Redistributive effects of different pension structures when longevity varies by socioeconomic status

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UCL-Belgium, January 28th, 2020
Motivation: Increasing longevity heterogeneity in the US (cohort and income)

Socio-economic differentials in life expectancy are widening in many countries.

Figure 1: US Male Life Expectancy at Age 50 by Midcareer Average Labor Income Quintile, as Estimated by NRC (2015), for Birth Cohorts of 1930 and 1960 (Extrapolated).

Mortality differences interact with government programs for the elderly like the pension system and may reduce or even reverse the direction of redistribution.
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**Figure 2:** Average total lifetime net benefits at age 50 for males (present value in thousands of dollars), by lifetime earnings quintile.

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- Studying the redistributive effects of public pension systems (NDC, DB) when longevity varies by SES
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- Studying the redistributive effects of public pension systems (NDC, DB) when longevity varies by SES
- Providing a general framework for comparing different pension systems
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The increase in the total number of pension points accumulated (pp) at the exact age \( t \) by an individual of type \( i \in I \) in any pension system can be formulated as follows

\[
\frac{\partial pp_i(t)}{\partial t} = (\bar{r} + \bar{\mu}(t))pp_i(t) + \phi \tau y_i(t) \text{ with } pp_i(0) = 0
\]  

(1)
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The increase in the total number of pension points accumulated (pp) at the exact age $t$ by an individual of type $i \in I$ in any pension system can be formulated as follows

$$\frac{\partial pp_i(t)}{\partial t} = (\bar{r} + \tilde{\mu}(t))pp_i(t) + \phi \tau y_i(t) \text{ with } pp_i(0) = 0$$

where
- $\bar{r}$ capitalization factor of pension points
- $\tilde{\mu}(t)$ mortality hazard rate at age $x$ used by the pension system
- $y_i(t)$ labor income of a worker belonging to group $i$ at age $x$
- $\tau$ contribution rate
- $\phi$ pension points earned per unit of social contribution paid, where $\phi = 1$ (in DC) and $\phi = \rho/\tau$ (in DB)
General pension model

Pension point (pp) system (Börsch-Supan, 2006; OECD, 2005)

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- \( \tau \) contribution rate
- \( \phi \) pension points earned per unit of social contribution paid, where \( \phi = 1 \) (in DC) and \( \phi = \rho / \tau \) (in DB)

Pension benefits

\[
b_i(R_i) = f_i(R_i)pp_i(R_i)
\]

(2)
Pension redistribution: Social security wealth

The evolution of the social security wealth (SSW) of an individual of type $i$ as 

$$\frac{\partial SSW_i}{\partial t} = \left( \tilde{r} + \tilde{\mu}(t) + 1 \right) P_i(t) \frac{\partial P_i(t)}{\partial t} + \tau y_i(t)$$

for $t \in (S_i, R_i)$ (3)

where $P_i(t)$ is the result of comparing at age $t$ the value of one dollar invested in the pension system to the value of investing the same dollar in the capital market.

The dynamics of $P_i$ is given by

$$\frac{1}{P_i(t)} \frac{\partial P_i(t)}{\partial t} =.$$
The evolution of the social security wealth (SSW) of an individual of type $i$ as

$$\frac{\partial SSW_i(t)}{\partial t} = \left( \tilde{r} + \tilde{\mu}(t) + \frac{1}{P_i(t)} \frac{\partial P_i(t)}{\partial t} \right) SSW_i(t) + \tau y_i(t) \text{ for } t \in (S_i, R_i) \quad (3)$$
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where

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The evolution of the social security wealth \((SSW)\) of an individual of type \(i\) as

\[
\frac{\partial SSW_i(t)}{\partial t} = \left( \bar{r} + \bar{\mu}(t) + \frac{1}{P_i(t)} \frac{\partial P_i(t)}{\partial t} \right) SSW_i(t) + \tau y_i(t) \text{ for } t \in (S_i, R_i) \tag{3}
\]

where

- \(P_i(t)\) is the result of comparing at age \(t\) the value of one dollar invested in the pension system to the value of investing the same dollar in the capital market.

The dynamics of \(P_i\) is given by \((P \text{ evolves differently by life expectancy})\)

\[
\frac{1}{P_i(t)} \frac{\partial P_i(t)}{\partial t} = (r - \bar{r}) + (\mu_i(t) - \bar{\mu}(t)). \tag{4}
\]
The evolution of the social security wealth \((SSW)\) of an individual of type \(i\) as

\[
\frac{\partial SSW_i(t)}{\partial t} = \left( \tilde{r} + \tilde{\mu}(t) + \frac{1}{P_i(t)} \frac{\partial P_i(t)}{\partial t} \right) SSW_i(t) + \tau y_i(t) \text{ for } t \in (S_i, R_i)
\]  

(3)

where

\(P_i(t)\) is the result of comparing at age \(t\) the value of one dollar invested in the pension system to the value of investing the same dollar in the capital market.

The dynamics of \(P_i\) is given by \((P\) evolves differently by life expectancy) \( \frac{1}{P_i(t)} \frac{\partial P_i(t)}{\partial t} = \frac{(r - \tilde{r})}{\text{Intergenerational effect}} + \frac{(\mu_i(t) - \tilde{\mu}(t))}{\text{Mortality differential effect}} \).  

(4)
The individual optimally chooses

the length of schooling, $S_i$
the retirement age, $R_i$
the consumption path, $c_i(t)$, and
hours worked path, $\ell_i(t)$,

maximizing a lifetime expected utility subject to a life cycle budget constraint and social security rules.
Table 1: Modeled PAYG pension systems

<table>
<thead>
<tr>
<th>Pension system</th>
<th>Acronym</th>
<th>Replacement Rate</th>
<th>Diff. in Life Expectancy across income groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defined Benefits</td>
<td>DB-I</td>
<td>Constant</td>
<td>No</td>
</tr>
<tr>
<td>Defined Benefits</td>
<td>DB-II</td>
<td>Progressive</td>
<td>No</td>
</tr>
<tr>
<td>Defined Benefits</td>
<td>DB-III</td>
<td>Progressive</td>
<td>Yes (at retirement)</td>
</tr>
<tr>
<td>Defined Contribution</td>
<td>NDC-I</td>
<td>Remaining life</td>
<td>No</td>
</tr>
<tr>
<td>Defined Contribution</td>
<td>NDC-II</td>
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<td>Yes (at retirement)</td>
</tr>
<tr>
<td>Defined Contribution</td>
<td>NDC-III</td>
<td>Remaining life</td>
<td>Yes (all ages)</td>
</tr>
</tbody>
</table>

Notes: (i) DB-II case matches the US pension system, (ii) NDC-II and DB-III cases implements the proposal of Ayuso, Bravo, and Holzmann (2017).
Those groups with an IRR < 2% transfer resources to those groups with an IRR > 2%
Redistributive effects of each pension system:
Internal rate of return by income quintile and pension system

Those groups with an IRR < 2% transfer resources to those groups with an IRR > 2%

**Figure 3:** US males, Mortality regime of 1930 cohort
Redistributive effects of each pension system: Internal rate of return by income quintile and pension system

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Figure 3: US males, Mortality regime of 1960 cohort
Redistributive effects of each pension system: Impact of each pension system on wealth by income quintile (relative to the NDC-III system)

**Figure 4:** US males, Mortality regime of 1930 cohort
Redistributive effects of each pension system: Impact of each pension system on wealth by income quintile (relative to the NDC-III system)

Figure 4: US males, Mortality regime of 1960 cohort
Redistributive effects of each pension system: Impact of each pension system on welfare by income quintile (relative to the NDC-III system)

<table>
<thead>
<tr>
<th>Income quintile</th>
<th>DBI</th>
<th>DBII</th>
<th>DBIII</th>
<th>NDCI</th>
<th>NDCII</th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>−0.75</td>
<td>−0.50</td>
<td>−0.25</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>q2</td>
<td>−0.75</td>
<td>−0.50</td>
<td>−0.25</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>q3</td>
<td>−0.75</td>
<td>−0.50</td>
<td>−0.25</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>q4</td>
<td>−0.75</td>
<td>−0.50</td>
<td>−0.25</td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td>q5</td>
<td>−0.75</td>
<td>−0.50</td>
<td>−0.25</td>
<td>0.00</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Figure 5: US males, Mortality regime of 1930 cohort
Redistributive effects of each pension system:
Impact of each pension system on welfare by income quintile (relative to the NDC-III system)

Figure 5: US males, Mortality regime of 1960 cohort
We have developed a general framework for analyzing any pension system.

We have assessed the direct and indirect effects of a variety of policy adjustments to DB and NDC pension programs in environments of more or less mortality heterogeneity.

Achieving progressivity in lifetime benefits would require more than current progressivity in annual benefits in combination with life tables for each group.
Conclusions

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Conclusions (cont’d)

Indirect effects on wealth:

NDCs small and regressive

Non-progressive DBs strong and positive (small LE diff.) and regressive (big LE diff.)

Progressive DBs strong and negative at high income

Indirect effect on welfare:

Losses for lower incomes and small gains for higher incomes

This is reduced with progressive DB plans
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Thank you!

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The value of $P_i$ at retirement is

$$P_i(t) = \phi f_i(R_i) A_i(R_i, r) e^{\int_{t}^{R_i} \tilde{r} + \tilde{\mu}(j) - (r + \mu_i(j)) dj}.$$  \hspace{1cm} (5)

The term $\phi f_i(R_i) A_i(R_i, r)$ PV of the stream of benefits from retirement until death that results from the contribution of a dollar.

The exponential term accounts for the difference from age $t$ until retirement between the rate of return of the pension system, $\tilde{r} + \tilde{\mu}$, and the rate of return of the capital market, $r + \mu_i$. 

back
Redistributive effects of each pension system:
Relative value of one additional dollar invested in the pension system

\[ \overline{P}_i(t) = P_i(t)(1 - \varepsilon_i) \]

is the relative value of investing one additional dollar in the pension system to the value of investing on additional dollar in the capital market.
Redistributive effects of each pension system: Relative value of one additional dollar invested in the pension system

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**Figure 6:** US males, **Mortality regime of 1930 cohort**
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**Figure 6:** US males, **Mortality regime of 1960 cohort**

**Average vs. Marginal P values**
Economic problem

Maximize the expected lifetime utility at age $x$

$$V_i(x) = \int_x^\omega e^{-\int_x^t \rho + \mu_i(j) \, dj} U(c_i(t)) \, dt - \int_{S_i}^{R_i} e^{-\int_x^t \rho + \mu_i(j) \, dj} \alpha_i \nu(\ell_i(t)) \, dt$$

$$- \int_x^{S_i} e^{-\int_x^t \rho + \mu_i(j) \, dj} \eta \, dt + \int_{S_i}^{\omega} e^{-\int_x^t \rho + \mu_i(j) \, dj} \varphi(t) \, dt.$$  \hspace{1cm} (6)
Maximize the expected lifetime utility at age $x$

$$V_i(x) = \int_x^\omega e^{-\int_x^t \rho + \mu_i(j) dj} U(c_i(t)) dt - \int_{S_i}^{R_i} e^{-\int_x^t \rho + \mu_i(j) dj} \alpha_i \nu(\ell_i(t)) dt$$

$$- \int_x^{S_i} e^{-\int_x^t \rho + \mu_i(j) dj} \eta dt + \int_x^{R_i} e^{-\int_x^t \rho + \mu_i(j) dj} \varphi(t) dt.$$  \hfill (6)

subject to

$$\int_x^\omega e^{-\int_x^t r + \mu_i(j) dj} c_i(t) dt = a_i(x) + \int_x^{R_i} e^{-\int_x^t r + \mu_i(j) dj} y_i(S_i, t) dt + SSW_i(x),$$  \hfill (7)
Maximize the expected lifetime utility at age $x$

$$V_i(x) = \int_x^\omega e^{-\int_x^t r + \mu_i(j) dj} U(c_i(t)) dt - \int_{S_i}^{R_i} e^{-\int_x^t r + \mu_i(j) dj} \alpha_i \nu(\ell_i(t)) dt$$

$$- \int_x^{S_i} e^{-\int_x^t r + \mu_i(j) dj} \eta dt + \int_\omega^{R_i} e^{-\int_x^t r + \mu_i(j) dj} \varphi(t) dt.$$  \hspace{1cm} (6)

subject to

$$\int_x^\omega e^{-\int_x^t r + \mu_i(j) dj} c_i(t) dt = a_i(x) + \int_{S_i}^{R_i} e^{-\int_x^t r + \mu_i(j) dj} y_i(S_i, t) dt + SSW_i(x),$$  \hspace{1cm} (7)

where

$$y_i(S_i, t) = w_i(S, t)\ell_i(t),$$  \hspace{1cm} (8)

$$w_i(S, t) = h_i(S)\bar{w}(t - S) = h_i(S) \exp(\beta_0(t - S) - \beta_1(t - S)^2),$$  \hspace{1cm} (9)

$$\frac{\partial h_i(t)}{\partial t} = \theta_i h_i(t)^\gamma - \delta h_i(t) \text{ for } t \in (x_0, S_i), \ h_i(x_0) = 1.$$  \hspace{1cm} (10)
The optimal consumption path and labor supply, conditional on a length of schooling $S_i$ and a retirement age $R_i$, are characterized by

\[
\frac{1}{c_i(t)} \frac{\partial c_i(t)}{\partial t} = \sigma_c (r - \rho),
\]

\[
\frac{1}{\ell_i(t)} \frac{\partial \ell_i(t)}{\partial t} = \sigma_l \left( \frac{\partial w(t-S_i)}{\partial t} \frac{w(t-S_i)}{w(t-S_i)} - (r - \rho) + \frac{\tau \bar{P}_i(t)}{1 - \tau + \tau \bar{P}_i(t)} \frac{\partial \bar{P}_i(t)}{\partial t} \right),
\]

where $\bar{P}_i(t) = P_i(t) (1 - \varepsilon_i)$ compares the value of one additional dollar invested in the pension system to the value of investing the same additional dollar in the capital market.
An **optimal length of schooling** satisfies

$$r_i^h(S^*) = r_i^w(S^*, R_i) + \frac{\eta}{U'(c_i(S^*))W_i(S^*, R_i)}.$$  \hspace{1cm} (13)
An **optimal length of schooling** satisfies

\[ r_i^h(S^*) = r_i^w(S^*, R_i) + \frac{\eta}{U'(c_i(S^*))W_i(S^*, R_i)}. \]  \hspace{1cm} (13)

An interior **optimal retirement age** satisfies

\[ U'(c_i(R^*))y_i(S, R^*)(1 - \tau_i^{GW}(R^*)) = \alpha_i v(\ell_i(R^*)) + \varphi(R^*). \]  \hspace{1cm} (14)
An **optimal length of schooling** satisfies

$$r^h_i(S^*) = r^w_i(S^*, R_i) + \frac{\eta}{U'(c_i(S^*))} W_i(S^*, R_i).$$  \hspace{1cm} (13)

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US OAI pension system (DB-II)

**Replacement rate**, \( \psi(p) \)  
\( p := \) Pension earnings or Average Indexed Monthly Earnings (AIME)  
\( y := \) Average Labor Income

Figure 7: Old-Age Insurance replacement rate in the US

Note: AIME is calculated as \(1/12\) of the mean of the 35 highest labor incomes over the working life, measured in real terms.
### Table 2: Marginal and average replacement rates at the normal retirement age

<table>
<thead>
<tr>
<th>Case</th>
<th>Marginal replacement rate $f_i(R_n, pp_i(R_n))(1 - \varepsilon_i)$</th>
<th>Replacement rate $f_i(R_n, pp_i(R_n))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB-II</td>
<td>$\begin{cases} 0.90 &amp; \text{for } pp_i \leq \bar{y}/6, \ 0.32 &amp; \text{for } \bar{y}/6 &lt; pp_i &lt; \bar{y}, \ 0.15 &amp; \text{for } \bar{y} &lt; pp_i \leq 2\bar{y}, \ 0.00 &amp; \text{for } 2\bar{y} &lt; pp_i, \end{cases}$</td>
<td>$\begin{cases} 0.90 &amp; \text{for } pp_i \leq \bar{y}/6, \ 0.32 + \frac{0.58}{6} \bar{y}<em>{pp_i} &amp; \text{for } \bar{y}/6 &lt; pp_i &lt; \bar{y}, \ 0.15 + \frac{1.60}{6} \bar{y}</em>{pp_i} &amp; \text{for } \bar{y} &lt; pp_i \leq 2\bar{y}, \ 3.40 \frac{\bar{y}}{6}_{pp} &amp; \text{for } 2\bar{y} &lt; pp_i, \end{cases}$</td>
</tr>
<tr>
<td>DB-III</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DB-I</td>
<td>0.417</td>
<td>0.417</td>
</tr>
<tr>
<td>NDC-I</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDC-II</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDC-III</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The term $\bar{y}$ denotes the average labor income of the economy.
Table 3: Optimal length of schooling by income quintile \((R_i)\), US male birth cohorts 1930 and 1960

<table>
<thead>
<tr>
<th>Cohort 1930</th>
<th>Defined Contribution (NDC)</th>
<th>Defined Benefit</th>
<th>Defined Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. LT</td>
<td>Corrected Avg. LT</td>
<td>(i)-th LT</td>
</tr>
<tr>
<td></td>
<td>NDC-I</td>
<td>NDC-II</td>
<td>NDC-III</td>
</tr>
<tr>
<td>Quintile 1</td>
<td>11.4</td>
<td>11.4</td>
<td>11.5</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>11.8</td>
<td>11.8</td>
<td>11.9</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>12.4</td>
<td>12.4</td>
<td>12.4</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>14.4</td>
<td>14.4</td>
<td>14.4</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>15.8</td>
<td>15.7</td>
<td>15.7</td>
</tr>
<tr>
<td>Cohort 1960</td>
<td>Quintile 1</td>
<td>11.3</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>Quintile 2</td>
<td>12.4</td>
<td>12.5</td>
</tr>
<tr>
<td></td>
<td>Quintile 3</td>
<td>15.7</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td>Quintile 4</td>
<td>18.8</td>
<td>18.8</td>
</tr>
<tr>
<td></td>
<td>Quintile 5</td>
<td>19.0</td>
<td>19.0</td>
</tr>
</tbody>
</table>
### Table 4: Optimal retirement age by income quintile ($R_i$), US male birth cohorts 1930 and 1960

<table>
<thead>
<tr>
<th></th>
<th>Defined Contribution (NDC)</th>
<th>Defined Benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. LT Corrected i-th LT</td>
<td>Non-progressive Progressive</td>
</tr>
<tr>
<td></td>
<td>NDC-I NDC-II NDC-III</td>
<td>DB-I DB-II DB-III</td>
</tr>
<tr>
<td><strong>Cohort 1930</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quintile 1</td>
<td>60.9 60.9 61.1</td>
<td>63.2 62.7 62.8</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>61.5 61.5 61.7</td>
<td>63.8 63.3 63.3</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>62.5 62.5 62.5</td>
<td>64.6 64.0 64.1</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>64.6 64.5 64.5</td>
<td>65.3 64.7 64.7</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>66.8 66.6 66.4</td>
<td>67.1 65.2 65.2</td>
</tr>
<tr>
<td><strong>Cohort 1960</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quintile 1</td>
<td>60.1 60.2 60.6</td>
<td>61.1 62.0 İ 62.0 İ</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>62.2 62.4 62.7</td>
<td>64.8 63.4 64.7</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>67.3 67.5 67.4</td>
<td>69.8 68.4 68.6</td>
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<tr>
<td>Quintile 4</td>
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<td>70.0 70.0 70.0</td>
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<tr>
<td>Quintile 5</td>
<td>70.0 70.0 70.0</td>
<td>70.0 70.0 70.0</td>
</tr>
</tbody>
</table>

*Note: Quintiles 1 to 5 represent increasing income levels.*
Figure 8: Stylized evolution over the working life of the value of one dollar contributed to the pension system for an individual who plans to retire at age $R_i$, $P_i(t)$. Case: when $\tilde{r} = r$. 

Back
Table 5: In-sample performance of the model: Optimal length of schooling ($S_i$), retirement age ($R_i$), and present value of lifetime benefits (PVB) by income quintile. US males born in 1930, US pension system (DB-II)

<table>
<thead>
<tr>
<th>Quintile</th>
<th>$S_i$</th>
<th>$R_i$</th>
<th>PVB at age 50 (in $1 000 s)$</th>
<th>Life expectancy at $S_i + 6$</th>
<th>Years-worked $e_i(S_i + 6)$</th>
<th>Years-retired $e_i(S_i + 6) - YW_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>q1</td>
<td>11.30</td>
<td>62.70</td>
<td>132</td>
<td>54.31</td>
<td>41.91</td>
<td>0.30</td>
</tr>
<tr>
<td>q2</td>
<td>11.80</td>
<td>63.30</td>
<td>149</td>
<td>54.63</td>
<td>42.12</td>
<td>0.30</td>
</tr>
<tr>
<td>q3</td>
<td>12.30</td>
<td>64.00</td>
<td>170</td>
<td>55.33</td>
<td>42.49</td>
<td>0.30</td>
</tr>
<tr>
<td>q4</td>
<td>13.20</td>
<td>64.70</td>
<td>198</td>
<td>56.25</td>
<td>42.50</td>
<td>0.32</td>
</tr>
<tr>
<td>q5</td>
<td>14.20</td>
<td>65.20</td>
<td>225</td>
<td>58.03</td>
<td>42.76</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Notes: Small figures highlighted in gray are data from the HRS on length of schooling and retirement age for males born in 1930, and from the NASEM (2015) on the present value of lifetime benefits for the same cohort.
### Table 6: Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td><strong>Demographics</strong></td>
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<td><strong>Preferences</strong></td>
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<tr>
<td>First age at entrance</td>
<td>$x_0$</td>
<td>14</td>
<td>Subjective discount factor</td>
<td>$\rho$</td>
<td>0.005</td>
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<tr>
<td>Maximum age</td>
<td>$\omega$</td>
<td>114</td>
<td>Utility cost of not being retired</td>
<td>$\varphi(t)$</td>
<td>186.29($e(t)$)$^{-1.8559}$</td>
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<tr>
<td>Annual population growth</td>
<td>$n$</td>
<td>0.005</td>
<td>Labor elasticity of substitution</td>
<td>$\sigma$</td>
<td>0.33</td>
</tr>
<tr>
<td>Minimum length of schooling</td>
<td>$S$</td>
<td>10</td>
<td>Utility weight of labor</td>
<td>$\alpha(q_1)$</td>
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<tr>
<td>Maximum length of schooling</td>
<td>$\overline{S}$</td>
<td>20</td>
<td></td>
<td>$\alpha(q_2)$</td>
<td>160</td>
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<td>$\alpha(q_3)$</td>
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<td>$\alpha(q_4)$</td>
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<td>$\alpha(q_5)$</td>
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<tr>
<td><strong>Technology</strong></td>
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<td><strong>Education</strong></td>
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<td>Market interest rate</td>
<td>$r$</td>
<td>0.030</td>
<td>Returns of scale in education</td>
<td>$\gamma$</td>
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<td>Labor-augmenting technological</td>
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<td>Disutility of schooling</td>
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<td>progress growth rate</td>
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<td>Mincerian eq.</td>
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<td>$\beta_1$</td>
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<tr>
<td><strong>Social security system</strong></td>
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<td></td>
<td>Learning ability</td>
<td>$\theta(q_1)$</td>
<td>0.110</td>
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<tr>
<td>Minimum retirement age</td>
<td>$R$</td>
<td>NDC=55, DB=62</td>
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<td>$\theta(q_2)$</td>
<td>0.110</td>
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<tr>
<td>Maximum retirement age</td>
<td>$\overline{R}$</td>
<td>70</td>
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<td>$\theta(q_3)$</td>
<td>0.110</td>
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<td>Capitalization factor</td>
<td>$\bar{r}$</td>
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<td>$\theta(q_4)$</td>
<td>0.115</td>
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<tr>
<td>Accrual rate in DB systems</td>
<td>$\phi$</td>
<td>1/45</td>
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<td>$\theta(q_5)$</td>
<td>0.115</td>
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<tr>
<td>Avg. replacement rate in DB systems</td>
<td>$f(pp)$</td>
<td>0.4167</td>
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<td>Social contribution rate</td>
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<td>Cohort 1930</td>
<td>$\tau_{1930}$</td>
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<td>Cohort 1960</td>
<td>$\tau_{1960}$</td>
<td>0.1460</td>
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</table>
Behavioral reactions vs. no behavioral reactions: Wealth

**Figure 9:** Difference in lifetime wealth between a model with and without behavioral reactions by pension system and mortality regime (1930 cohort vs 1960 cohort).
**Figure 10:** Relative difference in welfare between a model with and without behavioral reactions by pension system and mortality regime (1930 cohort vs 1960 cohort).