

Adjusting for a Mediator in Models With Two Crossed Treatment Variables

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In a simple mediation model, the effect of a manipulated variable X on a dependent variable Y over and above the effect of the mediator Me can be estimated by regressing Y on X and Me . The impact of X on Y in such a model is adjusted for the relationship both between X and Me and between Me and Y . The authors examine the adjustment function in the context of a 2×2 design with two manipulated variables. In such a situation, the mediator could be affected by either one of the main effects and/or the interaction between two manipulated variables. To adjust for the impact of the mediator, a standard procedure has been to rely on an ANCOVA that includes only the mediator. The authors show, both analytically and with simulations, that this leads to improper control of the mediator and to biased estimates of the model parameters.

Keywords: *mediation; ANCOVA; adjustment; regression*

In organizational settings, a common goal is to make customers more satisfied. One way to do this is to have employees appear friendlier so that they will be more positively regarded. It is thought that this will result in satisfied customers who will return in the future. Employees who smile are likely to be perceived as friendlier, and so one might be tempted to encourage smiles in customer relations. But are all smiles the same? And how do different types of smiles combine with the performance of the employees? In a recent study, Grandey, Fisk, Mattila, Jansen, and Sideman (2005) collected data to examine whether an authentic smile from hotel desk clerks leads to more customer satisfaction than an inauthentic one.¹ The study involved two manipulated independent variables, namely, the task performance of the clerks and the authenticity of their smile. The main dependent variable was customer satisfaction. The authors were also interested in the role of two other dependent variables (i.e., the perceived competence of the clerks and their perceived friendliness). For each of these intervening variables, one may wonder whether it plays a role in the observed effects of the two independent variables and their interaction on the main dependent variable. The present article shows that the answer to this question is not as straightforward as it may seem.

To introduce our argument, let us begin with a simpler question. Imagine for a moment that we are only interested in the impact of the authenticity of smile on customer satisfaction.

In line with predictions, our manipulation is shown to have an effect on the dependent variable. In the mediation literature, the effect of such a treatment (in this case authenticity of the smile) on the outcome variable (in this case customer satisfaction) is typically called the treatment's total effect (commonly designated as c). Imagine further that there is one intervening variable of interest: perceived competence of the clerk. One might then ask whether the increase in satisfaction in the authentic smile condition was because of an increase in perceived competence. In other words, one could ask whether competence mediated the treatment effect.² To examine this, two additional regression analyses would be conducted: the first one regressing perceived competence on the treatment variable and the second regressing the outcome variable, customer satisfaction, on both treatment and perceived competence (i.e., the mediator). The first regression model estimates the treatment effect on perceived competence (often referred to as a); the second one estimates the mediator effect on satisfaction controlling for treatment (commonly designated as b) and the treatment effect on satisfaction controlling for perceived competence (commonly designated as c').

In such an analysis, the equality $c' = c - ab$ could be conceptualized as the adjustment of the treatment effect when one moves from the total treatment effect to the treatment effect controlling for the mediator (i.e., perceived competence). Note that this adjustment function is also equivalent to $c - c' = ab$, a fundamental equality at the heart of the vast majority of mediational analyses (e.g., Baron & Kenny, 1986; MacKinnon, Lockwood, Hoffman, West, & Sheets, 2002; MacKinnon, Warsi, & Dwyer, 1995; Muller, Judd, & Yzerbyt, 2005).

In the above example, the key question concerns the mechanism that is responsible for the treatment effect. We want to know what produces the treatment effect. If perceived competence is, at least in part, responsible for the effect of the treatment on the dependent variable, the treatment effect should be reduced when adjusting for the role of perceived competence. Clearly thus, mediation analysis relies on the adjustment function of what is commonly called the analysis of covariance (ANCOVA) model. Here, ANCOVA means a model containing both categorical (e.g., a treatment) and continuous predictor variables (e.g., a mediator).

The goal of the present article is to examine this adjustment function in contexts where there are two crossed treatment variables of interest. In situations where one factor has been manipulated, we can readily identify what has to be included in the ANCOVA model. We simply need to control for the mediator in the ANCOVA model. As we will show, things are much less straightforward whenever one is dealing with two independent variables that may have both main and interactive effects on both the outcome variable and the mediator. We first analytically demonstrate that in a context with two crossed treatment variables, the simple inclusion of the mediator (or the covariate) often leads to biased estimates of the model parameters. We then rely on simulations to illustrate two cases of biased estimation.

Adjustment Function in the Context of One Treatment Variable

Although our goal is to draw researchers' attention to the intricacies of adjustment in the context of two crossed independent variables, we start with the formalities of the

simpler case that involves only one dichotomous treatment or independent variable (i.e., X_1). When the mediator Me is measured after manipulating X_1 and before measuring the dependent variable Y , the three models we need to deal with are those presented in Equations 1, 2, and 3.³ In Equation 1, Y is regressed on X_1 , with X_1 contrast coded (e.g., -1 for the first condition and $+1$ for the second one). Note that this model is what is commonly referred to as an ANOVA model. In Equation 2, the mediator Me is regressed on X_1 . In Equation 3, Y is regressed on both X_1 and Me . As we noted already, this latter model is what is known as an ANCOVA model:

$$Y = \beta_{10} + \beta_{11}X_1 + \varepsilon_1 \quad (1)$$

$$Me = \beta_{20} + \beta_{21}X_1 + \varepsilon_2 \quad (2)$$

$$Y = \beta_{30} + \beta_{31}X_1 + \beta_{32}Me + \varepsilon_3. \quad (3)$$

In Equation 1, β_{11} is the population parameter that represents the total effect of X_1 on Y . In Equation 2, β_{21} is the population parameter that represents the total effect of X_1 on Me . In Equation 3, β_{32} is the population parameter that represents the partialled Me effect on Y , and β_{31} is the population parameter that represents the partialled or direct effect of X_1 on Y . This last effect is said to control for Me , that is, it is the effect of X_1 on Y over and above the effect of Me on Y . In multiple regression terms, this partialled effect represents the effect of X_1 on Y partialling Me out of both (i.e., regressing the Y residual on the X_1 residual—see Judd & McClelland, 1989).

It is easy to demonstrate how these two adjustments combine and allow us to move from β_{11} to β_{31} . First, we combine Equations 2 and 3 by substituting for Me in Equation 3 according to Equation 2. This leads to

$$Y = \beta_{30} + \beta_{31}X_1 + \beta_{32}(\beta_{20} + \beta_{21}X_1 + \varepsilon_2) + \varepsilon_3,$$

which is given equivalently as,

$$Y = \beta_{30} + \beta_{31}X_1 + \beta_{32}\beta_{20} + \beta_{32}\beta_{21}X_1 + \beta_{32}\varepsilon_2 + \varepsilon_3.$$

Rearranged in terms of X_1 , we get

$$Y = (\beta_{30} + \beta_{32}\beta_{20}) + (\beta_{31} + \beta_{32}\beta_{21})X_1 + \beta_{32}\varepsilon_2 + \varepsilon_3. \quad (3')$$

As can be seen, the parameter associated with X_1 in Equation 3' ($\beta_{31} + \beta_{32}\beta_{21}$) must equal its parameter in Equation 1. Accordingly, in the population (and in sample estimates as well),

$$\beta_{11} = \beta_{31} + \beta_{32}\beta_{21}.$$

This last formulation is equivalent to

$$\beta_{11} - \beta_{31} = \beta_{32}\beta_{21}.$$

which is the same as the one we presented earlier, that is, $c - c' = ab$. Alternatively,

$$\beta_{31} = \beta_{11} - \beta_{32}\beta_{21}.$$

Hence, we can see that when we move from Equation 1 to Equation 3, the slope for X_1 is adjusted by the value of what could be labeled an “adjustment product,” that is, $\beta_{32}\beta_{21}$. Within this adjustment product, the first parameter (i.e., β_{32}) designates the effect of a variable that is controlled in Equation 3 (e.g., Me), and the second one (i.e., β_{21}) designates the effect of a variable on Me in Equation 2 that is being controlled for (e.g., a X_1 effect on Me). Said otherwise, the inclusion of Me in Equation 3 (i.e., β_{32}) controls for the effect of X_1 on Me in Equation 2 (i.e., β_{21}). As we shall see, things become quite a bit more complicated when two crossed independent variables (i.e., two X s) are being considered.

Adjustment Function in the Context of Two Crossed Treatment Variables

First, let us go back to our empirical example. Recall that two independent variables were manipulated: performance of the hotel desk clerk and authenticity of the smile. One potential mediator of the treatment main effects and/or their interaction is perceived competence of the clerk. The question we wish to address now is the following: Depending on the effect observed on the mediator (in the context of our example, an authenticity main effect, a performance main effect, or their interaction) and the effect of interest on the outcome variable (again in that case, an authenticity main effect, a performance main effect, or an interaction), what predictors must be included in the final regression model where the adjusted effect of the treatment or treatments and/or their interaction is to be estimated? For instance, the results of our study may reveal the presence of an authenticity main effect on satisfaction (the outcome variable) and an authenticity by performance interaction on perceived competence (the mediator). Understandably, we would be interested in testing the authenticity main effect on satisfaction controlling for the latter effect. Would it be satisfactory to add only perceived competence as a covariate in the model that has satisfaction as a dependent variable? That is, among others, the question we will address. We do so first analytically, and then we illustrate our results with relevant simulations.

In the general case, we are dealing with a situation in which there are two crossed manipulated dichotomous independent variables (i.e., X_1 and X_2), units being randomly assigned to the resulting four conditions. We also assume that the mediator, Me , has been measured after the independent variables have been manipulated and before measuring Y . Importantly, throughout our discussion, we are assuming that all variables have expectations of zero (their means being close to zero in any sample—and we recommend centering all variables based on sample means). Using these four variables, we will refer to the three models presented in Equations 4, 5, and 6: an ANOVA model in which Y is regressed on X_1 , X_2 , and their product X_1X_2 , capturing their interaction; a second ANOVA model in which Me is regressed on X_1 , X_2 , and their product X_1X_2 ; and finally a full model in which Y is regressed on X_1 , X_2 , Me , and all possible products (all three

Table 1
Interpretation of the Slope Parameters in Equations 4, 5, and 6

Slope Parameter	Interpretation of Slope Parameter
β_{41}	Total X_1 effect on Y on average across the two X_2 levels
β_{42}	Total X_2 effect on Y on average across the two X_1 levels
β_{43}	Change in total X_1 (or X_2) effect on Y as X_2 (or X_1) increases
β_{51}	Total X_1 effect on Me on average across the two X_2 levels
β_{52}	Total X_2 effect on Me on average across the two X_1 levels
β_{53}	Change in total X_1 (or X_2) effect on Me as X_2 (or X_1) increases
β_{61}	Residual direct X_1 effect on Y at the average level of Me
β_{62}	Residual direct X_2 effect on Y at the average level of Me
β_{63}	Change in residual direct X_1 (or X_2) effect on Y as X_2 (or X_1) increases at the average level of Me
β_{64}	Mediator effect on Y on average within the two X_1 and X_2 levels
β_{65}	Change in mediator effect on Y as X_1 increases
β_{66}	Change in mediator effect on Y as X_2 increases
β_{67}	Change in mediator effect on Y as X_1X_2 increases

products of pairs of these variables and the triple product among all three), capturing all three two-way interactions and the three-way interaction. The slope parameters in these three models can be interpreted as defined in Table 1:

$$Y = \beta_{40} + \beta_{41}X_1 + \beta_{42}X_2 + \beta_{43}X_1X_2 + \varepsilon_4 \quad (4)$$

$$Me = \beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5 \quad (5)$$

$$Y = \beta_{60} + \beta_{61}X_1 + \beta_{62}X_2 + \beta_{63}X_1X_2 + \beta_{64}Me + \beta_{65}MeX_1 + \beta_{66}MeX_2 + \beta_{67}MeX_1X_2 + \varepsilon_6. \quad (6)$$

Starting from these models and applying the same method as in the case of a single independent variable, we demonstrate in the appendix that the following three equalities hold (Equations 7, 8, and 9). These equalities specify how the various estimated effects in the overall ANOVA model (Equation 4) are adjusted by the inclusion of the various additional predictors in the full model (Equation 6):

$$\beta_{61} = \beta_{41} - (\beta_{64}\beta_{51} + \beta_{65}\beta_{50} + \beta_{66}\beta_{53} + \beta_{67}\beta_{52}) \quad (7)$$

$$\beta_{62} = \beta_{42} - (\beta_{64}\beta_{52} + \beta_{65}\beta_{53} + \beta_{66}\beta_{50} + \beta_{67}\beta_{51}) \quad (8)$$

$$\beta_{63} = \beta_{43} - (\beta_{64}\beta_{53} + \beta_{65}\beta_{52} + \beta_{66}\beta_{51} + \beta_{67}\beta_{50}). \quad (9)$$

As was the case with only one independent variable, these equations allow one to examine the adjustment that is produced by introducing Me and its interactions. In other words, the slopes of the two manipulated independent variables and their interaction in Equation 6

differ from those of Equation 4 because of various products of other parameters, representing the adjustments that are being made. Within each of these parameter-product terms (e.g., $\beta_{64}\beta_{51}$), the first parameter designates the effect of a variable that is controlled in Equation 6 (e.g., *Me* represented by β_{64}), and the second one designates the effect of a variable on *Me* in Equation 5 that is being controlled for (e.g., a X_1 effect on *Me*, that is β_{51}).

To illustrate, let us first focus on Equation 7, which deals with the adjustments in the effect of X_1 on Y . As was the case with one independent variable, we see that the inclusion of *Me* in Equation 6 (the corresponding parameter being β_{64}) controls for the effect of X_1 on *Me* in Equation 5 (i.e., β_{51}). The inclusion of the X_2 by *Me* interaction in Equation 6 (the parameter β_{66}) controls for the interactive effect X_1X_2 on *Me* in Equation 5 (i.e., β_{53}). Finally, the inclusion of the X_1 by X_2 by *Me* interaction in Equation 6 (the corresponding parameter being β_{67}) controls for the effect of X_2 on *Me* in Equation 5 (i.e., β_{52}). Note that the X_1 by *Me* interaction in Equation 6 (the corresponding parameter being β_{65}) controls for the intercept in Equation 5 (i.e., β_{50}). Because all variables are assumed to have expected values of zero, this intercept will equal zero.

Table 2 summarizes these adjustments. In this table, the rows refer to the variable whose effect is adjusted as we move from Equation 4 to Equation 6. The columns refer to the effects of the variables in Equation 5 that are being adjusted for. And the entries in the body of the table indicate which term in Equation 6 produces the respective adjustment. The first row of the table summarizes what we have just seen about the adjustment of the effect of X_1 on Y as we move from Equation 4 to Equation 6. First, the inclusion of *Me* in Equation 6 adjusts the X_1 effect on Y for the effect X_1 has on *Me*. Second, the inclusion of MeX_1X_2 in Equation 6 adjusts the X_1 effect on Y for the effect X_2 has on *Me*. Third, the inclusion of MeX_2 in Equation 6 adjusts the X_1 effect on Y for the effect X_1X_2 has on *Me*. Looking at Equation 8, or at the second row of Table 2, we can see that the inclusion of MeX_1X_2 in Equation 6 adjusts the X_2 effect on Y for the effect X_1 has on *Me* in Equation 5, the inclusion of *Me* in Equation 6 adjusts the X_2 effect on Y for the effect X_2 has on *Me* in Equation 5, and the inclusion of MeX_1 in Equation 6 adjusts the X_2 effect on Y for the effect of X_1X_2 on *Me* in Equation 5. Finally, looking at Equation 9, or the last row in Table 2, we can see that the inclusion of MeX_2 in Equation 6 adjusts the X_1X_2 effect on Y for the effect X_1 has on *Me* in Equation 5, the inclusion of MeX_1 in Equation 6 adjusts the X_1X_2 effect on Y for the effect X_2 has on *Me* in Equation 5, and the inclusion of *Me* in Equation 6 adjusts the X_1X_2 effect on Y for the effect X_1X_2 has on *Me* in Equation 5.

It is noteworthy that the diagonal in Table 2 reveals a straightforward correspondence: The inclusion of *Me* in Equation 6 permits us to examine the effects of X_1 , X_2 , and X_1X_2 on Y while controlling for the effects that X_1 , X_2 , and X_1X_2 , respectively, have on *Me*. For instance, we include *Me* in the model when one is interested in the X_1X_2 interaction on Y and this same interaction has been found to affect *Me*. But note that although the inclusion of *Me* as the sole predictor in Equation 6 accomplishes the adjustments specified on the diagonal of Table 2, its inclusion will not adjust for other effects, specified in the off-diagonal terms of Table 2. Suppose we were interested in the adjusted effects of X_2 on Y over and above the various factors that affect *Me*. If, for instance, X_1 affected *Me*, then we would need to include the three-way interaction, MeX_1X_2 , as a predictor in Equation 6 to properly estimate the adjusted X_2 effect on Y . Alternatively, suppose that we were interested in the adjusted effects of the X_1X_2 interaction on Y . If, for instance, both X_1

Table 2
Critical Variables to Be Added as a Function of the
Existing Effect on *Me* and the Factor of Interest

Factor of Interest	Effects on <i>Me</i>		
	X_1	X_2	X_1X_2
X_1	<i>Me</i>	<i>MeX₁X₂</i>	<i>MeX₂</i>
X_2	<i>MeX₁X₂</i>	<i>Me</i>	<i>MeX₁</i>
X_1X_2	<i>MeX₂</i>	<i>MeX₁</i>	<i>Me</i>

and X_2 had effects on *Me*, then we would need to include both the *MeX₂* interaction and the *MeX₁* interaction as predictors in the model to accomplish the appropriate adjustments (*MeX₂* to adjust for the effect of X_1 on Y and *MeX₁* to adjust for the effect of X_2 on Y).

We can now return to our example, asking the question that we posed there about competence as a mediator. Would it be sufficient to add only perceived competence (the potential mediator) as a covariate when one is interested in testing the authenticity main effect on the dependent variable and when an authenticity by performance interaction has been found on the potential mediator? In terms of Table 2, we are thus dealing with the column where X_1X_2 has an impact on *Me* and the row where the factor of interest is X_2 (we arbitrarily designate the performance factor as X_1 and the authenticity factor as X_2). Table 2 reveals that the crucial predictor that should be included is *MeX₁*. Hence, if the sole interest was in the authenticity main effect on satisfaction and the authenticity by performance interaction was the only significant effect on perceived competence, the model that should be used is the following:⁴

$$Satisf = \beta_0 + \beta_1 Perf + \beta_2 Auth + \beta_3 Perf*Auth + \beta_4 Comp + \beta_5 Perf*Comp + \epsilon.$$

Concretely, this means that two interactive terms would be computed by calculating the products of performance and authenticity on one hand and the product of performance and perceived competence on the other. A regression, with satisfaction as a criterion, would then be conducted with the five resulting predictors (i.e., performance, authenticity, perceived competence, and the performance products with authenticity and perceived competence). From this, it is clear that an ANCOVA model that only includes perceived competence as a predictor without the perceived competence by performance interaction would not accomplish the necessary adjustment.

For the sake of the example, imagine now that the results of our example study revealed the presence of a performance main effect (X_1) on perceived competence (*Me*). If we were interested in testing the authenticity main effect (X_2) on satisfaction, controlling for that effect, Table 2 shows that we would have to include the perceived competence by authenticity by performance three-way interaction (*MeX₁X₂*). In the next section, we will present simulated data that illustrate the bias that could result from not including this term in the model. A second simulation will examine another configuration that will also be illustrated with our example study, this time using perceived friendliness as our intervening variable.

Simulations

The persuasive power of the analytic demonstration notwithstanding, it is instructive to examine the consequences of misspecification in the context of simulated data. We chose to rely on Monte Carlo simulations. In this approach, one constructs a true model and then many samples (we will use 10,000) of a fixed sample size (we will use 100) are randomly drawn from the population in which the true model is known to hold. One then generates parameter estimates of various models in each sample and compares them to the known parameter values of the true model. We can then determine whether these parameter estimates, across samples, from various estimated models are biased or not. For instance, suppose in the true model, some parameter value is fixed at zero. Across samples, its estimated value ought to be zero on average and ought to be significant in any one sample only 5% of the time (given $\alpha = .05$). If the mean value of the estimate departs from zero and if it is significant more than 5% of the time, then it was estimated with bias.

In the present article, we decided to conduct two such simulations. The first one more concretely illustrates that a biased parameter estimate results for X_2 on Y whenever there is an impact of X_1 on Me and the MeX_1X_2 interaction term is not included in the model. The second simulation illustrates how a biased parameter estimate results for X_1X_2 on Y whenever there is an impact of X_1 and X_2 on Me and the MeX_1 and MeX_2 interactions are not included in the model.

First Illustration

The aim of this first simulation is to show that the X_2 effect on Y will be wrongly estimated whenever X_1 has an effect on Me and MeX_1X_2 is not included in the model (unless this three-way interaction equals zero). We are thus focusing on the case in the second row, first column, of Table 2. We generated data based on the following true model:

$$Y = \beta_{60} + \beta_{61}X_1 + \beta_{62}X_2 + \beta_{63}X_1X_2 + \beta_{64}Me + \beta_{65}MeX_1 + \beta_{66}MeX_2 + \beta_{67}MeX_1X_2 + \varepsilon_6, \quad (6)$$

constraining β_{60} , β_{61} , β_{62} , β_{64} , β_{65} , and β_{66} to equal zero while fixing β_{63} at 1 and β_{67} at 0.75. These effects were included because one usually conducts a study with two crossed treatment variables when an interaction is expected, and also the model specifies that this interaction is in turn moderated by Me . The variance of the residuals, $\sigma_{\varepsilon_6}^2$, was fixed at 5. Furthermore, in the true model for Me ,

$$Me = \beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5, \quad (5)$$

we constrained β_{50} , β_{52} , and β_{53} to equal zero while fixing β_{51} at 0.80. Thus, only X_1 was allowed to have an impact on Me . The variance of these residuals, $\sigma_{\varepsilon_5}^2$, was fixed at 1. We conducted 10,000 simulation trials, randomly sampling 100 cases each time.

Given the specific question with which we are dealing, we will be paying particular attention to the parameter estimates and significance for X_2 across four different models. Accordingly, in our simulations, we examined not only the mean values of the X_2 parameter estimate in the various models but also the relative frequency with which this parameter

was found to be significant across the 10,000 samples in each one of the models. The four models that will be presented are (a) the ANOVA model crossing X_1 and X_2 (i.e., Equation 4), (b) an ANCOVA model adding only Me to the ANOVA model, (c) a correct (full) model with the critical MeX_1X_2 interaction and its lower order interactions, and (d) an incomplete model including all the parameters but the critical (in this context) three-way interaction.

Reported in Table 3 are the mean values (and the rate of significance at $\alpha = .05$) of the coefficients for the various components of each model. The first row shows the misspecified ANOVA model (Equation 4), and as expected the mean value for the X_2 parameter is overestimated. In agreement with Equation 8, we would expect the effect of X_2 on Y to be biased by $\beta_{67}\beta_{51}$, that is, 0.75×0.80 or 0.60 . As a matter of fact, instead of being 0 as it should be, the simulation shows it to be 0.601 . Moreover, this parameter is found to be significant in almost 21% of the samples.

Turning to the second row, we notice that the standard ANCOVA model, which only includes Me , does nothing to correct the problem. The X_2 parameter and the Type I error rates are equivalent to those found in the ANOVA model.

Crucially, the third row presents the correct model. Here, we can see that the X_2 parameter is correctly estimated once the MeX_1X_2 (and its lower-order components) interaction is included. Moreover, this parameter is found to reach significance in only 5.2% of the samples, approximately what would be expected using a .05 level of significance.

The model presented in the last row is incomplete and was included to show that the critical term in the correct model is indeed the three-way interaction. As expected, the inclusion of all the parameters but the three-way interaction again leads to a slope for X_2 that is overestimated by the same amount (i.e., 0.601). Not surprisingly, the inflation in the Type I error is comparable to the one observed in the ANCOVA model (reaching 20.6% instead of 5%).

Finally, it should be noticed that the correct estimation of the X_2 parameters comes with a price. As a matter of fact, moving from the ANCOVA model to the correct model entails a loss of power for the test for the X_1X_2 interaction (significance rate falls from 48.3% of the samples to 31.1%). Note that the coefficient for this interaction is accurately evaluated as 1 across the different models. This is the case because β_{66} equals zero (see Table 2 and Equation 9).

What is the lesson of this simulation in terms of the empirical example we have been using in the introduction? If a performance main (X_1) effect is found on perceived competence (Me), and if one wants to test the authenticity main (X_2) effect on satisfaction over and above that effect on competence, one needs to include the perceived competence by authenticity by performance interaction in the model. As is clear from the simulation, in this context, the usual ANCOVA model would not accomplish the necessary adjustment. Hence, we would conclude that the effect is there when it is not. In more general terms, failing to include the necessary set of predictors in the model would lead to erroneous conclusions.

Second Illustration

Our second simulation again relies on our empirical example but focuses on the second potential mediator (i.e., perceived friendliness). Imagine that we found an authenticity by performance interaction on the main dependent variable (i.e., satisfaction) and both a

Table 3
Simulation 1: Parameter Estimates and Rates of Rejection of the Null Hypothesis
 ($\alpha = .05$) as a Function of the Type of Model

Model	Predictors															
	Intercept		X_1		X_2		X_1X_2		Me		MeX_1		MeX_2		MeX_1X_2	
	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR
ANOVA	0.002	0.049	0.007	0.046	0.601	0.209	1	0.487								
ANCOVA	0.002	0.049	0.002	0.049	0.601	0.209	0.999	0.483	0.007	0.053						
Correct	0.004	0.046	-0.002	0.050	0.003	0.052	0.996	0.311	0.009	0.049	-0.002	0.051	0.003	0.046	0.747	0.276
Incomplete	0.003	0.049	0	0.050	0.601	0.206	0.998	0.313	0.009	0.057	0	0.058	0.002	0.055		

Note: Y is the outcome variable. PE = parameter estimate; RR = rate of rejection of the null hypothesis.

performance effect and an authenticity effect on perceived friendliness. In other words, the effect of interest would be the X_1X_2 interaction on Y , and both X_1 and X_2 have significant effects on the potential mediator, Me . Again, would it be sufficient to include only Me in the model on Y ? Table 2 shows that this should not be the case. Our simulation nicely illustrates this.

The aim of the second simulation is thus to show that the X_1X_2 interaction effect on Y will be biased whenever X_1 and X_2 have an effect on Me and both MeX_2 and MeX_1 are not included in the model (unless these two interactions equal zero). Thus, we are dealing with the first two columns of the last row of Table 2. We relied on the true model,

$$Y = \beta_{60} + \beta_{61}X_1 + \beta_{62}X_2 + \beta_{63}X_1X_2 + \beta_{64}Me + \beta_{65}MeX_1 + \beta_{66}MeX_2 + \beta_{67}MeX_1X_2 + \varepsilon_6, \quad (6)$$

in which we constrained β_{60} , β_{62} , β_{63} , β_{64} , and β_{67} to equal 0 while fixing β_{61} at 1 and both β_{65} and β_{66} to 0.75. β_{61} was set at 1 to demonstrate that unnecessarily relying on the full model will lead to a loss in power without improving parameters estimation. The variance of the residuals, $\sigma_{\varepsilon_6}^2$, was again fixed at 5. Furthermore, in the true model for Me ,

$$Me = \beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5 \quad (5)$$

we constrained β_{50} and β_{53} to equal 0 while fixing β_{51} and β_{52} at 0.80. Thus, both X_1 and X_2 were allowed to have an impact on Me . The variance of these residuals, $\sigma_{\varepsilon_5}^2$, was fixed at 1. We conducted 10,000 simulation trials, randomly sampling 100 cases each time.

Given the specific question we are dealing with, we will be paying particular attention to the parameter estimates and significance for X_1X_2 across four different models. The four models that will be presented are (a) the ANOVA model crossing X_1 and X_2 (i.e., Equation 4), (b) an ANCOVA model adding only Me to the ANOVA model, (c) a correct model with the critical MeX_1 and MeX_2 interactions, and (d) a full model including all parameters.

Reported in Table 4 are the mean values (and the rate of significance at $\alpha = .05$) of the coefficients for the various components of each model. The first row shows the misspecified ANOVA model (Equation 4), and as expected the mean value for the X_1X_2 parameter is overestimated. In agreement with Equation 9, we would expect the effect of X_1X_2 on Y to be biased by $\beta_{65}\beta_{52} + \beta_{66}\beta_{51}$, that is by $(0.75 \times 0.80) + (0.75 \times 0.80)$, which equals 1.20. As a matter of fact, instead of being 0 as it should be, the simulation shows it to be 1.203. Moreover, this parameter is found to be significant in 63% of the samples.

Turning to the second row, we notice that the standard ANCOVA model, which only includes Me , hardly does a better job. As it turns out, the X_1X_2 parameter and the Type I error are equivalent to those found in the ANOVA model.

The third row presents the correct model that includes both MeX_1 and MeX_2 , in addition to their components. Now, the X_1X_2 parameter is correctly estimated as being 0. Moreover, this parameter is found to reach significance in only 5.1% of the samples.

The model presented in the last row is the full model. We present it to make clear that relying on the full model, when it is unnecessary, will be less efficient, even though it clearly yields unbiased estimates. As we have analytically demonstrated, the parameter values are equivalent to those estimated in the third model, but there is a definite loss of power, most notably for X_1 . We would argue that a loss of power in testing some

Table 4
Simulation 2: Parameter Estimates and Rates of Rejection of the Null Hypothesis
($\alpha = .05$) as a Function of the Type of Model

Model	Predictors																
	Intercept		X_1		X_2		X_1X_2		Me		MeX_1		MeX_2		MeX_1X_2		
	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	PE	RR	
ANOVA	1.203	0.641	1.006	0.482	0	0.051	1.203	0.631									
ANCOVA	1.203	0.635	1	0.322	-0.007	0.055	1.202	0.629	0.008	0.057							
Correct	0.002	0.050	0.999	0.322	-0.007	0.054	0	0.051	0.009	0.049	0.748	0.287	0.752	0.298			
Full	0.002	0.051	1	0.242	-0.004	0.052	-0.002	0.050	0.009	0.049	0.748	0.280	0.753	0.289	-0.003	0.049	

Note: Y is the outcome variable. PE = parameter estimate; RR = rate of rejection of the null hypothesis.

parameters is worth enduring when it comes with more accurate estimates of other parameters. However, when more powerful models yield equivalent unbiased estimates, then they are definitely to be preferred. Finally, note that in contrast to what happened in the previous simulation, X_2 is now accurately estimated across the different models. This is so because both β_{64} and β_{67} equal 0.

Going back to our example, this simulation again highlights that the usual ANCOVA model would do nothing to adjust the effect of interest (i.e., the authenticity by performance interaction on satisfaction). In other words, if the proper terms (in this case, the interactions between perceived friendliness and both factors) are not included in the model, the estimate of this critical interaction will be biased and often (erroneously) found to be significant when the incorrect model is estimated.

Discussion

Researchers in the behavioral sciences are increasingly confronted with so-called ANCOVA models. In the context of mediation analysis, one often relies on the ANCOVA model to test the mediator effect on the dependent variable controlling for the treatment and to evaluate the treatment effect controlling for the mediator effect. The ANOVA model, in which the treatment effect on some outcome is examined, is thus modified into a model that includes the mediator (or the covariate) as an additional predictor. Although this strategy is entirely justified when only one treatment variable is being considered, the inclusion of only *Me* as a covariate is likely to lead to bias when examining issues of mediation in the case of two treatment variables that are crossed (Muller et al., 2005; Yzerbyt, Muller, & Judd, 2004).

The present article examined factors that contribute to adjustment because of mediation in the context of two crossed treatment variables. We demonstrated analytically that, depending on the specific variable or variables that have an impact on *Me*, one should examine a fully specified model, which may include more predictors than simply the mediator as a covariate. In addition, our first simulation concretely showed how, in the context of an effect of X_1 on *Me*, the critical predictor that needs to be included to correctly estimate the effect of X_2 in the model of Y is the MeX_1X_2 interaction. Unless this precaution is taken, the X_2 parameter will be dangerously biased. Our second simulation showed how, in the context of effects of X_1 and X_2 on *Me*, the critical predictors that need to be included to correctly estimate the effect of X_1X_2 in the Y model are the MeX_1 and MeX_2 interactions. Unless these two interactions are included, the X_1X_2 parameter will be incorrectly estimated.

There are several cases in which the terms to be included in the full Y model are far from obvious. The consequence is that, more often than not, researchers who rely solely on the inclusion of *Me* in their ANCOVA model will end up with biased and misleading results. A seemingly reasonable way to handle the situation is to always estimate the full Y model, including *Me* along with all possible interactions. We would not recommend such a blanket strategy, at least in the case of two dichotomous independent variables. Indeed, although the use of such an “exhaustive” model allows for the proper estimation of the various parameters, the estimation of unnecessary predictors comes with a definite cost in efficiency. Even if researchers are not aware of the true underlying model

governing their data, we would therefore advise that they first examine a model in which Me is the criterion and the independent variables the predictors. Awareness of those specific predictors that exert an impact on Me should then allow them, relying on Table 2, to figure out which critical covariates need to be included in the Y model.

We have deliberately confined the discussion in this article to the case where there are two crossed independent variables, each having two levels. In this case, the analytic solution to the problem of adjustment is tractable. The bad news is that even in a very simple design as the one studied here, things are quite a bit more complex than one could think at first sight. The good news, however, is that embracing the framework presented here, these analyses can be conducted safely. But of course there are many other more complex cases that should and ought to be considered. For instance, one of the two predictors could be continuous (say, because it is measured rather than manipulated). In such a case, although the derivation would be more complex than the one presented in this article, the guidelines presented in Table 2 would hold.

Another example of a somewhat more complex situation occurs when one, or both, of the independent variables have more than two levels. In such a situation, simply including the covariate, but not the other terms, in the adjustment model will generally not accomplish the desired adjustment and will lead to biased estimates. Through simulations, such as those we have illustrated in this article, one could identify the necessary terms that ought to be included to yield unbiased estimates, given the pattern of treatment effects on the mediator and the adjusted effects of interest. But we have been unable to develop a general analytic solution that promises to yield the equivalent to Table 2 in these more complex situations. Although this goal remains elusive, the good news is that if one were to estimate the full model in such cases, including the mediator and all possible mediator by treatment interactions, then the full adjustment of all effects of interest would be accomplished and unbiased estimates would result. Therefore, in these more complicated scenarios, we encourage the researcher to estimate this full model. We also hope that the present work will provide a springboard for a more comprehensive and creative treatment of the issues we have raised in the context of more complex research designs.

Given the topic of this special issue, we restricted the present contribution to those situations in which there is a strong suspicion of the presence of mediation. It should be noted, however, that our analytical framework is more general. As a matter of fact, the same rationale applies in cases where researchers would like to show that some variable plays no mediating role in the relation between the treatments and the outcome. For instance, again in the context of our ongoing example, within a study such as Grandey et al.'s (2005), one could be interested in showing that the effect of the independent variables could not be explained by perceived competence or perceived friendliness. Here, one would be interested in showing that the effect on the dependent variable (e.g., satisfaction) persisted over and above the effect on the mediator (e.g., perceived competence). As we have seen, a tempting strategy might be to include only perceived competence as a covariate in the satisfaction model. We think that we have now made clear the risks associated with such a strategy.

To conclude, many researchers dealing with ANCOVA models involving more than one independent variable continue to overlook the costs of model misspecification. In the present article, we hope to have shed some light on the potential costs and possible remedies to this situation. A careful examination of the variables affecting the mediator should

prove most informative in specifying the correct Y model to achieve unbiased parameter estimates in the most efficient manner.

Appendix Derivation of the Adjustment Functions

Let us assume that Equations 5 and 6 represent the theoretical models that are responsible for generating the variance in both the mediator and the outcome. Accordingly, the model presented in Equation 4 is misspecified. It then becomes possible to derive the values of the parameters in this misspecified model in terms of the parameter estimates of Equations 5 and 6. To do so, we first combine Equations 5 and 6 by substituting for Me in Equation 6 according to Equation 5. This leads to the following result:

$$\begin{aligned}
 Y = & \beta_{60} + \beta_{61}X_1 + \beta_{62}X_2 + \beta_{63}X_1X_2 \\
 & + \beta_{64}(\beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5) \\
 & + \beta_{65}(\beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5)X_1 \\
 & + \beta_{66}(\beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5)X_2 \\
 & + \beta_{67}(\beta_{50} + \beta_{51}X_1 + \beta_{52}X_2 + \beta_{53}X_1X_2 + \varepsilon_5)X_1X_2 + \varepsilon_6,
 \end{aligned} \tag{6'}$$

which is given equivalently as

$$\begin{aligned}
 Y = & \beta_{60} + \beta_{61}X_1 + \beta_{62}X_2 + \beta_{63}X_1X_2 \\
 & + \beta_{64}\beta_{50} + \beta_{64}\beta_{51}X_1 + \beta_{64}\beta_{52}X_2 + \beta_{64}\beta_{53}X_1X_2 + \beta_{64}\varepsilon_5 \\
 & + \beta_{65}\beta_{50}X_1 + \beta_{65}\beta_{51}X_1^2 + \beta_{65}\beta_{52}X_1X_2 + \beta_{65}\beta_{53}X_1^2X_2 + \beta_{65}X_1\varepsilon_5 \\
 & + \beta_{66}\beta_{50}X_2 + \beta_{66}\beta_{51}X_1X_2 + \beta_{66}\beta_{52}X_2^2 + \beta_{66}\beta_{53}X_1X_2^2 + \beta_{66}X_2\varepsilon_5 \\
 & + \beta_{67}\beta_{50}X_1X_2 + \beta_{67}\beta_{51}X_1^2X_2 + \beta_{67}\beta_{52}X_1X_2^2 + \beta_{67}\beta_{53}X_1^2X_2^2 + \beta_{67}\varepsilon_5X_1X_2 + \varepsilon_6.
 \end{aligned} \tag{6'}$$

When X_1 and X_2 are contrast-coded dichotomous variables, then X_1^2 and X_2^2 are constant for all cases (i.e., their variances equal 0). In this case, Equation 6' reduces to

$$\begin{aligned}
 Y = & \beta_{60} + \beta_{61}X_1 + \beta_{62}X_2 + \beta_{63}X_1X_2 \\
 & + \beta_{64}\beta_{50} + \beta_{64}\beta_{51}X_1 + \beta_{64}\beta_{52}X_2 + \beta_{64}\beta_{53}X_1X_2 + \beta_{64}\varepsilon_5 \\
 & + \beta_{65}\beta_{50}X_1 + \beta_{65}\beta_{51} + \beta_{65}\beta_{52}X_1X_2 + \beta_{65}\beta_{53}X_2 + \beta_{65}X_1\varepsilon_5 \\
 & + \beta_{66}\beta_{50}X_2 + \beta_{66}\beta_{51}X_1X_2 + \beta_{66}\beta_{52} + \beta_{66}\beta_{53}X_1 + \beta_{66}X_2\varepsilon_5 \\
 & + \beta_{67}\beta_{50}X_1X_2 + \beta_{67}\beta_{51}X_2 + \beta_{67}\beta_{52}X_1 + \beta_{67}\beta_{53} + \beta_{67}\varepsilon_5X_1X_2 + \varepsilon_6.
 \end{aligned} \tag{6'}$$

Finally, this can be reformulated as,

$$\begin{aligned}
 Y = & (\beta_{60} + \beta_{64}\beta_{50} + \beta_{65}\beta_{51} + \beta_{66}\beta_{52} + \beta_{67}\beta_{53} + \beta_{64}\varepsilon_5) \\
 & + (\beta_{61} + \beta_{64}\beta_{51} + \beta_{65}\beta_{50} + \beta_{66}\beta_{53} + \beta_{67}\beta_{52} + \beta_{65}\varepsilon_5)X_1 \\
 & + (\beta_{62} + \beta_{64}\beta_{52} + \beta_{65}\beta_{53} + \beta_{66}\beta_{50} + \beta_{67}\beta_{51} + \beta_{66}\varepsilon_5)X_2 \\
 & + (\beta_{63} + \beta_{64}\beta_{53} + \beta_{65}\beta_{52} + \beta_{66}\beta_{51} + \beta_{67}\beta_{50} + \beta_{67}\varepsilon_5)X_1X_2 + \varepsilon_6.
 \end{aligned} \tag{6'}$$

Accordingly, the parameter associated with X_1 , X_2 , and their interaction X_1X_2 in the respecified Equation 6' must equal their parameters in the misspecified Equation 4. Note that products involving ε_5 drop out because this residual is uncorrelated with X_1 , X_2 , and X_1X_2 , and hence its product with them is as well. We are left with

$$\beta_{41} = \beta_{61} + \beta_{64}\beta_{51} + \beta_{65}\beta_{50} + \beta_{66}\beta_{53} + \beta_{67}\beta_{52} \quad (\text{A1})$$

$$\beta_{42} = \beta_{62} + \beta_{64}\beta_{52} + \beta_{65}\beta_{53} + \beta_{66}\beta_{50} + \beta_{67}\beta_{51} \quad (\text{A2})$$

$$\beta_{43} = \beta_{63} + \beta_{64}\beta_{53} + \beta_{65}\beta_{52} + \beta_{66}\beta_{51} + \beta_{67}\beta_{50}. \quad (\text{A3})$$

These three equations are equivalent to, respectively,

$$\beta_{61} = \beta_{41} - (\beta_{64}\beta_{51} + \beta_{65}\beta_{50} + \beta_{66}\beta_{53} + \beta_{67}\beta_{52}) \quad (7)$$

$$\beta_{62} = \beta_{42} - (\beta_{64}\beta_{52} + \beta_{65}\beta_{53} + \beta_{66}\beta_{50} + \beta_{67}\beta_{51}) \quad (8)$$

$$\beta_{63} = \beta_{43} - (\beta_{64}\beta_{53} + \beta_{65}\beta_{52} + \beta_{66}\beta_{51} + \beta_{67}\beta_{50}) \quad (9)$$

Notes

1. We use the variables examined in this article as an example throughout this article. However, it is not our intention to follow the work conducted by Grandey, Fisk, Mattila, Jansen, and Sideman (2005) in every aspect. Our choice in this specific study is solely based on the fact that the issues it addresses allow us to illustrate our points in an organizational context, although they apply to many other contexts as well.

2. Note that Grandey et al. (2005) did not explicitly talk about mediation although their discussion and analyses are mediational.

3. Throughout we will use β s to refer to unknown population parameters and b s to refer to their sample estimates. We have omitted the i subscript, referring to observations, from all variables.

4. Within this example, the critical term to be included is the competence by performance interaction. The reader may wonder why we also included the competence term. The reason here and throughout is that a product term is never included without its lower-order components (here, competence and performance; cf. Judd & McClelland, 1989). Only partialled products examine interactions.

References

- Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*, 1173-1182.
- Grandey, A. A., Fisk, G. M., Mattila, A. S., Jansen, K. J., & Sideman, L. A. (2005). Is "service with a smile" enough? Authenticity of positive displays during service encounters. *Organizational Behavior and Human Decision Processes*, *96*, 38-55.
- Judd, C. M., & McClelland, G. H. (1989). *Data analysis: A model comparison approach*. San Diego, CA: Harcourt, Brace, Jovanovich.

- MacKinnon, D. P., Lockwood, C. M., Hoffman, J. M., West, S. G., & Sheets, V. (2002). A comparison of methods to test mediation and other intervening variable effects. *Psychological Methods, 7*, 83-104.
- MacKinnon, D. P., Warsi, G., & Dwyer, J. H. (1995). A simulation study of mediated effect measures. *Multivariate Behavioral Research, 30*, 41-62.
- Muller, D., Judd, C. M., & Yzerbyt, V. (2005). When moderation is mediated and mediation is moderated. *Journal of Personality and Social Psychology, 89*, 852-863.
- Yzerbyt, V., Muller, D., & Judd, C. M. (2004). Adjusting researchers' approach to adjustment: On the use of covariates when testing interactions. *Journal of Experimental Social Psychology, 40*, 424-431.

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