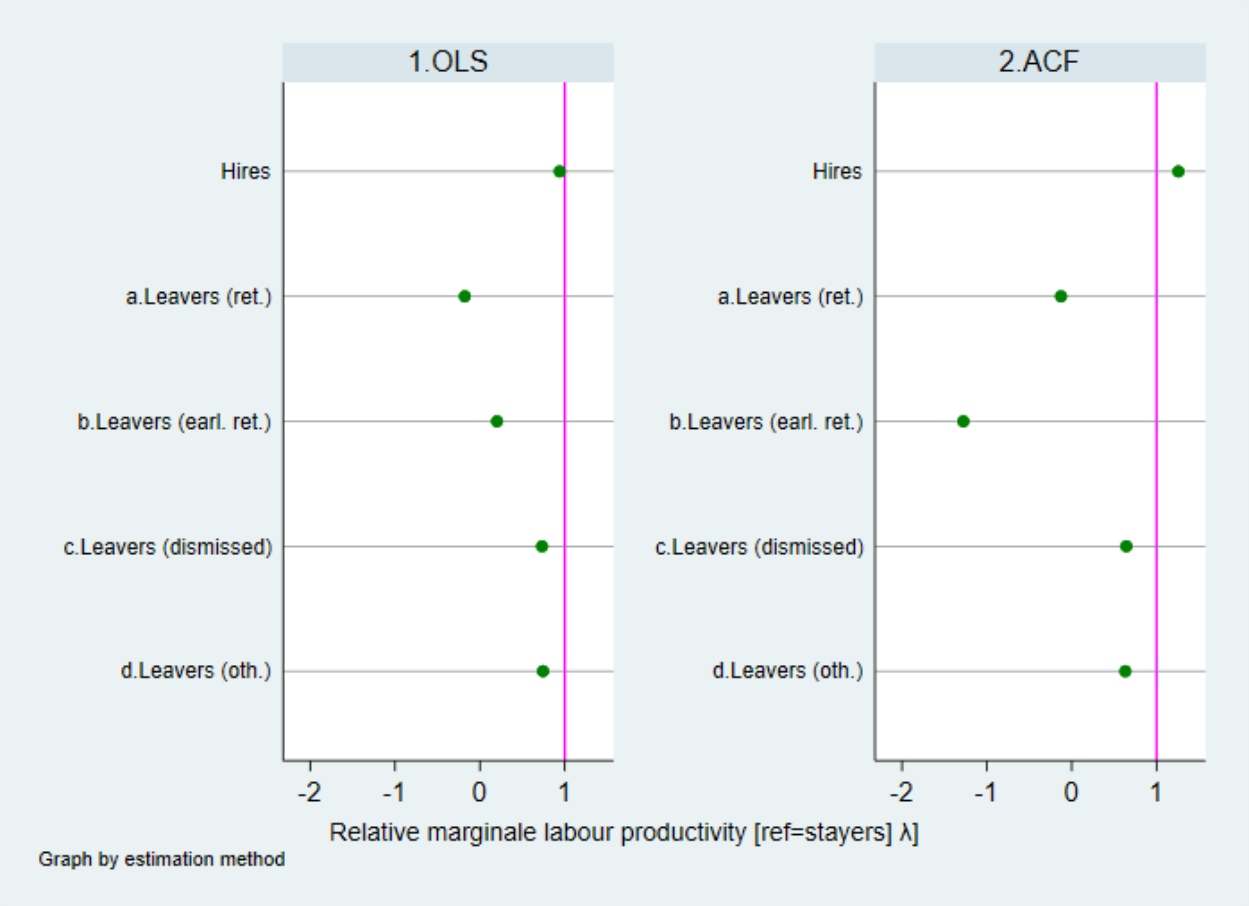

Figure 3: Productivity-related LACs (ref. stayers' productivity) - Assessing the contaminating role of NET adjustment costs

Reported values correspond to λ. They are obtained with the subsample of firms displaying a lower level of change in their overall labour force; thus those where, presumably, so-called net LACs (strictly related to changes in the overall level of employment/hours) are minimal.

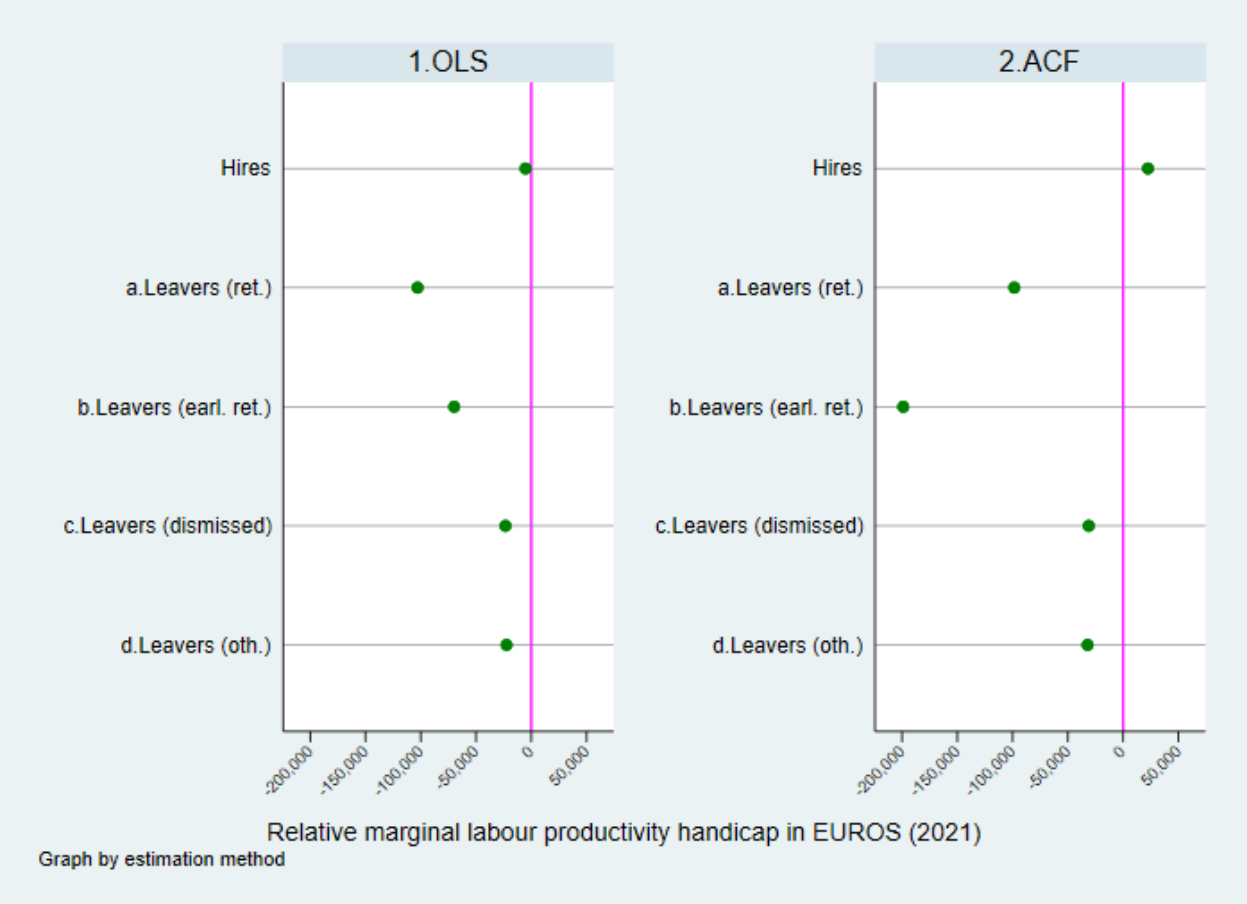

Figure 4: Productivity-related LACs in EUROs (ref. stayers' productivity)- Assessing the contaminating role of NET adjustment costs

Reported values are obtained by multiplying the estimates of (marginal) labour productivity losses (λ) by the year 2021 average labour productivity of stayers i.e. 87,300 euros. They are obtained with the subsample of firms displaying a lower level of change in their overall labour force; thus those where, presumably, so-called net LACs (strictly related to changes in the overall level of employment/hours) are minimal.