Should we differentiate the retirement age by socioeconomic status? A tagging problem*

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Abstract

Differences of life expectancy across socioeconomic status are well documented and lead to unintended regressive transfers in the pension system. Up to now, economists' policy recommendation has been to index the retirement age to the socioeconomic group specific life expectancy. However, this response only focuses on differences across categories, neglecting the longevity heterogeneity within them. This paper analyses the usefulness of using socioeconomic characteristics to tag people for pension policy. Using US mortality rates assembled by Chetty et al. (2016), we simulate the realized longevity distribution inside each socioeconomic group. Then, we assess numerically the capacity of a tagging-based pension system to match the longevity distribution, using an uniform retirement age policy as a benchmark. Results suggest that even with 200 different pension ages (2 genders $\times$ 100 income percentiles), capacity to match is of limited magnitude. This result is robust to higher value of an "error" aversion parameter or to higher weight put on early death. The take-home message is that, at the individual level, differentiating the retirement age based on socioeconomic characteristic has limited capacity for matching the full distribution and others policy should be thought of.

Keywords:
Pension design, Heterogeneous longevity, Pension progressivity, Tagging
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1. Introduction

It is now well established that life expectancy differs across socioeconomic categories, with low-educated or low-income people living, on average, significantly less years that their better-endowed and wealthier peers. Moreover, there is also evidence that the gap could be on the rise (Chetty et al., 2016). This issue has recently received considerable critical attention for its impact on the pension system (OECD, 2018). Many analysts (unionists, academics,...) question the fairness of policies that uniformly raise the retirement age, neglecting the importance of life expectancy differences. The General Secretary of the Trades Union Congress in the UK (Brendan Barber) said: "We remain opposed to helping pay for more generous state pensions by increasing the state pension age. This means that the poor and those with stressful jobs will end up paying for better pensions of the better off with longer life expectancies" (Whitehouse and Zaidi, 2008, p. 8) and Piketty (2019) criticized the recent French pension reform proposal for "taking no account of social inequalities in life expectancy". Indeed, if people at a higher level in the income distribution tend to live longer than those at a lower level, an implicit and unwitting redistribution will be done in their favor, due to the longer length of perceiving retirement benefits. This issue has been thoroughly investigated in the empirical (see e.g. Haan et al., 2019; Whitehouse and Zaidi, 2008; Bommier et al., 2005; Liebman, 2002; Coronado et al., 2000; Garrett, 1995) as well as in the theoretical (see e.g. Fleurbaey et al., 2016; Pestieau and Ponthiere, 2016; Pestieau and Racionero, 2016; Bommier et al., 2011) economic literature. The empirical conclusion is that the pension system progressivity is more or less diminished, or even reversed, when the life expectancy differentials are factored in. The policy recommendation is often to differentiate the retirement age by socioeconomic status (see e.g. Ayuso et al., 2016) or another policy derived from this principle, like making pension benefits inversely proportional to the remaining socioeconomic life expectancy (Breyer and Hupfeld, 2010).

However, one limitation of the policy recommendation is its systematic focus on the difference of life expectancy across groups, while forgetting the longevity distribution inside them. Figure 1 shows a simulation of American longevity for male in the 1st income percentile and female in the 100th.[1] Although, the two look quite different, they both have people dying at any given age. Therefore, stating that a woman located in A should have a different retirement age that a man located in B, while they both die at the same age could be problematic. Figure 1 reflects the most extreme difference. Considering now a less pronounced example, Figure 2 shows that the distribution for women in the 25th and the 75th income percentile quite overlap, although there exists a gap of life expectancy. The usefulness of "tagging" by socioeconomic characteristics seems rather weak when assessing those two pictures. The key objective of this paper is to analyse

[1] Details on the simulation will be provided in Section 4.1.

thoroughly the effectiveness of "tagging" for pension policy. To this aim, we will simulate American longevity distribution (based on the mortality rates collected by Chetty et al., 2016) and then construct an index of adequacy of the pension system. To the best of our knowledge, there is no empirical paper addressing this particular problem and, with the noteworthy exception of Pestieau and Racionero (2016), not much work either in the theoretical literature.

The rest of the paper is organized as follows; Section 2 provides a review of the empirical and theoretical literature. In the Section 3, we will construct an index of pension system adequacy which fundamentally measures the distance between the "fair" retirement age[2] of an individual and the retirement age(s) imposed by the pension system. Those distances represent the errors due to the imperfect tagging. In the Section 4, we present our data that are built on the mortality rate collected by Chetty et al. (2016) for the US. We generate a distribution of realized longevity for each sex, state and income rank. We compute our index when the age of retirement varies by state and/or gender and/or income. A key result is that the introduction of up to 200 different ages of retirement, capturing significant life expectancy differences across socioeconomic categories does not considerably improve the degree of adequacy of the pension system; primarily due to enormous variation in realized longevity within each socioeconomic group. This finding is robust to several specifications of the index, which suggests that "tagging" has limited power for pension policy. In Section 5, we discuss our results and the underlying normative principles behind our paper and then, we conclude.

2. Literature review

A considerable amount of literature has been published on the role of the socioeconomic gradient on life expectancy. In particular, various studies have shown its impact, as well as its evolution, in the case of the United States. Meara et al. (2008) found an increase in the gap between low- and high-educated white men, aged 25, from 6.2 to 7.8 years between 1990 and 2000. Using a more fine-grained approach, Olshansky et al. (2012) reported that, in 2008, Black Americans with less than 12 years of education have a lower life expectancy, at birth, of 14.2 years compared to White ones with 16 or more years of education. Using income as a proxy for socioeconomic status, Cristia (2009) established that the advantage of belonging to the top income quintile (compared to the bottom one) has grown by 30 % for men between 1983 and 2003. Recently, Chetty et al. (2016) reassessed the mortality gap and showed that a man in the top income quartile has gained, on average, 0.2 years of life expectancy (at 40) each year between 2001 and 2014, whereas the gain for one in the bottom was only, on average, 0.08. According to their results, there exists a life expectancy

[2] We define the "fair" retirement age of an individual as α times his longevity.

gap between the top 1 % and the bottom 1 % of the income distribution of 14.6 years for men and 10.1 years for women. Data from other countries confirm the mortality differential over the world. Attanasio and Emmerson (2001) showed that the move for a British man, aged 65, from the 40$^{\text{th}}$ to the 60$^{\text{th}}$ wealth percentile increases his survival probability by 2.4-3.4 percentage points, using the British Retirement Survey (1988/89 & 94). Evidence for several European countries (Finland, Norway, Denmark, UK, Belgium, France, Switzerland and Austria) are provided in Huisman et al. (2004) and in Steingrímsdóttir et al. (2012) for Norway.

As explained in the introduction, the socioeconomic gradient has lead pension economists to investigate its implication for the pension system progressivity. The overall evidence is that it makes the system less progressive, but there is no consensus to the magnitude of the reduction. For the US, some authors conclude that the system becomes regressive (e.g. Coronado et al., 2000), neutral (e.g. Garrett, 1995) or even stays progressive (e.g. Liebman, 2002). In France, the retirement scheme progressivity has been shown to be reduced by one quarter to one half (Bommier et al., 2005). Whitehouse and Zaidi (2008) confirmed that the pension system progressivity is always reduced, when life expectancy differential are factored in, with an impact ranging from staying progressive (in e.g. Norway), becoming neutral (in e.g. Germany) or even becoming regressive (in e.g. Poland). Contrary to their findings, Haan et al. (2019) suggested that the German system becomes regressive, using data from cohorts born between 1926 and 1949. The absence of consensus on the level of the progressivity reduction stems from differences pertaining to the techniques, assumptions and cohorts analyzed. Nevertheless, it should also be remembered that the utility gained from the insurance against longevity risk of the pension system can still make it desirable, even if it makes regressive transfers (see e.g. Brown, 2003, who discussed it for the case of annuities).

The policy recommendations of the empirical literature have either been to propose that the retirement age should vary by socioeconomic status (see e.g. Ayuso et al., 2016), or suggest other indirect solutions, like linking the retirement benefit to the remaining life expectancy (Breyer and Hupfeld, 2010). Although, it is never really stressed those approaches use the "tagging" principle introduced by Akerlof (1978). He explained in his seminal paper that policy taxation should be based on "tag", which help to identify the needy. Several proposals have been made in the subsequent years; for example, gender (Alesina et al., 2011), age (Weinzierl, 2011; Blomquist and Micheletto, 2008) or even height (Mankiw and Weinzierl, 2010). However, tagging is far from being perfect as people could end up missclassified and Parsons (1996); Diamond and Sheshinski (1995) discussed the design of security system with imperfect tagging. Their solution was to create a system such that people have an incentive to self-revealed their type if they are mistagged. Another way is an individual screening; however, it has been shown that this latter could lead to strong errors (see e.g. Benitez-Silva et al., 2004, who showed that 20 % of US disability recipients are missclassified). In our particular case, individuals could not know in advance their longevity which rules out the self-revealed approach. Therefore, policy makers are left to find a tag which brings enough relevant information about the longevity.

In the recent years, the longevity differential has also attracted attention in the theoretical literature (see e.g. Leroux et al., 2015, for a survey). The main emphasis has been put on the optimal policy to compensate the short-lived individuals. It has first been shown that the standard utilitarian framework is not adequate because it concludes to transfer resources from short- to long-lived individuals (Bommier et al., 2011). Pestieau and Ponthiere (2016) have even shown that a social planner following the standard utilitarian framework will arrive at a worse solution than the laissez-faire one. Bommier et al. (2011) adopted a concave transformation to deal with this problem, which they justified as either "aversion for multiperiod inequality" or "risk aversion with respect to length of life". One of their findings is that the retirement age should be different for short- and long-lived individuals. In another paper, Fleurbaey et al. (2016) stressed the difference between "ex ante" (based on life expectancy) and "ex post" egalitarianism (based on realized longevity). They showed that a social planner can let the economy reaches its decentralized outcome if he opts for an ex ante focus; on the contrary, if he adopts for an ex post perspective, it is not efficient anymore and an increase of consumption when young is required to achieve efficiency. However, those papers do not consider the use of a proxy (e.g. income) to know the type of the individual; the longevity is either perfectly known, private information or a random variable. To the best of our knowledge, there exists only one theoretical paper dealing with this issue. Pestieau and Racionero (2016) considered a framework with two jobs (one harsh and one soft) with a mix of short- and long-lived individuals working in them (with the harsh having a higher proportion of short-lived). They showed the short-comings of establishing the retirement age based on the profession as a proxy for longevity. They considered different frameworks and showed that heterogeneity in longevity inside each profession always lead to undesirable results. For example, a maximin criterion on the utility of the short-lived in both occupations leads to difference in utility between the long-lived in the soft and harsh profession. At the end, they conclude that a pension system "should be sufficiently flexible to separate the lucky from the unlucky within each occupation" (Pestieau and Racionero, 2016, p. 201).

Although extensive research has been carried out on the impact of the socioeconomic gradient on the pension system progressivity, no single study has *thoroughly* analyzed the possibility of a policy solution to it. Empirical literature has proposed to differentiate the retirement age based on socioeconomic life expectancy, but failed to notice the relevance of the longevity distribution. In this paper, we analyze in depth the usefulness of tagging by socioeconomic status, when taking into account the longevity distribution. Our paper could be seen as the first empirical investigation of the problem raised by Pestieau and Racionero (2016).

3. Index

In the following pages, we will establish the relevance of using socioeconomic characteristics to differentiate the retirement age. To do so, we will construct an adequacy index consisting of the sum of deviations of each individual between his fair retirement age (defined as a given percentage of his realized longevity) and the one of the pension system (that could differ according to the socioeconomic group to which the individual belongs). If the tags were fully informative about individual's longevity, the social planner would perfectly be able to differentiate the retirement age and those departures would not exist. However, the achievement of this goal is clearly utopic due to the longevity distribution inside each socioeconomic status, which will always generate some unwitting departures. It is important to notice that those departures are the *only* root of the regressive financial transfers identified in the literature. The magnitude of those transfers depends on many of the features of the pension system; but, they fundamentally stem from the social planner's ex ante ignorance of the individuals' realized longevity. What follows illustrates how difficult (and perhaps how unrealistic) is it to reduce this ignorance.

We assume that the society is composed of a set of agents with exogenous heterogeneous longevity depending, among others, on state, sex and income level. The social planner is egalitarian, so it would like that each member inside the society spends the same share α of his life at work (and thus, the same share $(1-\alpha)$ in retirement). Consequently, the fair retirement age of each individual is equal to α times his longevity.

Definition 1. The fair retirement age of an individual is α times his longevity.

This policy follows the principle of the recommendations of the European Commission (2012) of linking, at the national level, the retirement age to the life expectancy and echoes the current concerns, raised e.g. by Piketty (2019), that pension reforms unilaterally raising the retirement age are unfair because they ignore the important life expectancy gradient. Our social planner would like to produce a retirement scheme that equalizes as much as possible the share of life spent in retirement (so, it amounts to individualising the retirement age using the full distribution of realised longevities). Therefore, his strategy consists of moving away from a unique retirement age regime, to one where there are several retirement ages (based on tags like e.g. sex and/or income percentile) to match with the utmost precision the heterogeneity in realized longevity inside the society.

In order to introduce our index, let us first consider a simplified example. Society consists of n members, living a life that varies in length, who can be tagged into two subsets (H and L) of identical size. The H one enjoys a high life expectancy with half of its member living 4.5 periods and the other half 3.5. Subset L has a lower life expectancy with half of its member living 2.5 periods and the other half 1.5. As explained before, the social planner has for policy that individuals should spend a share α of their life working. Consequently, he is willing to introduce different retirement ages for different groups of individuals to reflect their longevities. The problem comes from the impossibility of dividing further the different subsets due to its incapacity to distinguish the short- vs long-lived within each of group. However, there is no doubt that the use of subset-specific retirement ages is a better arrangement than the use of an uniform one. In the first case, the social planner would set the retirement age at $\alpha \times 3$.[3] This will create a sum of deviations between the retirement age and the fair ones of $\alpha\, n$. In contrast, he would set it at 4 for subset H and at 2 for subset L, generating a sum of deviations of $0.5\, \alpha\, n$. Our adequacy index will measure the percentage of *improvement* between those two policies; so the decrease in the sum of deviations between the fair retirement age and the one(s) of the system. In this simple example, it is equal to $\frac{0.5\, \alpha\, n}{\alpha\, n}$ suggesting a gain of 50 %. This amounts to saying that the mean error is reduced by half. It is crucial to notice that our index measures the *improvement* between the worst policy (only one retirement age) and the one proposed (e.g. a different one for each income percentile). The value of α does not play any role; but it facilitates the explanation.

Let's us now generalize to a society constituted of a set of individuals i who could be split into j mutually exclusive subsets defined by a vector of socioeconomic characteristics (income,...). Their realized longevity (i.e. age of death) is denoted $m_{i,j}$. Their retirement age is $\alpha\, \mu\left(\mathbf{m}\right)$ in case of an unique retirement age and $\alpha\, \mu_j\left(\mathbf{m_j}\right)$ for the subset j when the social planner chooses an age that is subset specific. Our improvement index of the pension system adequacy is formulated as follows:

$$I\left(\mathbf{m}\right) = \frac{\sum_{j=1}^{k}\sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\, \mu_j\left(\mathbf{m_j}\right)|^{\beta}}{\sum_{j=1}^{k}\sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\, \mu\left(\mathbf{m}\right)|^{\beta}} \tag{1}$$

$$s.t. \quad \mu\left(\mathbf{m}\right) \in \underset{\mu(\mathbf{m})}{\arg\min} \sum_{j=1}^{k}\sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\, \mu\left(\mathbf{m}\right)|^{\beta} \tag{2}$$

$$\mu_j\left(\mathbf{m_j}\right) \in \underset{\mu_j(\mathbf{m_j})}{\arg\min} \sum_{i=1}^{n_j} |\alpha\, m_{i,j} - \alpha\, \mu_j\left(\mathbf{m_j}\right)|^{\beta}, \forall j \tag{3}$$

[3] The retirement age is set at $\alpha \times 3$ because it minimizes the sum of the deviations between the retirement age (i.e. $\alpha \times 3$) and the fair ones ($\alpha \times 4.5$; $\alpha \times 3.5$; $\alpha \times 2.5$ and $\alpha \times 1.5$); more details will be provided in the general formulation below.

The index is the sum, over each individuals, of the absolute deviations between the fair retirement age and the one(s) chosen by the social planner to the power β, divided by the sum of deviations in case of an uniform retirement age. Constraint (2) (resp. (3)) assures that the retirement age set by the social planner is the optimal one for the case of an unique (resp. each of the different) retirement age. It can be shown (see Appendix A) that $(\alpha\mu(\mathbf{m}); \alpha\mu_j(\mathbf{m_j}))$ correspond to the median longevity if β is equal to 1 and to the mean longevity for a value of β of 2. Above 2, the most appropriate values do not correspond to a statistical concept and are identified numerically.[4] The parameter β represents the aversion to error of the social planner. A value of 1 corresponds to an absence of error aversion and a higher degree of error aversion is reflected by a higher value of β. No error aversion amounts to saying that a policy generating 10 deviations of 1 is not more appropriate than one producing one deviation of 10. This is ethically questionable as the first moderately affects 10 individuals whereas the second imposes a large cost to one individual. Values of $\beta > 1$ solves this issue by weighting more the larger deviations. The properties of our improvement index are:

- The index is relative to the default option of an unique retirement age and is below (resp. above) one for a better policy (resp. worse);
- The index has a value of zero if each individual has his retirement age set optimally (i.e. $\alpha\, m_{i,j} = \alpha\, \mu_j(\mathbf{m_j}), \forall\, i, j$);
- The index does not depend on α;
- The index puts more weight on the worst off if and only if $\beta > 1$. In particular, the higher β is, the more extreme deviations matter;
- The index is population invariant;
- The index is invariant with respect to the length of longevity (multiplicative and additive);
- The index respects the anonymity principle.

4. Results

4.1. Data construction

This Section presents estimates of our index based on US simulation of longevity for several tagging policy. We first need to explain how we simulate our data. The computation of the index calls for a complete distribution of longevity of a population, which could be split into subsets based on some

[4] In practice, we use Stata$_{16}$ to compute the deviations brought by every possible retirement age and select the most suitable one.

socioeconomic characteristics (e.g. sex, income,...). We have used the mortality rates assembled by Chetty et al. (2016) for the US to simulate it. Their data details them either by sex and income percentile or by state, sex and income quartile. We input them into the lifetable techniques, explained in Chiang (1984), to simulate the longevity distribution of our population (see Appendix B for more details). Consequently, we have a longevity distribution that could either be split by sex and income percentile or by state, sex and income quartile. Figure 3 displays two longevity distributions; the dashed one corresponds to women in the 20^{th} income percentile while the solid one to those forming the 80^{th} percentile. Simulations start at age 40 because Chetty et al. (2016) provide mortality rates from that age. More deaths occur at a younger age for the women in the 20^{th} percentile which is a reflection of the mortality gradient. Two stylised facts emerge. First, the 20^{th} percentile distribution is less negatively skewed, illustration of the negative impact of lower income on life expectancy. Second, the distribution of realized longevity is more disperse for the 20^{th} percentile, reflecting the well-known demographic fact of a higher longevity dispersion for lower socioeconomic categories (see e.g. Brønnum-Hansen, 2017; Sasson, 2016; van Raalte et al., 2014, 2011).

4.2. No error aversion ($\beta = 1$)

We will now empirically explore the behavior of our index with US data for several policies. They will be based on state, sex and income rank. The choice of those tags is partially data driven; but also because gender and income are the most obvious starting points. Note that sex is not prone to moral hazard. The problem is of limited magnitude for income if the policy uses average lifetime income. This last point could raised some doubt and is more discussed after presentations of our results. State is naturally prone to moral hazard as individuals would have an incentive to move for benefiting of a different pension policy. To reveal in advance our result, it is not a powerful tag; so there is no need to discuss in depth its moral hazard problem. We present it more as an example in a first best framework.

As explained before, it is assumed that each individual should spend a fraction $(1-\alpha)$ of his life in retirement. The social planner tries to adjust with the upmost precision the retirement age to fulfill that goal. We can observe in the Figure 4 that the life expectancy is lower for men than for women and, therefore, imposing a common/uniform retirement age appears inadequate. Nevertheless, and this is *the crucial point*, the effect of splitting the retirement age would be ambiguous. Figure 5 shows us the longevity distribution of men and two different policies: the solid line corresponds to the common retirement and the dashed one to the differentiated one. The change from the former to the latter reduces the mistagging of the people who will die sooner than expected; for example, those located in the point A. The distance between them and the dashed line is less than that with the solid line. However, the change will lead to a higher mistagging for the people who will live longer, for example those located in the point B. The solid line is indeed closer to their position than the dashed one.

We will first assume that our social planner has no error aversion (i.e. β equals 1). The median longevity for the whole population is 85 and the common retirement age is therefore set at $\alpha \times 85$. The aim of this paper is not to discuss the particular value that α should take and, therefore, we normalized it at 1. Thus, the reader should remember that the retirement ages presented in the Figures and Tables below should always be multiplied by a particular value of α (α equal 1 is more a mortality age than a retirement age). We will first illustrate the tagging policy based on state, which is represented in the Figure 6. The map shows that the retirement ages vary a lot between places. People in Nevada could retire at $\alpha \times 83$ whereas those in Minesotta could end their working lives at only $\alpha \times 87$. Nevertheless, this illustration does not bring more information than the previous papers about life expectancy heterogeneity. The aim of this paper is to take into account the overall longevity distribution by computing our index. We can see in the Table 1 that our index reveals that 99.59 % of the deviations remain after having split the retirement age by state. This indicates a very small decrease of only 0.41 % of the errors generated by the social planner due to his inability to better distinguish the longevity of the people inside each state. This number is surprisingly small; we could indeed have hoped that differentiating the retirement age between states would lead to a higher improvement. However, the impact of the distribution leads to only a small diminution in the errors due to the high heterogeneity in the realized longevity. The mistagging changes differently across the different members inside the society. The deviations of the short-lived in Nevada are now smaller as the life expectancy on which the retirement age is calculated is now closer to theirs. However, the ones of the long-lived is now higher as the mistagging increases for them. The contrary arises in Minesotta. The only deviations that are unaffected are those of the individuals with a longevity comprised between the common- and differentiated retirement ages, as a change from one to the other does not lead to any change in the mistagging for them. As the effect of tagging by state is relatively small, we will add other tags to see their relevance. If we differentiate the retirement age based on state, sex and income quartile, we arrive at a value of our index of 95.08 %, which means that the mean error between the "fair" retirement age and the ones of the system is decreased by almost 5 % compared to a situation without any tagging. This improvement is higher than the previous one; however, we arrive at a situation with more than 400 different retirement ages. Moreover, as explained before, the tagging by state is subject to moral hazard. This is the reason why we will now try to differentiate only based on sex and/or income percentile.

The policy recommendation of the empirical literature has often been to differentiate the retirement age by income. We have simulated this policy by tagging people based on their income percentile. Table 2 displays the difference of retirement age for people in the 10^{th}, 20^{th}, 30^{th}, 40^{th}, 50^{th}, 60^{th}, 70^{th}, 80^{th}, 90^{th}, 100^{th}. Poor people can retire 4 years before the one that would have prevailed in the case of an uniform retirement age whereas the richest one can end their working life only six years after. Although those differences seems important; the improvement is not more substantial as our index indicates now a value of 96.80 %. This demonstrates that the policy recommendation of splitting the retirement age by income due to the difference of life expectancy is misleading as only 3,20 % of the deviations disappears with such a policy. A further split by sex and by percentile leads to a change of 4.96 %. Two conclusions could emerge either the decrease seems to weak and one concludes that tagging is not an interesting idea, or the decrease looks sufficiently important and therefore, one concludes in the opposing direction. To our opinion, this assessment is highly subjective. One could argue that even a small diminution is always worthy enough, whereas another could disagree by saying that a huge effect is required for a policy to be implemented. We left those considerations for later and inquire about the reason of the surprising small decrease.

The small diminution of the index is due to the high distribution of longevity inside each group. To better found this statement, we have decomposed the variance of the distribution as well as its Theil index (when the retirement ages is done with respect to sex and income percentile).[5] The decomposition of the Theil index tells us that 95.22 % is due to the within component and the one of the variance gives us a figure of 92.35 %. Those numbers are not surprising as van Raalte et al. (2012) have shown that differences of longevity between education status (elementary, lower secondary, higher secondary and tertiary) can explained not more than 4 % of the overall variance, the rest being from the within group component. In a policy paper, Deaton (2002) already pointed out that health policy should not be targeted directly at the gradient in health, but rather at sick people, due to the high variation of health among individuals forming a particular socioeconomic group. However, one could wonder if creating more categories would have provided a better index. In order to answer this question, we have plotted the index (see Figure 7) for 1 to 10 income categories.[6] We see that the marginal improvement is decreasing and, that the gain becomes almost nihil after 8 income percentile brackets. Therefore, further differentiating the retirement age should have a very small effect as we are already approaching the asymptote. This suggests that if one would really differentiate the retirement ages, he does not need to create a lot of them. Furthermore, if it is sufficient to create only 8 income categories, the moral hazard problem of trying to change one's income category is diminished as it will be more difficult to move from one category to another.

[5] We use the formulas provided in the Additional File from van Raalte et al. (2012).

[6] If there are two categories, the retirement age is different for the people in the 1-50^{th} and the 51-100^{th} income percentiles. If there are three categories, the split is done between those in the 1-33^{th}, 34-66^{th} and the 67-100^{th} income percentiles. And so on.

4.3. Impact of risk aversion ($\beta \to 10$)

We have assumed in our previous computation that β is equal to 1 (i.e. the social planner is not error averse). The sensitivity of our result to his degree of error aversion is something deserving attention. Table 3 displays the index as well as the common and some differentiated retirement ages. A value above 2 is already quite extreme (1 deviation of 10 equals 100 deviations of 1), and almost implausible; nevertheless, their computation remain interesting to notice the tendency implied by a growing β. Let us first concentrate ourselves on β equal to 2. The common retirement age decreases to $\alpha \times 83$ (compared to 85) and the ones tag by gender to $\alpha \times 81$ for men (compared to 83) and $\alpha \times 85$ for women (compared to 88). However, although our retirement ages have changed, the values of our index do not improve that much, as they are now at 97,64 % (compared to 98.32 % with $\beta = 1$) for a retirement age differentiated by sex, 92.35 % (compared to 95.04 %) by sex and by percentile, 99.53 % by state (compared to 99.59 %) and 92.57 % by state, sex and income quartile (compared to 95.08 %). This indicates that, with a reasonable value of error aversion, differentiating the retirement age does not lead to significant improvement(s). A system with 200 different retirement ages (by sex and percentile) reduces our index by only 7.65 %. The root of the problem remains in the shape of the distribution, which is not concentrated enough for the mean to be meaningful.

Three interesting results come into view in Table 3. First, a higher error aversion implies a lower index. Second, the retirement age decreases as β increases. Finally, a higher error aversion reduces the gap between the different retirement age (e.g. between the one of the 25^{th} percentile and the one of the 75^{th} percentile). This stylized fact is well-noticed in the comparison between the Figure 6 and 8. Whereas the former displays a gap of 4 years across the different retirement ages, the latter shows a difference of only 1 year. The explanation of the first point is rather straightforward. The common retirement age generates more deviations (and more extreme ones) than the differentiated ones at the numerator. By consequence, as β increases, the denominator grows more than the numerator; as a result, the index diminishes as the error aversion grows. This point is also quite intuitive; a very error averse social planner will care more about deviations from the situation of equal treatment and, therefore, be more prone to establish several retirement ages. The second feature (the diminution of the retirement age) stems from the fact that i) by definition of error aversion, larger deviations matter more than small ones ii) and that these large deviations are more frequent on left than on the right side of the distribution (see Figure 9). The third point (the decrease of the gap between the retirement ages) is due to the difference of longevity dispersion between the income percentiles. Figure 10 shows that the distribution characterising men belonging the 25^{th} percentile is more dispersed that the one of women in the 75^{th} income percentile. It is well-documented in demography that lower income percentiles display more longevity dispersion (see e.g. Brønnum-Hansen, 2017;

Sasson, 2016; van Raalte et al., 2014, 2011). By consequence, the change of retirement age for low income is smaller due to the importance of right-hand side deviations. As a result, as one retirement age decreases more than the other, the gap drops.

4.4. Different weights for positive versus negative deviations

Thus far, we have shown that the value of β has some impact on our index and on the different retirement ages. However, we have not investigated the possibility that the social planner cares more about negative deviations than about positive ones. This amount to saying that mistagging should have a different value depending if the individual retires too late or to soon. It is indeed likely that people prefers end their working life too early than too late. We will therefore modify (1) as follows:

$$I(\mathbf{m}) = \frac{\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}|\alpha m_{i,j} - \alpha\mu_j(\mathbf{m_j})|^{\beta}|\, m_{i,j} \leq \mu_j(\mathbf{m_j})\right) + \left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sigma|\alpha m_{i,j} - \alpha\mu_j(\mathbf{m_j})|^{\beta}|\, m_{i,j} \geq \mu_j(\mathbf{m_j})\right)}{\left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}|\alpha m_{i,j} - \alpha\mu(\mathbf{m})|^{\beta}|\, m_{i,j} \leq \mu(\mathbf{m})\right) + \left(\sum_{j=1}^{k}\sum_{i=1}^{n_j}\sigma|\alpha m_{i,j} - \alpha\mu(\mathbf{m})|^{\beta}|\, m_{i,j} \geq \mu(\mathbf{m})\right)} \tag{4}$$

The difference between (1) and (4) is the presence of σ that weights differently positive and negative deviations. In some sense, σ can be related to a sort of poverty line, which purpose is to "partition the population into two groups that we want to treat differently" (Cowell, 2016, p. 49). A value below 1 means that deviations corresponding to those who retire too early are less important than the deviations reflecting the situation of individuals who retire too late. The Table 4 shows the effect of various σ on the retirement age when β equals 1. The retirement age decreases with a diminishing σ. This result is quite intuitive; as the social planner cares more about the deviations of those who retire to late than too early, he lowers the retirement age. The extreme result is obtained when he does not care at all about the deviations of those who retire too early ($\sigma = 0$). In this case, the retirement age is set at 40 with no deviation on the left; all are on the right, but he does not care about them. This situation is obviously not realistic. The second question is the impact of σ on our index. The effect is rather small as the Table 5 suggests. This comes from the fact than the structure of the deviation is not altered by the weighting parameter σ. This parameter changes the retirement age and the weight on some deviations; but the behaviour of our index remains primarily driven by the very important within category dispersion of optimal retirement ages.

4.5. Truncation

We have shown that our results are robust to extra weight put on later retirees or to an higher degree of error aversion. The last specification that we try is to remove the simulations below a certain age threshold. Some people raised the doubts about calculating the index based on the whole distribution. Retirement can only happen at a certain age and, therefore, some have argued that the presence of people dying at, e.g., 40 should not be taken into account.

Although, we are skeptic about this remark, we have added this subsection to show the relevance of this question. We disagree with it because the retirement age is *obviously* set with regard to the whole distribution. If more people would die at 40, there will be a consensus for lowering the retirement age. The Table 6 shows the computation of our index with different level of truncation. Truncation at 40 is our benchmark result from previous Section (simulations start at that age). Those at 55, 60 and 70 are some arbitrary value below which the dead "should not be relevant". The Table shows that our results are robust to this last remark. Although, the retirement age increases (see Table 7) as we do not care about people below a certain threshold, the values of our index are relatively stable.

In this Section, we have shown the behavior of our index with various specifications. The improvement of the pension system, without error aversion, was small and it took 200 different retirement ages to decrease the index by 5 %. However, error aversion did not change the conclusion much as long as a very high level was not assumed. Weighting differently positive and negative deviations did not alter neither our index. Truncation at a given threshold did not modify more our results. Together those different specifications show that our results are robust. Overall, this suggests that differentiating the retirement age by socioeconomic status is not a very relevant policy and that other solutions should be think of. This problem is likely to be also found in other policy proposal. For example, some current pension discussions raise the question of recognizing the job hazardousness in the formula of the retirement age (see e.g. Zaidi and Whitehouse, 2009). Although, some "average" indicators could be found (see e.g. Baurin and Hindriks, 2019), it is likely that a huge heterogeneity inside each profession also lies in.

5. Discussion

The previous Section has presented our results without analyzing the normative content behind our index. The role of this Section is to take a step back and discuss its underlying philosophical principles. Two points should be raised: the choice of an ex post or an ex ante setting and the choice to shape retirement policy with respect to individuals or to groups.

5.1. Ex post or ex ante?

The first point is to choose between assessing differences of life expectancy (de facto, ex ante) or realized longevities (de facto, ex post). The existing empirical literature is quite ambiguous as to which of the perspective it adopts. It is not uncommon to read policy recommendations that are intrinsically ex ante (focusing on differences of life expectancy) from papers that are based on ex post mortality data. Several papers in the theoretical literature emphasize the importance of using an ex post approach. The starting argument is that "at the end of the day, what matters is what people achieved, not what they expected to achieve" (Fleurbaey et al., 2016, p. 201). Let us explain the fundamental flaw of using an ex ante approach and proposing to differentiate the retirement age. Assume that a social planner wants to be egalitarian. To do so, he has to take into account every factor influencing the life expectancy of an individual. Starting from a situation with an unique retirement age, he introduces different ones by, e.g., average lifetime income. However, this is not the end of the story, as there is no justification why he should stop at this step if he wants to be *truly* egalitarian. He should then, for example, further split the retirement age based on lifetime income of the parents (as we know that little childhood has also an impact for later in life). Again, this is not the end of the story and he should now differentiate, for example, based on sex. He will become truly egalitarian only when it will have taken all the factors affecting the longevity of the individual.[7] However, at that moment, our social planner will end up with an individualized retirement age, and with enough scientific knowledge, the one reflecting the true realized longevity of the individual. Fleurbaey et al. (2016, p. 201) already stated it, slightly differently: "ex ante egalitarianism that is based on average mortality statistics is not really ex ante egalitarian if it fails to track individual's true life expectancy. In a deterministic world, the true ex ante perspective coincides with the ex post perspective". Therefore, either one says that differences in life expectancy should be taken into account and, by doing so, should agree on using an ex post framework or one argues that this is not relevant and he can use an ex ante setting.

The argumentation above ends up with a conclusion that ex ante and differentiating the retirement age is incoherent. However, it does not say which framework was the most relevant. To our opinion, this is a subjective choice that should be left to the population. Problematically, empirical economists have shown that people do not behave consistently with respect to ex ante or ex post choice. Andreoni et al. (2016) made an experiment demonstrating that people reverse their *own* choices when the framework of the question changes from ex ante to ex post. At the end, they conclude that people are "deontologically naive" and use the fairness related to the framework they are dealing with. This is troubling if one wants to know how to shape policy. Others experiments have been done with similar conclusions (Brock et al., 2013; Krawczyk and Le Lec, 2010). To conclude, both ex ante and ex post theories of fairness appeal to people and there is no clear preferences over one of the other.

5.2. Across individuals or groups?

The second choice to make is to assess the difference across individuals or across groups. The empirical literature has focused on differences across groups whereas this paper concentrates on tagging individuals. The reason comes from

[7] Notice the resemblance of this argument with the one in the Fishkin (2014)'s introduction.

the impossibility of putting someone *uncontroversially* in a particular group due to the non-existence of "natural" ones. By consequence, those created by statisticians will always be somehow arbitrary, undermining the legitimacy of using them in public policy. Moreover, for an individual what matters is his situation and not what may have caused it (Murray, 2001). As such, it makes no difference for an individual if his longevity is smaller due to some between- or within-group factors; what matters for him is his realized longevity and how it compares with other individuals. Nevertheless, some (e.g. Gakidou et al. (2000)) argued that differences in health across groups are more informative than differences across individuals because the former removes the component due to luck. However, knowing that there are differences in average health between socioeconomic status is interesting, but not sufficiently informative for pension policies. Statistical significance is indeed not the same as economic significance (Ziliak and McCloskey, 2004). Focusing on average, with such a huge dispersion around it, will lead to what Cornia and Stewart (1993) have called F-mistake (failure of coverage) and E-mistake (excessive coverage). The first one is the case of a short-lived rich individuals who would be tag as long lived and the second case is the one of long-lived poor individuals who would be tag as short-lived. This could easily be related to the economics of statistical discrimination. Arrow (1973); Phelps (1972) explained in their seminal works that a decision maker, with time constraint, could based his decision on average characteristics and, by doing so, some high-performing members belonging to an under-performing group are discriminated against. The same arises when the social planner focus only on life expectancy, the short-lived rich individuals are penalized because the social planner does not take the time to look into each group.

The goal of this Section was to explain the underlying philosophical principles behind our index. Although, we have explained and discussed them, we did not state a particular choice as this is, for us, subjective. At the end of the day, what fair or not fair is the eye of the beholder. However, whatever the philosophical choice made, the sort of the short-lived is something that deserves attention. As retirement policy could not be too much rely on for solving it; other solutions should be thought of. To that aim, the papers of Ponthière (2018) and Fleurbaey et al. (2014) are an obvious starting point.

6. Conclusion

The pension literature has established that the mortality differential between socioeconomic categories reduces or completely eliminates the progressivity of retirement system. The policy recommendation has often been to differentiate the retirement age by socioeconomic status in order to cope with this problem. In this paper, we have reviewed the usefulness of tagging by socioeconomic status to differentiate the retirement age. We have shown that the distribution of realized longevity inside the different subgroups is such that splitting the retirement age between state and/or sex and/or income rank does not lead to a substantial improvement. The conclusion is robust to several specifications: error aversion of the social planner, different weights for positive and negative deviations or truncation below an age threshold. The take-home message is that pension policy is relatively unable to differentiate optimally the retirement age. The best tag would be by sex and a small amount of income categories. However, it would not be sufficient to solve the problem of the longevity differences.

At individual level, many factors are causing enormous variation in realized longevity; and although state, gender and socioeconomic categories are important predictors of longevity, they are no panacea for a pension policy that would want to treat individuals equally. Achieving equity requires adopting a lifetime perspective which implies that the social planner needs to be able to know *in advance* the longevity of people. This papers show that the gains that may be achieved by abandoning a uniform retirement age policy are limited, when the social planner's information is imperfect in the sense that is consists of estimates of group-specific life expectancies.

Nevertheless, we expect the question of unfairness of the pension system due to (in some cases growing) longevity heterogeneity to remain a very hot topic in the future. There is therefore a real need for more research in this field to better understand the determinants of that heterogeneity and how it can be addressed within the pension system or via other policies.

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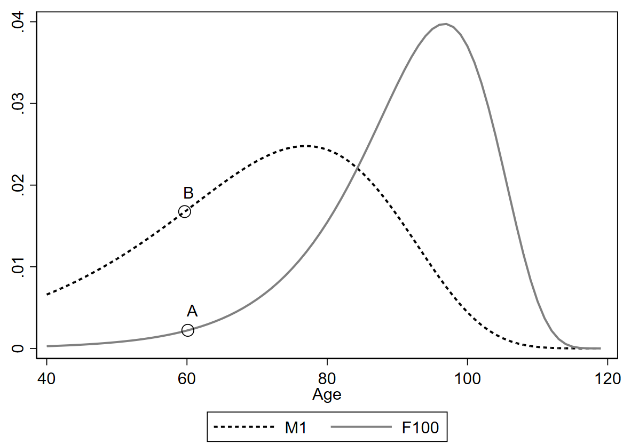
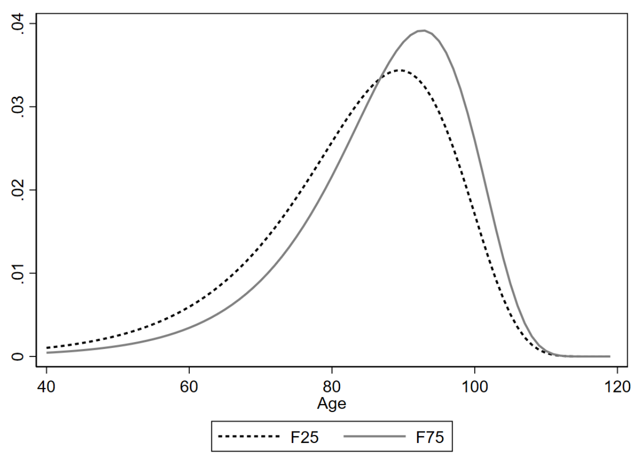


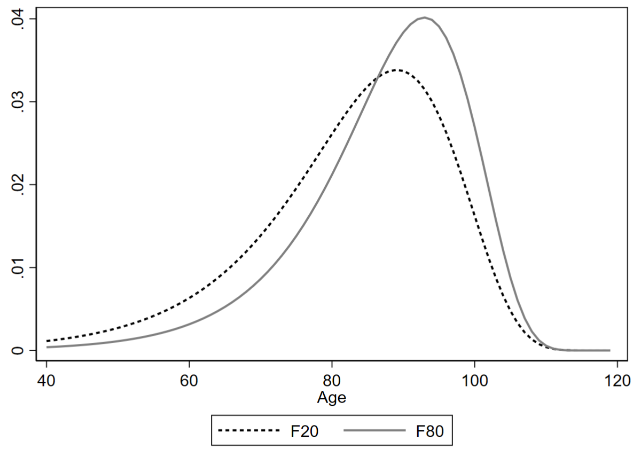
The Figure shows the simulations of US longevity distribution for men in the 1st income percentile and women in the 100th. Simulations are done using the mortality rates provided by Chetty et al. (2016).

Figure 1: Distribution of longevity for men in the 1st income percentile and women in the 100th



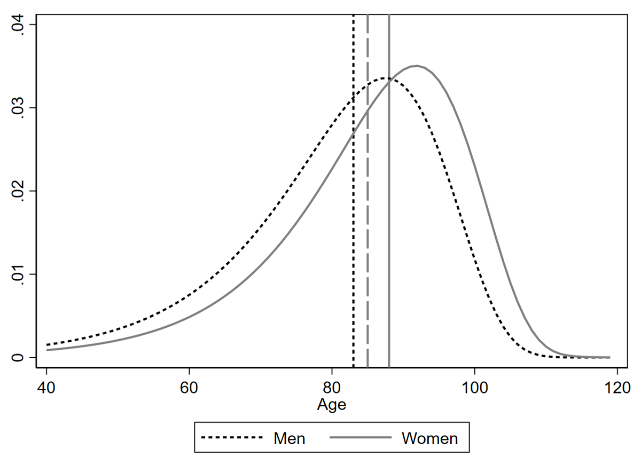
The Figure shows the simulations of US longevity distribution for women in the 25th and 75th income percentile. Simulations are done using the mortality rates provided by Chetty et al. (2016).

Figure 2: Distribution of longevity for women in the 25th and the 75th percentile



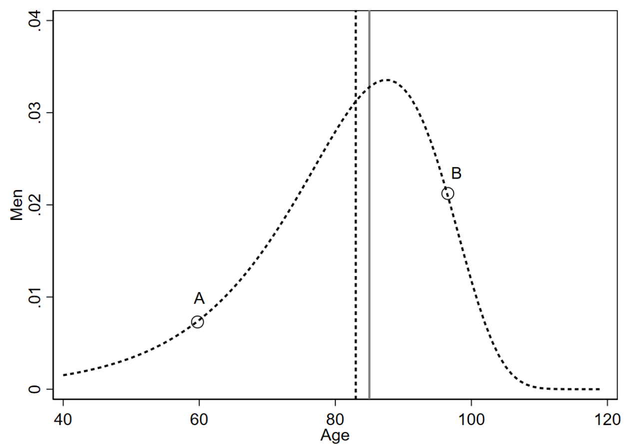
The Figure shows the simulations of US longevity distribution for women in the 20$^{\text{th}}$ and 80$^{\text{th}}$ income percentile. Simulations are done using the mortality rates provided by Chetty et al. (2016). The longevity distribution of the richer women is more negatively skewed which reflects their higher life expectancy. The longevity of the poorer women is more dispersed which is a stylized demographic fact.

Figure 3: Distribution of longevity for women in the 20$^{\text{th}}$ and the 80$^{\text{th}}$ percentile



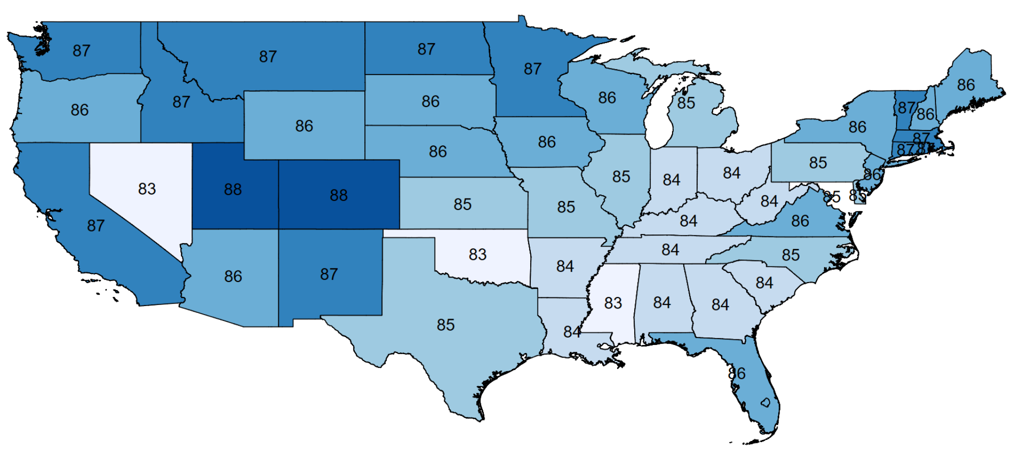
The Figure shows the simulations of US longevity distribution for men and women. Simulations are done using the mortality rates provided by Chetty et al. (2016) . The retirement age (with α and β equal to one) for men is the small dashed line and the grey solid line for women. The central dashed line is the common retirement age.

Figure 4: Distribution of longevity for men and women



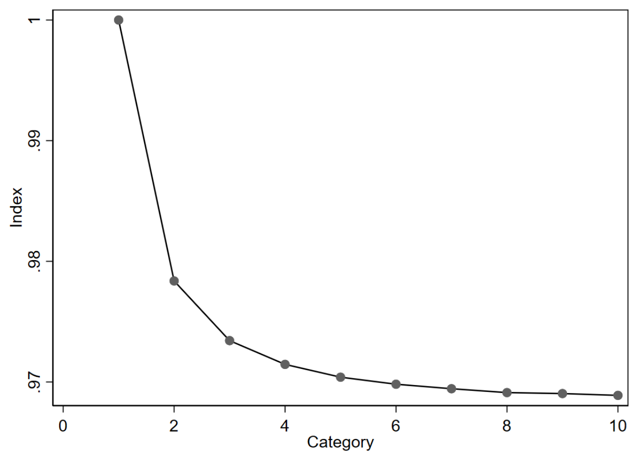
The Figure shows the simulations of US longevity distribution for men. Simulations are done using the mortality rates provided by Chetty et al. (2016) . The common retirement age (with α and β equal to one) is the solid line and the differentiated one (for men) is the dashed one.

Figure 5: Impact of differentiating the retirement age by sex for men



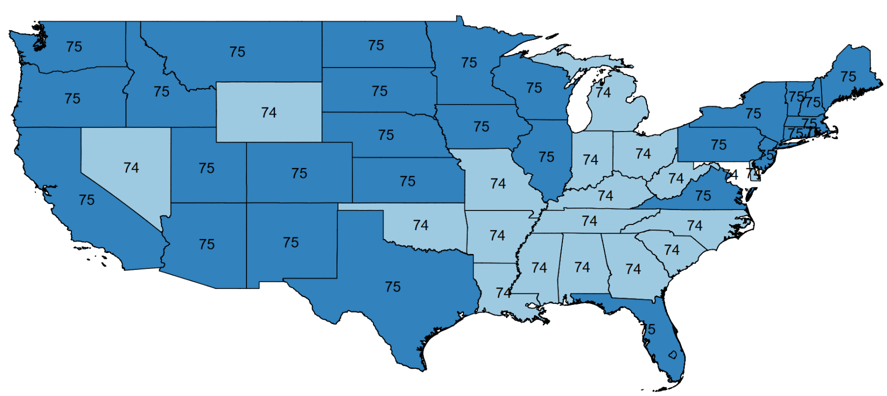
The Figure shows the retirement age by state with β and α equal to 1. Simulations are done using the mortality rates provided by Chetty et al. (2016) .

Figure 6: Impact of differentiating the retirement age by state (β and $\alpha = 1$)



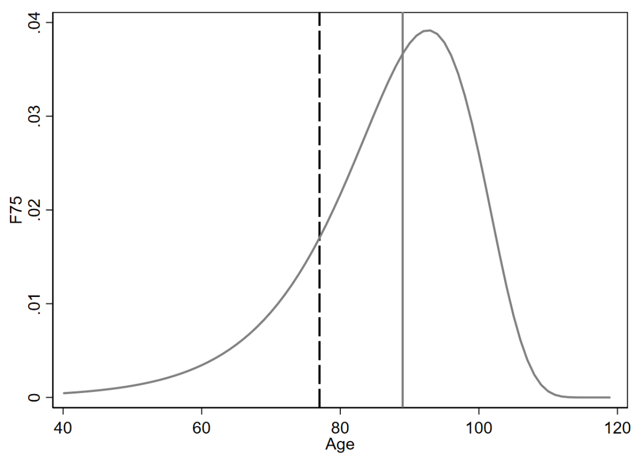
The Graph shows the value of the index with the increase in the number of income brackets. One category means that there is only one retirement age, two categories means that the retirement age is different for the individuals having their income in the 1-50$^{\text{th}}$ percentile and in the 51-100$^{\text{th}}$, three categories means that it is differentiated between the individuals in the 1-33$^{\text{th}}$, 34-66$^{\text{th}}$ and 67-100$^{\text{th}}$ income percentile; and so on.

Figure 7: Decrease of the index with the number of category



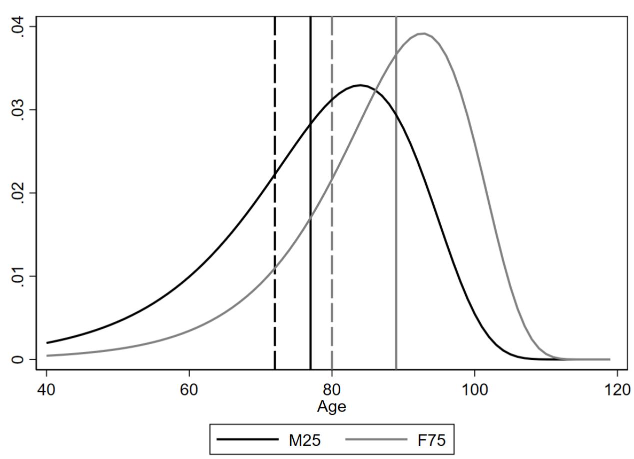
The Figure shows the retirement age by state with β equal to 10 (and α equal to 1). Simulations are done using the mortality rates provided by Chetty et al. (2016) .

Figure 8: Impact of differentiating the retirement age by state ($\beta = 10$, $\alpha = 1$)



The Figure shows the decrease in the retirement age when β increases. Simulations are done using the mortality rates provided by Chetty et al. (2016). The retirement age (with α equal to 1) is the solid line when β equal 1 and the dashed one when β equal 10.

Figure 9: Explanation of the decrease of the retirement age (Women, 75$^{\text{th}}$ percentile)



The Figure shows the decrease in the gap between the retirement ages when β increases. Simulations are done using the mortality rates provided by Chetty et al. (2016). The retirement age for the men in the 25$^{\text{th}}$ income percentile (with α equal to one) is the solid black line when β equal 1 and the dashed black one when β equal 10. The retirement age for the women in the 75$^{\text{th}}$ income percentile (with α equal to 1) is the solid grey line when β equal 1 and the dashed grey one when β equal 10.

Figure 10: Explanation of the narrowing gap

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Index												
	By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
	98.32 %		96.80 %		95.04 %		99.59 %		97.92 %		95.08 %	
Retirement age ($\alpha = 1$)												
Unique	Male	Female	25^{th}	75^{th}	M, 25^{th}	F, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
85	83	88	83	87	80	89	87	83	85	85	79	88

The Table provides the value of the index when β equal 1. The index is done for tagging by sex, by income percentile, by sex and percentile, by state, by state and sex and by state, sex and income quartile. The line below shows different retirement ages depending on the policy (with α equal to 1). The retirement ages provided are the common, the ones for each gender, for individuals in the 25^{th} and the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile and for women in Nevada in the 4^{th} income quartile.

Table 1: Index and retirement age for several policies ($\beta = 1$)

Common retirement age	85
Income percentile	
10^{th}	-4
20^{th}	-3
30^{th}	-2
40^{th}	0
50^{th}	0
60^{th}	+1
70^{th}	+2
80^{th}	+3
90^{th}	+4
100^{th}	+6

The Table shows the difference in retirement age (with β and α equal to 1) between a common retirement age and ones differentiated based on income percentiles. For example, people in the 20^{th} could retire at 82 if the retirement is differentiated.

Table 2: Retirement age by percentiles (β and $\alpha = 1$)

β	Index					
	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & quartile
1	98.32 %	96.80 %	95.04 %	99.59 %	97.92 %	95.08 %
2	97.64 %	94.97 %	92.35 %	99.53 %	97.15 %	92.57 %
3	96.89 %	93.38 %	90.16 %	99.38 %	96.40 %	90.54 %
4	96.72 %	92.54 %	88.83 %	99.47 %	96.03 %	89.30 %
5	95.88 %	91.68 %	87.56 %	99.41 %	95.41 %	88.18 %
10	94.39 %	90.18 %	84.30 %	99.07 %	93.17 %	85.14 %

β	Retirement age ($\alpha = 1$)												
	Unique	Male	Female	25^{th}	75^{th}	M, 25^{th}	F, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
2	83	81	85	80	85	78	86	84	81	82	83	77	86
3	81	79	82	79	83	76	84	82	79	80	81	75	84
4	79	77	80	77	81	75	82	80	78	79	79	74	82
5	78	76	79	76	79	74	81	79	77	77	78	74	80
10	75	73	76	74	76	72	77	75	74	74	75	72	76

The Table provides the value of the index with different values of β. The index is done for tagging by sex, by income percentile, by sex and percentile, by state, by state and sex and by state, sex and income quartile. The lines below shows different retirement ages depending on the policy (with α equal to 1). The retirement ages provided are the common, the ones for each gender, for individuals in the 25^{th} and the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile and for women in Nevada in the 4^{th} income quartile.

Table 3: Index and retirement age for various value of β

σ	Unique	By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
		Male	Female	25^{th}	75^{th}	Male, 25^{th}	Female, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	87	83	80	89	83	87	85	85	79	88
0.75	83	81	85	80	85	77	87	81	85	83	83	76	86
0.5	80	77	82	77	82	74	84	81	77	79	80	72	83
0.25	73	71	76	70	76	68	78	75	71	73	74	65	81
0	40	40	40	40	40	40	40	40	40	40	40	40	40

The Table provides the retirement ages with different values of σ. The retirement ages provided are the common, the ones for each gender, for individuals in the 25^{th} and the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile and for women in Nevada in the 4^{th} income quartile. σ equals to 1 is our benchmark from previous Table.

Table 4: Retirement age by σ (β and $\alpha = 1$)

σ/β	By sex			By percentile			By sex & percentile		
	1	2	5	1	2	5	1	2	5
1	98.32 %	97.64 %	95.88 %	96.80 %	94.97 %	91.68 %	95.04 %	92.35 %	87.56 %
0.75	98.48 %	97.65 %	96.33 %	96.70 %	94.71 %	91.73 %	95.06 %	92.18 %	87.67 %
0.5	98.55 %	97.72 %	96.22 %	96.42 %	94.41 %	91.50 %	94.96 %	92.00 %	87.54 %
0.25	98.78 %	97.86 %	96.50 %	96.09 %	93.91 %	91.29 %	94.89 %	91.74 %	87.50 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
σ/β	By state			By state & sex			By state, sex & quartile		
	1	2	5	1	2	5	1	2	5
1	99.59 %	99.53 %	99.41 %	97.92 %	97.15 %	95.41 %	95.08 %	92.57 %	88.18 %
0.75	99.65 %	99.48 %	99.48 %	98.11 %	97.13 %	95.61 %	95.15 %	92.38 %	88.26 %
0.5	99.65 %	99.51 %	99.46 %	98.25 %	97.24 %	95.61 %	95.14 %	92.28 %	88.18 %
0.25	99.67 %	99.48 %	99.45 %	98.48 %	97.36 %	95.77 %	95.16 %	92.10 %	88.16 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %

The Table provides the index with different values of σ and β. σ varies from 0 to 1 and β takes the value of 1,2 and 5. σ equal to 1 is our benchmark from previous Table.

Table 5: Index by σ and by β

Truncation	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & percentile
40	98.32 %	96.80 %	95.04 %	99.59 %	97.92 %	95.08 %
55	98.32 %	97.13 %	95.33 %	99.66 %	97.95 %	95.32 %
60	98.30 %	97.22 %	95.42 %	99.57 %	97.83 %	95.36 %
70	98.28 %	97.67 %	95.88 %	99.64 %	97.87 %	95.85 %

The Table provides the index with different level of truncation. Truncation at 40 is our benchmark from the previous Table, truncation at 55, 60, 70 are arbitrary values of truncation.

Table 6: Index with different level of truncation

		By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
Truncation	Unique	Male	Female	25	75	M, 25	F, 75	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1	Nev, F, 4
40	85	83	88	83	87	80	89	87	83	85	85	79	88
55	86	84	88	83	88	81	89	88	84	86	86	80	89
60	86	84	88	84	88	81	89	88	84	86	86	81	89
70	88	86	89	86	89	84	90	89	86	87	87	84	90

The Table provides the retirement ages with different level of truncation. The retirement ages provided are the common, the ones for each gender, for individuals in the 25^{th} and the 75^{th} income percentile, for men in the 25^{th} income percentile, for women in the 75^{th} income percentile, for residents in Minesotta, Nevada, for men in Minesotta, for women in Nevada, for men in Minesotta and in the 1^{st} income quartile and for women in Nevada in the 4^{th} income quartile. Truncation at 40 is our benchmark from the previous Table, truncation at 55, 60, 70 are arbitrary values of truncation.

Table 7: Retirement ages with different level of truncation ($\alpha = 1$)

Appendix A. The parameter $(\boldsymbol{\mu}(\mathbf{m}), \boldsymbol{\mu_j}(\mathbf{m_j}))$

The social planner desires to minimize the sum of the deviations between the retirement age and the longevity of the observations in each subset/the entire set. Let us study $\mu(\mathbf{m})$:

$$\sum_{i=1}^{n} |\alpha m_i - \alpha\mu(\mathbf{m})|^{\beta} \tag{A.1}$$

The derivative of (A.1) with respect to $\mu(\mathbf{m})$ is:

$$\sum_{i=1}^{n} -\alpha\beta|\alpha m_i - \alpha\mu(\mathbf{m})|^{\beta-2}(\alpha m_i - \alpha\mu(\mathbf{m})) \tag{A.2}$$

If β is odd, then the root is located at the following condition:

$$\sum_{i=1}^{n} |\alpha m_i - \alpha\mu(\mathbf{m})|^{\beta-1}\frac{(\alpha m_i - \alpha\mu(\mathbf{m}))}{|\alpha m_i - \alpha\mu(\mathbf{m})|} \tag{A.3}$$

Condition (A.3) states that μ is the median when β equals 1. If β is odd and above 1, there exists no statistical name to the condition (A.3) and the social planner finds the root by computation and by applying condition (A.3). For an even value of β, the root is located at the following condition:

$$\sum_{i=1}^{n} (\alpha m_i - \alpha\mu(\mathbf{m}))^{\beta-1} = 0 \tag{A.4}$$

Condition (A.4) states that μ is the mean when β equals 2. There exists no statistical name to the condition (A.4) when β is even and above 2 and the social planner finds the root by computation and by applying condition (A.4).

The same principles apply for $\mu_j(\mathbf{m_j}), \forall j$.

Appendix B. Data construction

Our data have been constructed based on the life table technique detailed in Chiang (1984) and on the mortality rate provided by Chetty et al. (2016). The life-table method starts with a normalized population (called the "radix") and, at each interval of time, a fraction of the population "died" based on the empirically observed mortality rate. The division of the total years lived beyond age x by the population alive at that age gives the life expectancy of the population at age x. Our interest lies in the number of people dying at each age, which provides us our distribution of longevity. We used the mortality rate provided by Chetty et al. (2016)[8] which they computed based on a sample of

[8] They provided adjusted and non-adjusted race mortality rate, we use the non-adjusted to approach more the real distribution.

1,4 billion observations from deidentified tax records between 1999 and 2014. Their mortality rates are the empirical one between 40 and 75 years old, then they computed an interpolation using the Gompertz curve for the ages between 76 and 90 and finally, used the income-independent mortality rates based on NCHS and SSA data for the ages between 91 and 120. Figure B.1 shows us the longevity distribution for women in the 20^{th} and in the 80^{th} percentile. One can notice the presence of a spike at 91, which is the result of the change from the Gompertz curve to the NCHS-SSA mortality rate. It is more important for the 80^{th} percentile because there are more person alive at 91 in it than in the 20^{th} and the change to the NCHS-SSA mortality rate is therefore more reflected in it.

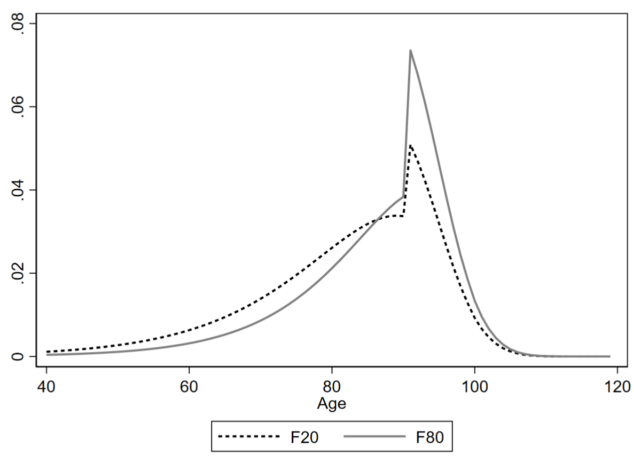

Figure B.1: Distribution of longevity for women in the 20^{th} and the 80^{th} percentile

The presence of this spike could raises the controversy of its importance for the results that we obtain. As a robustness test, we have also computed the distribution using ony the Gompertz curve. There is some empirical debate about the limit age until which it can be used; for example, Gavrilov and Gavrilova (2011) found that it could be extended until 105 without any major issues. Above 105, its relevance is difficult to test due to the small number of observations and the quality of the data.[9] As there is no much individuals living older than 105, extending the Gompertz curve until the end is not a strong assumption. In the main part of the paper, we use graphs based on the Gompertz curve for reading easiness (no spike), but the results reported are those using the assumptions of Chetty. Robustness of results using only the Gompertz curve could be found in Appendix C. A little limitation of our database is that we

[9] An individual older than 105 has his birth date at the start of the 1900's and the record are often of poor quality. For example, Gavrilov and Gavrilova (2011) showed that the ratio male-female observed in their data above 105 could not be the true one.

could not decompose longevity by months. We think that it would only change slightly our results as the main part is already captured by using years. There remains some seasonality of death[10], but this should not impact dramatically our results. Another limitation of our study is that we have to take as exogeneous the longevity distribution; although some papers (see e.g. Dave et al., 2006) have shown that retirement has an effect on the health of individuals.

Appendix C. Table with the data using only the Gompertz curve

Appendix C.1. Index and retirement age for various value of β

β	Index					
	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & quartile
1	98.44 %	97.02 %	95.36 %	99.62 %	98.07 %	95.43 %
2	97.22 %	94.53 %	91.58 %	99.29 %	96.51 %	91.75 %
3	96.27 %	92.71 %	88.69 %	99.15 %	95.36 %	89.03 %
4	95.24 %	91.46 %	86.46 %	98.81 %	94.08 %	86.70 %
5	94.40 %	90.69 %	84.78 %	98.67 %	93.06 %	85.00 %
10	89.79 %	89.20 %	78.85 %	97.55 %	87.95 %	79.19 %

β	Retirement age ($\alpha = 1$)											
	Unique	Male	Female	25^{th}	75^{th}	M, 25^{th}	F, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
2	83	81	86	81	85	78	87	85	81	83	83	77	87
3	82	80	84	79	84	77	85	83	80	81	82	76	85
4	80	78	82	78	82	76	84	82	78	80	80	75	83
5	79	77	81	77	81	75	82	80	77	79	79	74	82
10	77	75	78	75	77	73	78	77	75	76	76	73	78

Table C.1: Index and retirement age for various value of β

[10] The seasonality of death is not felt uniformly across the population; old people and those at a low socioeconomic level tend to be more impacted by it (Rau, 2006).

Appendix C.2. Retirement age by σ (β and $\alpha = 1$)

σ	Unique	By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
		Male	Female	25^{th}	75^{th}	Male, 25^{th}	Female, 75^{th}	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1^{st}	Nev, F, 4^{th}
1	85	83	88	83	87	80	89	87	83	85	85	79	88
0.75	83	81	85	80	85	77	87	85	81	83	83	76	86
0.5	80	77	82	77	82	74	84	81	77	79	80	72	83
0.25	73	71	76	70	76	68	78	75	71	73	74	65	78
0	40	40	40	40	40	40	40	40	40	40	40	40	40

Table C.2: Retirement age by σ (β and $\alpha = 1$)

Appendix C.3. Index by σ and by β

σ/β	By sex			By percentile			By sex & percentile		
	1	2	5	1	2	5	1	2	5
1	98.44 %	97.22 %	94.40 %	97.02 %	94.53 %	90.69 %	95.36 %	91.58 %	84.78 %
0.75	98.57 %	97.37 %	94.46 %	96.89 %	94.43 %	90.45 %	95.35 %	91.62 %	84.69 %
0.5	98.62 %	97.47 %	94.70 %	96.59 %	94.13 %	90.48 %	95.20 %	91.50 %	84.95 %
0.25	98.83 %	97.67 %	95.01 %	96.23 %	93.76 %	90.33 %	95.07 %	91.44 %	85.17 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %
σ/β	By state			By state & sex			By state, sex & quartile		
	1	2	5	1	2	5	1	2	5
1	99.62 %	99.29 %	98.67 %	98.07 %	96.51 %	93.06 %	95.43 %	91.75 %	85.00 %
0.75	99.68 %	99.43 %	98.47 %	98.22 %	96.78 %	93.03 %	95.45 %	91.86 %	84.94 %
0.5	99.67 %	99.42 %	98.76 %	98.33 %	96.89 %	93.43 %	95.38 %	91.83 %	85.25 %
0.25	99.70 %	99.44 %	98.86 %	98.54 %	97.14 %	93.82 %	95.33 %	91.86 %	85.58 %
0	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %

Table C.3: Index by σ and by β

Appendix C.4. Index with different level of truncation

Truncation	By sex	By percentile	By sex & percentile	By state	By state & sex	By state, sex & percentile
40	98.44 %	97.02 %	95.36 %	99.62 %	98.07 %	95.43 %
55	98.45 %	97.35 %	95.67 %	99.69 %	98.12 %	95.70 %
60	98.44 %	97.45 %	95.77 %	99.61 %	98.02 %	95.76 %
70	98.47 %	97.89 %	96.23 %	99.68 %	98.12 %	96.28 %

Table C.4: Index with different level of truncation

Appendix C.5. Retirement ages with different level of truncation (α = 1)

		By sex		By percentile		By sex & percentile		By state		By state & sex		By state, sex & quartile	
Truncation	Unique	Male	Female	25	75	M, 25	F, 75	Minesotta	Nevada	Min, M	Nev, F	Min, M, 1	Nev, F, 4
40	85	83	88	83	87	80	89	87	83	85	85	79	88
55	86	84	88	83	88	81	89	88	84	86	86	80	89
60	86	84	88	84	88	81	89	88	84	86	86	81	89
70	88	86	89	86	89	84	90	89	86	87	87	84	90

Table C.5: Retirement ages with different level of truncation (α = 1)