

Fair retirement and premature mortality: towards a theory of reverse retirement?

Gregory Ponthiere¹

UCLouvain, January 2020.

¹Université Paris 12, PSE and IUF.

- Retirement systems are often presented as giving a *fair reward* for a long working career.
- However, only workers who have a sufficiently long life benefit from that reward, but not workers who die prematurely.
 - In France, about 10 % of men and 4 % of women die before reaching the age of 60.
- **This paper reexamines the fairness of retirement systems in an economy with unequal lifetime.**
 - **The capacity of retirement systems - differing on ages of entry/exit of labor - to compensate the unlucky short-lived.**

Why should we care about the short-lived?

- Principle of Compensation *versus* Principle of Liberal Reward (Fleurbaey and Maniquet 2004, Fleurbaey 2008)
 - well-being inequalities due to circumstances should be abolished.
 - well-being inequalities due to efforts should be left unaffected.
- Inequalities in the duration of life due to:
 - genetic background: 25-33 % of inequalities (Christensen et al 2006)
PURE CIRCUMSTANCES
 - environmental factors: 23-40 % of premature deaths (Pimentel et al 1998) MIXED
 - lifestyles: 25 % of inequalities (Balía and Jones 2008)
RESPONSIBILITY / MIXED
- The Principle of Compensation more relevant under unequal lifetime.

- Fleurbaey et al (2016): focus on the age at retirement.
- *Ex post* egalitarian criterion (priority to the worst-off in realized terms).
- **Compensation of the short-lived pushes towards postponing retirement (wrt utilitarian criterion).**
- Intuition:
 - Postponing retirement allows to transfer more resources towards young individuals, who include those who will turn out to be short-lived.

This paper: reverse retirement

- Fleurbaey et al (2016) assumed the usual life cycle: individuals work at the young age, and become retiree at the old age.
- On the contrary, this paper considers a - purely hypothetical - alternative retirement system: *reverse retirement*.
 - Individuals are *first retirees* when being young, and, then, become *workers* once they reach a higher age.
- **This paper aims at examining the economic feasibility and the social desirability of reverse retirement.**

- We develop a 4-period OLG model.
 - Production involves capital and young/old labor.
 - Perfect substitutability between young/old labor (but with age-dependent labor productivity).
 - Old workers face a higher marginal disutility of labor than young ones.
- We study:
 - the temporary equilibrium and the long-run dynamics of the economy under standard/reverse retirement,
 - the social desirability of reverse retirement under the utilitarian and the *ex post* egalitarian social criteria.

Three main results

1. Under standard assumptions, the economy with reverse retirement - *once in place* - converges towards a unique stationary equilibrium.
2. If productivity decreases with age, reverse retirement is never optimal under the utilitarian criterion, but can be optimal under the *ex post* egalitarian criterion.
3. Although the transition from standard to reverse retirement would make the economy collapse at the laissez-faire, there exists a set of policy instruments that allow governments to organize a successful transition to reverse retirement.

- **On retirement and redistribution**

- Schokkaert and Van Parijs (2003), Cremer and Pestieau (2011), Schokkaert et al (2017).

- **On compensation for unequal lifetimes**

- Fleurbaey et al (2016): *extensive margin* of labor: age of exit from labor market (static).
- Leroux and Ponthiere (2018): *intensive margins* of labor (number of hours worked per week) (static).

- 1 The model
- 2 The laissez-faire
 - 1 The temporary equilibrium
 - 2 Long-run dynamics
- 3 The long-run social optimum
 - 1 The long-run utilitarian optimum
 - 2 The long-run ex post egalitarian optimum
- 4 Decentralization
- 5 Discussions
- 6 Concluding remarks

The model: basics

- We consider a 4-period OLG economy with risky lifetime. The length of each period is normalized to 1.
- Each cohort has a size $N > 0$ (replacement fertility).
- The lifecycle:
 - Period 1 (childhood): no decision.
 - Period 2 (young adult): plan their life, have one child, consume and save.
 - Period 3 (old adult), reached with probability $0 < \pi < 1$: consume and save.
 - Period 4 (very old age), reached with probability $0 < \pi p < 1$: consume.
- Labor does not take place in periods 1 and 4.
 - Standard retirement: entry of labor at age 1, exit at age $1 + \ell_t$.
 - Reverse retirement: entry of labor at age 2, exit at age $2 + \tilde{\ell}_{t+1}$.

The model: production

- Production takes place by using physical capital K_t and labor L_t :

$$Y_t = F(K_t, L_t)$$

where $F(\cdot)$ is increasing and concave and exhibits CRS.

- Capital fully depreciates after one period of use.
- Perfect substitutability between young-age and old-age labor:

$$L_t = aN\ell_t + b\pi N\tilde{\ell}_t$$

- Mixed results on age/productivity ($a \gtrsim b$):
 - Haegeland and Klette (1999): productivity grows with age;
 - Crepon et al (2003): an inverted U shaped curve in age;
 - Aubert and Crépon (2007), Gobel and Zwick (2009): productivity grows then stabilizes.

The model: preferences

- In young adulthood, well-being U_t^y is equal to:

$$U_t^y = u(c_t) - v\ell_t$$

where c_t is consumption in young adulthood, $v > 0$ is the (marginal) disutility of working. There exists $\bar{c} > 0$ such that $u(\bar{c}) = 0$.

- At the old age, individual well-being U_t^o is equal to:

$$U_t^o = u(d_t) - \tilde{v}\tilde{\ell}_t$$

where d_t is old-age consumption, $\tilde{v} > v$ is the (marginal) disutility of old-age labor.

- At the very old age (period 4), well-being U_t^{vo} is equal to:

$$U_t^{vo} = u(e_t)$$

where e_t is consumption at the very old age.

The laissez-faire

- We consider a perfectly competitive economy, where production factors are paid at their marginal productivity:

$$w_t = aF_L(K_t, aNl_t + b\pi N\tilde{l}_t)$$

$$\tilde{w}_t = bF_L(K_t, aNl_t + b\pi N\tilde{l}_t)$$

$$R_t = F_K(K_t, aNl_t + b\pi N\tilde{l}_t)$$

where w_t is the wage rate for the young worker, \tilde{w}_t is the wage rate for the old worker, and R_t equals 1 *plus* the interest rate.

- There exists a perfect annuity market, which yields an actuarially fair return. The return on savings for young adults is:

$$\hat{R}_t = \frac{R_t}{\pi}, \text{ where } \hat{R}_t \text{ denotes the gross interest factor.}$$

- The return on savings for old adults is equal to:

$$\check{R}_t = \frac{R_t}{\rho}, \text{ where } \check{R}_t \text{ denotes the gross interest factor.}$$

The laissez-faire: the temporary equilibrium

- The problem of the young adult at time t is:

$$\begin{aligned} \max_{s_t, z_t, \ell_t, \tilde{\ell}_{t+1}} & \left[\begin{aligned} & u(w_t \ell_t - s_t) - v \ell_t \\ & + \pi \left[u \left(\tilde{w}_{t+1}^{E_t} \tilde{\ell}_{t+1} + \frac{R_{t+1}^{E_t} s_t}{\pi} - z_{t+1} \right) - \tilde{\ell}_{t+1} \tilde{v} \right] \\ & + \pi p u \left(\frac{R_{t+2}^{E_t} z_{t+1}}{p} \right) \end{aligned} \right] \\ \text{s.t. } \ell & \geq 0 \text{ and } 1 - \ell \geq 0 \\ \text{s.t. } \tilde{\ell} & \geq 0 \text{ and } 1 - \tilde{\ell} \geq 0 \end{aligned}$$

The laissez-faire: the temporary equilibrium

Proposition (laissez-faire temporary equilibrium)

Consider the temporary equilibrium at time t given anticipations

$$\left\{ R_t^{E_{t-1}}, R_{t+1}^{E_{t-1}}, R_{t+1}^{E_t}, R_{t+2}^{E_t}, \tilde{w}_t^{E_{t-1}}, \tilde{w}_{t+1}^{E_t} \right\}.$$

- If $\frac{v}{\tilde{v}} < \frac{R_t^{E_{t-1}} w_{t-1}}{\tilde{w}_t^{E_{t-1}}}$ and $\frac{v}{\tilde{v}} < \frac{R_{t+1}^{E_t} w_t}{\tilde{w}_{t+1}^{E_t}}$, standard retirement prevails ($\tilde{\ell}_t = \tilde{\ell}_{t+1} = 0$).
- If $\frac{v}{\tilde{v}} > \frac{R_t^{E_{t-1}} w_{t-1}}{\tilde{w}_t^{E_{t-1}}}$ and $\frac{v}{\tilde{v}} > \frac{R_{t+1}^{E_t} w_t}{\tilde{w}_{t+1}^{E_t}}$, reverse retirement prevails ($\ell_{t-1} = \ell_t = 0$).

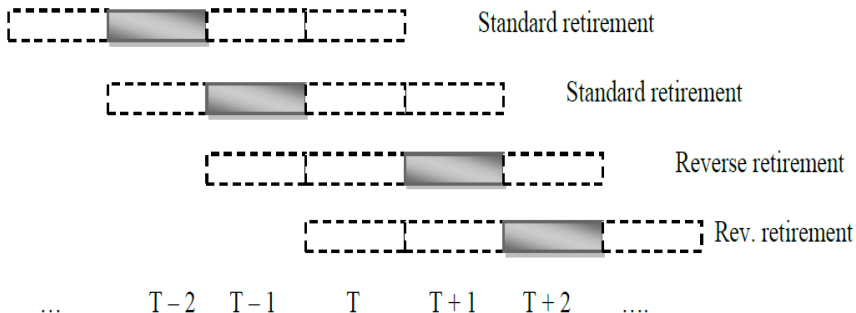
- Whether there is standard or reverse retirement depends on:
 - preferences: $v \ll \tilde{v}$.
 - age-productivity relation: $a \gtrsim b$.
 - there is under- or over-accumulation of capital: $R_{t+1}^{E_t} \gtrsim 1$.

Laissez-faire: a transition problem

- There is a *transition* from standard to reverse retirement when:

$$\frac{v}{\tilde{v}} < \frac{R_{t+1}^E w_t}{\tilde{w}_{t+1}^E} \quad \text{and} \quad \frac{v}{\tilde{v}} > \frac{R_{t+2}^E w_{t+1}}{\tilde{w}_{t+2}^E}$$

- The transition from standard to reverse retirement at the laissez-faire would lead the economy to collapse.



Laissez-faire: long-run dynamics

- In order to avoid difficulties raised by retirement regime shifts at the laissez-faire, we will assume that expectations are such that regime shifts cannot arise in the laissez-faire.
- For that purpose, we impose the following non-regime shift condition.

Definition (the non-regime shift condition)

Individual expectations on future factor prices $\left\{ \tilde{w}_{t+1}^{E_t}, R_{t+1}^{E_t} \right\}$ satisfy the conditions:

$$\text{If, at } t = 0, \frac{v}{\tilde{v}} < \frac{R_1^{E_0} w_0}{\tilde{w}_1^{E_0}}, \text{ then, for all } t > 0, \text{ we have } \frac{v}{\tilde{v}} < \frac{R_{t+1}^{E_t} w_t}{\tilde{w}_{t+1}^{E_t}};$$

$$\text{If, at } t = 0, \frac{v}{\tilde{v}} > \frac{R_1^{E_0} w_0}{\tilde{w}_1^{E_0}}, \text{ then, for all } t > 0, \text{ we have } \frac{v}{\tilde{v}} > \frac{R_{t+1}^{E_t} w_t}{\tilde{w}_{t+1}^{E_t}}.$$

Laissez-faire: long-run dynamics

- Under that non-regime shift condition, we can study the dynamics of the economy *conditionally on a given retirement regime*.
- The dynamics of capital is given by the law:

$$K_{t+1} = \underbrace{Ns_t}_{\text{the saving of the young}} + \underbrace{\pi Nz_t}_{\text{the saving of the old}}$$

- The saving s_t is positive under standard retirement, but negative under reverse retirement (young adults are then borrowing to finance young-age consumption).
- The saving z_t is always positive, whatever we consider standard or reverse retirement.

Proposition (laissez-faire stationary equilibrium)

Consider the stationary equilibrium with perfect foresight with $\max \{\ell_t, \tilde{\ell}_t\} < 1$. Assume the non-regime shift condition, as well as $u(c_t) = \log(c_t) - \beta$ and $Y_t = AK_t^\alpha (aN\ell_t + b\pi N\tilde{\ell}_t)^{1-\alpha}$ with $0 < \alpha < \frac{1}{2}$.

- If the laissez-faire temporary equilibrium at $t = 0$ involves standard retirement (i.e. $\ell_0 > 0, \tilde{\ell}_0 = 0$), there exist only two stationary equilibria $K^{s*} = 0$ and $K^{s**} > 0$, where K^{s*} is unstable, while K^{s**} is locally stable.
- If the laissez-faire temporary equilibrium at $t = 0$ involves reverse retirement (i.e. $\ell_0 = 0, \tilde{\ell}_0 > 0$), there exist only two stationary equilibria $K^{r*} = 0$ and $K^{r**} > 0$, where K^{r*} is unstable, while K^{s**} is locally stable.

The long-run utilitarian optimum

- The utilitarian social planner chooses $\{c, d, e, \ell, \tilde{\ell}, K\}$ so as to maximize the sum of individual utilities at the stationary equilibrium, subject to the resource constraint of the economy:

$$\begin{aligned} \max_{c, d, e, \ell, \tilde{\ell}, K} \quad & N [u(c) - v\ell + \pi [u(d) - \tilde{v}\tilde{\ell}] + \pi p u(e)] \\ \text{s.t.} \quad & F(K, aN\ell + b\pi N\tilde{\ell}) = Nc + \pi Nd + \pi p Ne + K \\ \text{s.t.} \quad & \ell \geq 0 \text{ and } 1 - \ell \geq 0 \\ \text{s.t.} \quad & \tilde{\ell} \geq 0 \text{ and } 1 - \tilde{\ell} \geq 0 \end{aligned}$$

Proposition

Consider the long-run utilitarian social optimum $\{c^u, d^u, e^u, \ell^u, \tilde{\ell}^u, K^u\}$.

- If young workers are weakly more productive than old workers (i.e. $a \geq b$), then standard retirement prevails (i.e. $\tilde{\ell}^u = 0$), and we have:

$$\begin{aligned}u'(c^u) &= u'(d^u) = u'(e^u) \\ u'(c^u)F_L(K^u, aN\ell^u)a &\geq v \text{ and } F_K(K^u, aN\ell^u) = 1\end{aligned}$$

- If old workers are more productive than young workers (i.e. $a < b$), then:

- If $\frac{v}{a} < \frac{\tilde{v}}{b}$, standard retirement prevails (i.e. $\tilde{\ell}^u = 0$) (see above);
- If $\frac{v}{a} > \frac{\tilde{v}}{b}$, reverse retirement prevails (i.e. $\ell^u = 0$), and we have:

$$\begin{aligned}u'(c^u) &= u'(d^u) = u'(e^u) \\ u'(c^u)F_L(K^u, b\pi N\tilde{\ell}^u)b &\geq \tilde{v} \text{ and } F_K(K^u, \pi N\tilde{\ell}^u) = 1\end{aligned}$$

The ex post egalitarian optimum

- The utilitarian criterion does not do justice to the idea of compensating the unlucky short-lived.
- The utilitarian optimum involves perfect smoothing of consumption, which leads to large well-being losses in case of premature death.
- The utilitarian optimum involves, when $\frac{v}{a} < \frac{\tilde{v}}{b}$, standard retirement, which is a major source of deprivation for the short-lived.

The ex post egalitarian optimum

- Under the *ex post* egalitarian criterion, the social planner chooses $\{c, d, e, \ell, \tilde{\ell}, K\}$ that maximize the realized lifetime well-being of the worst off living at the stationary equilibrium:

$$\begin{aligned} \max_{c,d,e,\ell,\tilde{\ell},K} \quad & \min \left\{ \begin{array}{l} u(c) - v\ell, u(c) - v\ell + u(d) - \tilde{v}\tilde{\ell}, \\ u(c) - v\ell + u(d) - \tilde{v}\tilde{\ell} + u(e) \end{array} \right\} \\ \text{s.t.} \quad & F(K, aN\ell + b\pi N\tilde{\ell}) = Nc + \pi Nd + p\pi Ne + K \\ \text{s.t.} \quad & \ell \geq 0 \text{ and } 1 - \ell \geq 0, \tilde{\ell} \geq 0 \text{ and } 1 - \tilde{\ell} \geq 0 \end{aligned}$$

- That planning problem can be rewritten as:

$$\begin{aligned} \max_{c,d,e,\ell,\tilde{\ell},K} \quad & N[u(c) - v\ell] \\ \text{s.t.} \quad & F(K, aN\ell + b\pi N\tilde{\ell}) = Nc + \pi Nd + p\pi Ne + K \\ \text{s.t.} \quad & u(d) - \tilde{v}\tilde{\ell} = 0 \text{ and } u(e) = 0 \\ \text{s.t.} \quad & \ell \geq 0 \text{ and } 1 - \ell \geq 0, \text{ s.t. } \tilde{\ell} \geq 0 \text{ and } 1 - \tilde{\ell} \geq 0 \end{aligned}$$

The ex post egalitarian optimum

Proposition

Consider the long-run ex post egalitarian optimum $\{c^e, d^e, e^e, \ell^e, \tilde{\ell}^e, K^e\}$. Define $\mu \equiv \frac{\pi N u'(c^e)}{u'(d^e)}$ as the shadow value of relaxing the old-age egalitarian constraint.

- If $\frac{v}{a} < \frac{\mu \tilde{v}}{\pi N b}$, then standard retirement holds ($\tilde{\ell}^e = 0$), and we have:

$$\begin{aligned} c^e &> d^e = \bar{c} = e^e \\ u'(c^e) F_L(K^e, aN\ell^e) &\geq \frac{v}{a} \text{ and } F_K(K^e, aN\ell^e) = 1 \end{aligned}$$

- If $\frac{v}{a} > \frac{\mu \tilde{v}}{\pi N b}$, then reverse retirement prevails ($\ell^e = 0$), and we have:

$$\begin{aligned} c^e &> d^e = u^{-1}(\tilde{v}\tilde{\ell}^e) > e^e = \bar{c} \\ u'(c^e) F_L(K^e, b\pi N\tilde{\ell}^e) &\geq \frac{\mu \tilde{v}}{\pi N b} \text{ and } F_K(K^e, b\pi N\tilde{\ell}^e) = 1 \end{aligned}$$

The ex post egalitarian optimum

Proposition

Assume $u(c_t) = \log(c_t) - \beta$ and $Y_t = AK_t^\alpha (aN\ell_t + b\pi N\tilde{\ell}_t)^{1-\alpha}$ with $0 < \alpha < \frac{1}{2}$. Assume that $\max\{\ell, \tilde{\ell}\} < 1$. Define $\bar{c} = \exp(\beta)$ and $\Xi \equiv A(1-\alpha)(A\alpha)^{\frac{\alpha}{1-\alpha}}$, as well as $\Phi \equiv \log\left(\frac{b}{\tilde{v}}\Xi\right) - \beta - 1$. Define also: $\eta \equiv \log\left(\frac{a}{\tilde{v}}\Xi\right) - \beta - 1 - v\frac{\pi(1+p)\bar{c}}{a\Xi}$ and $\xi \equiv \log\left(\frac{b\pi}{\tilde{v}}\Xi\Phi - \pi p\bar{c}\right) - \beta$.

- If $\max\{\eta, \xi\} = \eta$, the optimum involves standard retirement, and:

$$c^e = \frac{a\Xi}{v}; \ell^e = \frac{1 + \frac{v\bar{c}}{a\Xi}\pi(1+p)}{v}; K^e = aN(A\alpha)^{\frac{1}{1-\alpha}} \frac{\left(1 + \frac{v\pi(1+p)\bar{c}}{a\Xi}\right)}{v}$$

- If $\max\{\eta, \xi\} = \xi$, the optimum involves reverse retirement, and:

$$c^e = \frac{b\Xi}{\tilde{v}}\pi\Phi - \pi p\bar{c}; \tilde{\ell}^e = \frac{1 + \Phi}{\tilde{v}}; K^e = \pi bN(A\alpha)^{\frac{1}{1-\alpha}} \frac{\pi(1 + \Phi)}{\pi\tilde{v}}$$

Proposition

The long-run utilitarian optimum $\{c^u, d^u, e^u, \ell^u, \tilde{\ell}^u, K^u\}$ with standard retirement can be decentralized by means of an intergenerational lumpsum transfer device leading to a capital stock $K = K^u$ such that:

$$\begin{aligned}F_K(K^u, aN\ell^u) &= 1 \\F_L(K^u, aN\ell^u) u'(c^u) &= v\end{aligned}$$

Proposition

The long-run ex post egalitarian optimum with reverse retirement can be decentralized by means of:

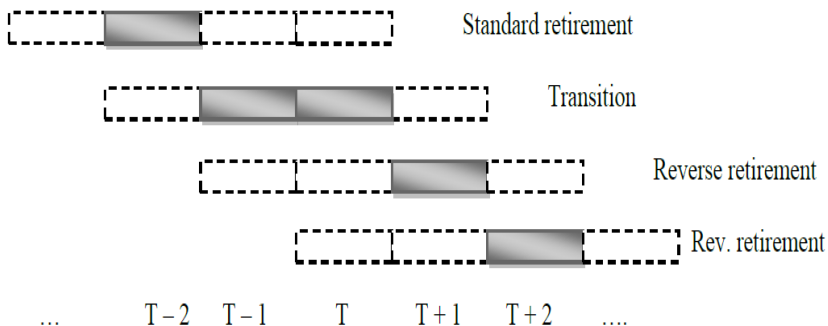
- *a prohibition of young-age labor: $\ell = \ell^e = 0$;*
- *a legal retirement age fixed at $2 + \tilde{\ell} = 2 + \tilde{\ell}^e$;*
- *a subsidy θ on young-age borrowing satisfying: $\theta^e = \frac{u'(d^e)}{u'(c^e)} - 1 > 0$;*
- *a tax τ on old-age savings satisfying: $\tau^e = 1 - \frac{u'(d^e)}{u'(e^e)} > 0$;*
- *an intragenerational lumpsum transfer device leading to the egalitarian constraint at the old age: $T^e = d^e - d^{LF}$;*
- *an intragenerational lumpsum transfer device leading to the egalitarian constraint at the very old age: $\tilde{T}^e = e^e - e^{LF}$;*
- *an intergenerational lumpsum transfer device leading to a capital stock $K = K^e$ such that: $F_K(K^e, \pi b N \tilde{\ell}^e) = 1$.*

- One may regard standard retirement as based on the Principle of Liberal Reward (inequalities due to efforts should be left unaffected).
 - Shifting from standard to reverse retirement would lead to a "free lunch" for the prematurely dead.
- The "free lunch" for the prematurely dead under reverse retirement is less unfair than the "no reward" under standard retirement.

- The standard retirement system is based on the *insurance motive* (Barr and Diamond 2006, Cremer and Pestieau 2011).
 - Standard retirement would provide insurance against old-age poverty (e.g. in case of myopia).
 - From that perspective, shifting from standard to reverse retirement might seem to go against the insurance motive.
- But the largest life-damage is not old-age poverty, but premature death without retirement.
- Hence reverse retirement *does justice to the insurance motive*, by insuring individuals against the largest life-damage.

Discussions: the transition

- The case of a raw transition.



Discussions: the transition

- Increasing the length of the transition divides the burden on more cohorts.



- We examined the economic feasibility and the social desirability of reverse retirement.
- **Economic feasibility:**
 - -: a laissez-faire transition would lead the economy to collapse;
 - +: once in place, the economy with reverse retirement converges towards a unique steady-state.
- **Social desirability:**
 - -: reverse retirement never optimal under utilitarianism (when labor productivity decreases with age);
 - +: reverse retirement can be optimal under *ex post* egalitarian criterion (if π large, \tilde{v} low and b close to a);
 - +: there exists a set of policy instruments allowing for a successful transition to reverse retirement.